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RESEARCH ARTICLE

# THE RESULTS ON FIXED POINTS IN DISLOCATED QUASI-METRIC SPACE

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**A B S T R A C T**

The aim of this paper is to obtain a fixed point theorem in generalized form for continuous contracting mappings in dislocated quasi-metric space. In this paper, we extended the work of Isufati [5] and then show that the result of Zeyada [4] is special case of our theorem.



## INTRODUCTION

The Polish mathematician Stefan Banach [1922] proved a theorem which ensures, under appropriate conditions, the existence and uniqueness of a fixed point. It is well known as a Banach fixed point theorem. This result provides a technique for solving variety of applied problems in mathematical science and engineering. The existence of a fixed point is therefore of paramount importance in several area of mathematics, physics and chemistry. Isufati [5] and Zeyada [4] have extended, generalized and improved Banach fixed point theorem in different ways.

The aim of this paper is to obtain a fixed point theorem in the generalized form for continuous contracting mappings in dislocated quasi-metric space.

### *Preliminaries*

**Definition 2.1** [4] Let X be a non empty set and let $d : X \times X \rightarrow [0, \infty)$ be a function satisfying following conditions:

1. $d(x, y) = d(y, x) = 0$, implies $x = y$,
2. $d(x, y) \leq d(x, z) + d(z, y)$, for all $x, y, z \in X$.

Then d is called a dislocated quasi-metric on X. If d satisfies $d(x, y) = d(y, x)$, then it is called dislocated metric.

**Definition 2.2** [4] A sequence $\{x_n\}$ in dq-metric space (dislocated quasi-metric space) (X, d) is called Cauchy sequence if for, given ε   0, there exist $n_0 \in N$, such that ∀ m, $n \geq n_0$, implies $d(x_m, x_n)$   ε or $d(x_n, x_m)$   ε i.e. min $\{d(x_m, x_n), d(x_n, x_m)\}$   ε.

**Definition 2.3** [4] A sequence $\{x_n\}$ dislocated quasi-convergent to x if
$\lim_{n\rightarrow\infty} d(x_n, x) = \lim_{n\rightarrow\infty} d(x, x_n) = 0$
In this case x is called a dq-limit of $\{x_n\}$ and we write $x_n \rightarrow x$.

**Definition 2.4** [4] A dq-metric space (X, d) is called complete if every Cauchy sequence in it is a dq-convergent.

**Definition 2.5** [4] Let (X, d) be a dq-metric space. A map $T : X \rightarrow X$ is called contraction if there exists $0 \leq \lambda \leq 1$ such that $d(Tx, Ty) \leq \lambda\, d(x, y)$, for all $x,y \in X$.

### Main results

**Theorem 3.1** Let (X, d) be a complete dq-metric space and let $T : X \rightarrow X$ be a continuous mapping satisfying the following conditions

$$d(Tx, Ty) \leq \alpha \frac{d(y,Ty)[1+d(x,Ty)]}{1+d(x,y)} + \beta\, d(x,y) + \gamma\, d(x, Tx)$$

for all $x,y \in X$, $\alpha>0$, $\beta>0$, $\gamma>0$, $\alpha + \beta + \gamma <1$. Then T has a unique fixed point.

**Proof** : Let $x_o \in X$ and define a sequence $\{x_n\}$ in X such that

$T(x_0) = x_1,\ T(x_1) = x_2$ ............, $T(x_n) = x_{n+1}$.............

Consider, $d(x_n, x_{n+1}) = d(Tx_{n-1}, Tx_n)$

$$\leq \alpha \frac{d(x_n,Tx_n)[1+d(x_{n-1},Tx_{n-1})]}{1+d(x_{n-1},x_n)} + \beta\, d(x_{n-1},x_n) + \gamma\, d(x_{n-1}, Tx_{n-1})$$

$$\leq \alpha \frac{d(x_n,x_{n+1})[1+d(x_{n-1},x_n)]}{1+d(x_{n-1},x_n)} + \beta\, d(x_{n-1},x_n) + \gamma\, d(x_{n-1}, x_n)$$

Therefore, $d(x_n, x_{n+1}) \leq \frac{\beta+\gamma}{1-\alpha} d(x_{n-1}, x_n)$

$= \lambda\, d(x_{n-1}, x_n)$

Where $\lambda = \frac{\beta+\gamma}{1-\alpha}$ with $0 \leq \lambda <1$. In a similar way we will show that

$$d(x_{n-1}, x_n) \leq \lambda\, d(x_{n-2}, x_{n-1})$$

and $d(x_n, x_{n+1}) \leq \lambda^2 d(x_{n-2}, x_{n-1})$

Thus $d(x_n, x_{n+1}) \leq \lambda^n d(x_1, x_0)$

Since $0 \leq \lambda<1$,as $n \to \infty$, $\lambda^n \to 0$. Hence $\{x_n\}$ is a dq-cauchy sequence in X. Thus $\{x_n\}$ dislocated quasi-convergences to some $t_0$. Since T is continuous, we have

$$T(t_0) = \lim T(x_n) = \lim x_{n+1} = t_0.$$

Thus $T(t_0) = t_0$. Hence T has a fixed point.

**Uniqueness:** Let x be a fixed point of T. Then by given condition, we have

$$d(x, x) \leq d(Tx, Tx)$$

$$\leq \alpha \frac{d(x,Tx)[1+d(x\ ,Tx)]}{1+d(x,x)} + \beta\, d(x, x) + \gamma\, d(x, x)$$

$\leq (\alpha + \beta + \gamma)d(x, x)$. Which gives $d(x, x) = 0$, since $0 \leq \alpha + \beta + \gamma<1$ and $d(x, x) \geq 0$. Thus $d(x, x) \geq 0$, if x is fixed point of T.

Let $x,y \in X$ be fixed points of T, i.e. $Tx = x$, $Ty = y$.
Then by given condition,

$$d(x, y) = d(Tx, Ty)$$

$$\leq \alpha \frac{d(x,Tx)[1+d(x,Tx)]}{1+d(x,y)} + \beta\, d(x, y) + \gamma\, d(x, Tx)$$

$\leq \beta\, d(x, y)$. Which gives $d(x, y) = 0$, since $0 \leq \beta \quad 1$ and $d(x, y) \geq 0$. Similarly $d(y, x) = 0$ and hence $x = y$.

Thus fixed point of T is unique.

***Remark***

1. If we put $\gamma = 0$ we obtained Theorem 3.1 of [5].
2. If we put $\alpha = \gamma = 0$ we obtain Theorem 2.8 of [4].

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