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MORE ON INTUITIONISTIC FUZZY VOLTERRA SPACES

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A B S T R A C T **RESEARCH ARTICLE**

In this paper, we investigate several characterizations of Intuitionistic fuzzy Volterra spaces.

Keywords:

Intuitionistic Fuzzy dense set, Intuitionistic Fuzzy nowhere dense set, Intuitionistic fuzzy G_δ set, Intuitionistic fuzzy F_σ set, Intuitionistic Volterra spaces.

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1. Introduction

The fuzzy concept has invaded almost all branches of mathematics ever since the introduction of fuzzy set by L.A. Zadeh [16]. The theory of fuzzy topological spaces was introduced and developed by C.L. Chang [6]. The idea of *Intuitionistic fuzzy set* was first published by K.T. Atanassov [1]. Later, this concept was generalised to *Intuitionistic L-fuzzy set* by Atanassov and Stoneva. The concept of volterra spaces have been studied extensively in classical topology in [8], [9], and [12]. The concept of fuzzy Volterra space was introduced and studied by Thangaraj and Soundararajan [15]. The concept of Intuitionistic fuzzy Volterra spaces was introduced and studied by Soundararajan, Rizwan and Syed Tahir Hussainy[13]. In this paper, we discuss several characterizations of Intuitionistic fuzzy Volterra spaces. Intuitionistic fuzzy p-spaces and Intuitionistic fuzzy strongly irresolvable spaces are considered for this work.

2 Preliminaries

Definition 2.1 [3]: Let X be a non-empty set. An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$, where $\mu_A(x): X\to[0,1]$ and $\nu_A(x): X\to[0,1]$ denote the membership and non-membership functions of A respectively, and $0\le\mu_A(x)+\nu_A(x)\le 1$, for each $x\in X$.

Definition 2.2 [3]:Let A and B be two IFSs of the non-empty set X such that $A = \{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$, $B = \{\langle x,\mu_B(x),\nu_B(x)\rangle : x\in X\}$.

We define the following basic operations on A and B.

(i) $A\subseteq B$ iff $\mu_A(x)\le\mu_B(x)$ and $\nu_A(x)\ge\nu_B(x), \forall x\in X$

(ii) $A\supseteq B$ iff $\mu_A(x)\ge\mu_B(x)$ and $\nu_A(x)\le\nu_B(x), \forall x\in X$

(iii) $A = B$ iff $\mu_A(x)=\mu_B(x)$ and $\nu_A(x)=\nu_B(x), \forall x\in X$

(iv) $A\cup B=\{\langle x,\mu_A(x)\vee\mu_B(x),\nu_A(x)\wedge\nu_B(x)\rangle : x\in X\}$

(v) $A\cap B=\{\langle x,\mu_A(x)\wedge\mu_B(x),\nu_A(x)\vee\nu_B(x)\rangle : x\in X\}$

(vi) $A^c=\{\langle x,\nu_A(x),\mu_A(x)\rangle : x\in X\}$.

Definition 2.3[7]: An Intuitionistic fuzzy topology (IFT) on X is a family T of IFSs in X satisfying the following axioms.

$(i) 0,1\in T$
$(ii) G_1\cap G_2\in T, for\ any\ G_1, G_2\in T$
$(iii) \cup G_i\in T\ for\ any\ family\ \left\{\frac{G_i}{i}\in J\right\}\subseteq T$

In this case, the pair (X,T) is called an Intuitionistic fuzzy topological space (IFTS) and any IFS in T is known as Intuitionistic fuzzy open set (IFOS) in X.

The complement A^c of an IFOS A in an IFTS (X,T) is called an Intuitionistic fuzzy colsed set (IFCS) in X.

Definition 2.4[7]: Let (X,T) be an IFTS and $A=\langle X,\mu_A,\nu_A\rangle$ be an IFS in X. Then the Intuitionistic fuzzy interior and an Intuitionistic fuzzy closure are defined by

$$int(A) = \cup\{G/G \text{ is an IFOS in } X \text{ and } G\subseteq A\},$$
$$cl(A) = \cap\{K/K \text{ is an IFCS in } X \text{ and } A\subseteq K\},$$

Theorem 2.5[7]: Let (X,T) be any Intuitionistic fuzzy topological space. Let A be an IFS in (X,T). Then

$$(i) \quad 1-IFcl(A) = IFint(1-A),$$
$$(ii) \quad 1-IFint(A) = IFcl(1-A).$$

Definition 2.6[10]: An Intuitionistic Fuzzy set A in an Intuitionistic fuzzy topological space (X,T) is called Intuitionistic fuzzy dense if there exists no Intuitionistic fuzzy closed set B in (X,T) such that $A\subset B\subset 1$.

3. More on Intuitionistic Fuzzy Volterra Spaces

Definition 3.1[13]: An Intuitionistic fuzzy set A in an Intuitionistic fuzzy topological space (X,T) is called an Intuitionistic fuzzy G_δ set in (X,T) if $A=\bigcap_{i=1}^{\infty} A_i$ where $A_i \in T, \forall i$

Definition 3.2[13]: An Intuitionistic fuzzy set A in an Intuitionistic fuzzy topological space (X,T) is called an Intuitionistic fuzzy F_σ set in (X,T) if $A=\bigcup_{i=1}^{\infty} A_i$ where $1-A_i \in T, \forall i$.

Lemma 3.3[13]: A is an Intuitionistic fuzzy G_δ set in an Intuitionistic fuzzy topological space (X,T) if and only if $1-A$ an Intuitionistic fuzzy F_σ set in (X,T)

Definition 3.4[13]: Let (X,T) be an Intuitionistic fuzzy topological space. Then (X,T) is called an Intuitionistic fuzzy Volterra space, if $IFcl\left(\bigcap_{i=1}^{N} A_i\right)=1$, where A_i's are Intuitionistic fuzzy dense and Intuitionistic fuzzy G_δ sets in (X,T).

Proposition 3.5: If $A=\bigcap_{i=1}^{\infty} A_i$, where A_i's are Intuitionistic fuzzy dense and Intuitionistic fuzzy G_δ sets in an Intuitionistic fuzzy Volterra space (X,T) then A is not an Intuitionistic fuzzy closed set.

Proof: Let $A=\bigcap_{i=1}^{N} A_i$, where A_i's are Intuitionistic fuzzy dense and Intuitionistic fuzzy G_δ sets in (X,T). Since (X,T) is an Intuitionistic fuzzy Volterra space, we have $IFcl(\bigcap_{i=1}^{N} A_i)=1$. That is, $IFcl(A)=1$. This implies that $IFcl(A)\neq A$. Hence, A is not an Intuitionistic fuzzy closed set in (X,T).

Definition 3.6[14]**:** An Intuitionistic fuzzy set A in an Intuitionistic fuzzy topological space (X,T) is called an Intuitionistic fuzzy nowhere dense set, if there exists no Intuitionistic fuzzy open set U in (X,T) $U\subseteq IFcl(A)$ such that That is, $IFintIFcl(A)=0$.

Proposition 3.7: If A is an Intuitionistic fuzzy nowhere dense set in $IFTS(X,T)$, then $1-A$ is an Intuitionistic fuzzy dense set in (X,T)

Proof: Let A be an Intuitionistic fuzzy nowhere dense set in (X,T), then $IFint\{IFcl(A)\}=0$. That is, $1-IFint\{IFcl(A)\}=1$. That is,

$IFcl\{IFint(1-A)\}=1$. That is, $IFcl(1-A)=1$. Hence, $1-A$ is an Intuitionistic fuzzy dense set in (X,T)

Proposition 3.8: Let (X,T) be an Intuitionistic fuzzy topological space. An IFS A is an Intuitionistic fuzzy dense and Intuitionistic fuzzy open set in (X,T), then $1-A$ is an Intuitionistic fuzzy nowhere dense set in (X,T).

Proof: Since, A is an Intuitionistic fuzzy dense set in (X,T), we have $IFcl(A)=1$. Also, since A is an Intuitionistic fuzzy open set, we have $IFint(A)=A$. Now,

$$\begin{aligned} IFintIFcl(1-A) &= 1-IFclIFintA \\ &= 1-IFclA \\ &= 1-1 \\ &= 0 \end{aligned}$$

Hence, $1-A$ is an Intuitionistic fuzzy nowhere dense set in (X,T)

Proposition 3.9: If $B=\bigcup_{i=1}^{N} B_i$, where B_i's are Intuitionistic fuzzy nowhere dense and Intuitionistic fuzzy F_σ sets in an Intuitionistic fuzzy Volterra space (X,T), then B is not an Intuitionistic fuzzy open set in (X,T).

Proof: Let $B=\bigcup_{i=1}^{N} B_i$, where B_i's are Intuitionistic fuzzy dense and Intuitionistic fuzzy F_σ sets in an Intuitionistic fuzzy Volterra space (X,T). Then

$$1-B=1-\bigcup_{i=1}^{N}B_i=\bigcap_{i=1}^{N}(1-B_i).$$

Since, B_i's are Intuitionistic fuzzy nowhere dense sets, by the theorem, $(1-B_i)$'s are Intuitionistic fuzzy dense sets. Also, B_i's are Intuitionistic fuzzy F_σ sets, $(1-B_i)$'s are Intuitionistic fuzzy G_δ sets. By the proposition [3.5], $1-B$ is not an Intuitionistic fuzzy closed set in (X,T). Hence, B is not an Intuitionistic fuzzy open set in (X,T).

Proposition 3.10: In an IFTS (X,T), an Intuitionistic fuzzy set A is Intuitionistic fuzzy σ-nowhere dense if and only if $1-A$ is Intuitionistic fuzzy dense and Intuitionistic fuzzy G_δ set.

Proposition 3.11: In the Intuitionistic fuzzy σ-nowhere dense sets A_i's are Intuitionistic fuzzy closed sets in an Intuitionistic fuzzy Volterra space (X,T) then $IFclIFint\left\{\bigcap_{i=1}^{\infty}A\right\}=1.$

Proof: The sets A_i's are Intuitionistic fuzzy σ-nowhere dense sets in (X,T). Hence $1-A_i$'s are Intuitionistic fuzzy dense and Intuitionistic fuzzy G_δ-sets in (X,T). Since (X,T), is Intuitionistic fuzzy Volterra space, $IFcl\left\{\bigcap_{i=1}^{N}(1-Ai)\right\}=1$. Also, given that Intuitionistic fuzzy σ-nowhere dense sets are Intuitionistic fuzzy closed sets in (X,T). Therefore, A_i's are Intuitionistic fuzzy closed sets in (X,T). That is, $1-A_i$'s are Intuitionistic fuzzy open sets in (X,T). Hence, $\bigcap_{i=1}^{N}(1-A_i)$ is an Intuitionistic fuzzy open set in (X,T). This implies that, $IFint\left\{\bigcap_{i=1}^{N}(1-A_i)\right\}=\bigcap_{i=1}^{N}(1-A_i)$. Hence, $IFclIFint\left\{\bigcap_{i=1}^{N}(1-A_i)\right\}=IFcl\left\{\bigcap_{i=1}^{N}(1-A_i)\right\}=1.$

Proposition 3.12: If A_i's are the Intuitionistic fuzzy σ-nowhere dense sets in an Intuitionistic fuzzy Volterra space (X,T), then $IFint\left\{\bigcup_{i=1}^{N}A_i\right\}=0.$

Proof: The sets A_i's are Intuitionistic fuzzy σ-nowhere dense sets in (X,T). This gives $1-A_i$'s are Intuitionistic fuzzy dense and Intuitionistic fuzzy G_δ-sets in (X,T). Since (X,T) is Intuitionistic fuzzy Volterra space, therefore, $IFcl\left\{\bigcap_{i=1}^{N}(1-Ai)\right\}=1$. That is, $1-IFcl\left\{\bigcap_{i=1}^{N}(1-Ai)\right\}=0$. Hence, $IFint\left\{\bigcup_{i=1}^{N}A_i\right\}=0.$

Proposition 3.13: If A_i's (i = 1 to N) are the Intuitionistic fuzzy nowhere dense and Intuitionistic fuzzy F_σ sets in an Intuitionistic fuzzy Volterra space (X,T), then $IFint\left\{\bigcup_{i=1}^{N}A_i\right\}=0.$

Proof: The sets A_i's are Intuitionistic fuzzy nowhere dense and Intuitionistic fuzzy F_σ sets. Therefore, A_i's are Intuitionistic fuzzy nowhere dense sets. That is, $IFintIFclA_i=0$. But, $IFintA\subseteq IFint\,IFclA=0$. Therefore, $IF\,intA=0$. Hence, A_i's are Intuitionistic fuzzy F_σ sets with $IFintA_i=0$. Therefore, A_i's are Intuitionistic fuzzy σ-nowhere dense sets in an Intuitionistic fuzzy Volterra space (X,T). Hence, by proposition [3.12], $IFint\left\{\bigcup_{i=1}^{N}A_i\right\}=0.$

Definition 3.14[10]**:** An Intuitionistic fuzzy topological spaces (X,T) is said to be an Intuitionistic fuzzy strongly irresolvable space, if $IFclIFintA=1$, for each Intuitionistic fuzzy dense set A in (X,T).

Definition 3.15[13]**:** [13] Let (X,T) be an Intuitionistic fuzzy topological space. An Intuitionistic fuzzy set A in (X,T) is called an Intuitionistic fuzzy σ-nowhere dense set in (X,T) if A is an Intuitionistic fuzzy F_σ-set such that $IFintA=0$.

Proposition 3.16: If $A=\bigcup_{i=1}^{N}A_i$, where A_i's are Intuitionistic fuzzy σ-nowhere dence sets in an Intuitionistic fuzzy strongly irresolvable space (X,T), then A is an Intuitionistic fuzzy nowhere dense set in (X,T).

Proof: Let $A=\bigcup_{i=1}^{N}A_i$, where A_i's are Intuitionistic fuzzy σ-nowhere sets in (X,T). Since, (X,T) is an Intuitionistic fuzzy Volterra space, by theorem[3.12], $IFint(\bigcup_{i=1}^{N} A_i) = 0$. This implies that, $1 - IFint(\bigcup_{i=1}^{N} A_i) = 1$. That is, $IFcl\{\bigcap_{i=1}^{N}(1-A_i)\} = 1$. Since, (X,T) is an Intuitionistic fuzzy strongly irresolvable space, $IFcl\{\bigcap_{i=1}^{N}(1-A_i)\} = 1$, implies that,

$$IFclIFint\{\bigcap_{i=1}^{N}(1-A_i)\} = 1.$$

This implies that,

$$1 - IF\,clIF\,int\left\{\bigcap_{i=1}^{N}(1-A_i)\right\} = 0$$
$$IF\,intIF\,cl\left\{\bigcup_{i=1}^{N} A_i\right\} = 0$$
$$IF\,intIF\,clA = 0$$

Hence, A is an Intuitionistic fuzzy nowhere dense set in (X,T).

Proposition 3.17: If $IFint\left(\bigcup_{i=1}^{N}(1-A_i)\right) = 0$, where $A_i's$ are Intuitionistic fuzzy G_δ- sets in an IFTS (X,T), then (X,T) is an Intuitionistic fuzzy volterra space.

Proof: Let $A_i's$ are Intuitionistic fuzzy G_δ- sets in an IFTS (X,T) such that $IFint\left(\bigcup_{i=1}^{N}(1-A_i)\right) = 0$

Now, $\bigcup_{i=1}^{N} IFint(1-A_i) \subseteq IFint\left(\bigcup_{i=1}^{N}(1-A_i)\right) = 0$ implies that $\bigcup_{i=1}^{N} IFint(1-A_i) = 0$. This implies that $IF\,int(1-A_i) = 0$, for all $i = 1,2,..,N$. This implies that $1 - IFint(1-A_i) = 1$. Hence $IF\,cl(A_i) = 1$, $i = 1,2,\ldots,N$. Therefore, $A_i's$ are Intuitionistic fuzzy nowhere dense sets in (X,T). $IFint\left(\bigcup_{i=1}^{N}(1-A_i)\right) = 0$ implies $Ifcl(\bigcap_{i=1}^{N} A_i) = 1$ where $A_i's$ are Intuitionistic fuzzy dense and Intuitionistic fuzzy G_δ- sets in (X,T). Hence, (X,T) is an Intuitionistic fuzzy volterra space.

Proposition 3.18: If $IFcl \bigcap_{i=1}^{N} A_i = 1$, where $A_i's$ are intuitionistic fuzzy G_δ- sets in an IFTS(X, T) then (X, T) is an intuitionistic fuzzy volterra space.

Proof: IF cl $\bigcap_{i=1}^{N} A_i = 1$, $A_i's$ are intuitionistic fuzzy G_δ-sets in (X, T). Then

$$1 - IF\,cl\left(\bigcap_{i=1}^{N} A_i\right) = 0$$

$$IF\,int\left(\bigcup_{i=1}^{N}(1-A_i)\right) = 0$$

Then by proposition 3.16 (X,T) is an intuitionistic fuzzy volterra space.

Proposition 3.19: If (X, T) is an intuitionistic fuzzy volterra space then there exists an intuitionistic fuzzy F_δ-set A in (X, T) such that $IF\ int(A) \neq 0$

Proof: Let $B = \bigcap_{i=1}^{N} B_i$, where $B_i's$ are intuitionistic fuzzy dense and intuitionistic fuzzy G_δ-set in (X, T). Since (X, T) is an intuitionistic fuzzy volterra space, we have

$$IF\,cl(B) = IF\,cl\left(\bigcap_{i=1}^{N} B_i\right) = 1$$

$$IFcl(B) = 1$$

Now, $1 - IFint(B_i)$ is an intuitionistic fuzzy closed set in (X, T).

Let $A = \bigcup_{i=1}^{\infty} A_i$, where $A_i's$ are intuitionistic fuzzy closed sets in (X, T) in which the first N intuitionistic fuzzy closed sets be $1 - IF\,int(B_i)$. Then A is an intuitionistic fuzzy F_δ- set in (X, T). Hence

$$\bigcup_{i=1}^{N}(1 - IF\ int(B_i)) \subseteq \bigcup_{i=1}^{N} A_i$$

$$1 - \bigcap_{i=1}^{N} IF\ int(B_i) \subseteq \bigcup_{i=1}^{N} A_i$$

Also,

$$1 - \bigcap_{i=1}^{N}(B_i) \subseteq 1 - \bigcap_{i=1}^{N} IF\ int(B_i)$$

$$\subseteq \bigcup_{i=1}^{N} A_i$$

This implies that,

$$1 - B \subset A$$

$$IF\ int(1-B) \subset IF\ int(A)$$

$1 - IFcl(B) \subset IF\,int(A)$

$1 - 1 \subset IF\,int(A)$

$0 \subset IF\,int(A)$

$IF\,int(A) \neq 0.$

Definition 3.20[19]**:** Let (X, T) be an IFTS then (X, T) is called an intuitionistic fuzzy P - space if countable intersection of intuitionistic fuzzy open sets in(X, T) is intuitionistic fuzzy open.

Proposition 3.21: If the intuitionistic fuzzy topological space (X, T) is an intuitionistic fuzzy volterra and intuitionistic fuzzy P - space and if $A = \bigcap_{i=1}^{N} A_i$, where A_i's are intuitionistic fuzzy dense and intuitionistic G_δ- sets in X, then $IFcl(A) = 1$ but $IF\,int(A) \neq 0$.

Proof: Let $A = \bigcap_{i=1}^{N} A_i$, where A_i's are intuitionistic fuzzy dense and intuitionistic fuzzy G_δ- sets in (X, T). Since (X, T) is an intuitionistic fuzzy volterra space, we have

$$IF\,cl(A) = IF\,cl\left(\bigcap_{i=1}^{N} A_i\right)$$

$$=1,$$

implies that $IFcl(A) = 1$.

As in the proof of proposition 3.19, there exists an intuitionistic fuzzy F_δ- set B in (X, T) such that $1 - A \subset B$.

Since B is an intuitionistic fuzzy F_δ- set, 1-B is an intuitionistic fuzzy G_δ- set in (X, T).

Since (X, T) is an intuitionistic fuzzy P - space, the intuitionistic fuzzy G_δ- set (1 - B) is an intuitionistic fuzzy open set in (X, T).

Therefore, B is an intuitionistic fuzzy closed set in (X, T). $IFcl(B) = B$. Now,

$IFcl(1 - A) \subset IFcl(B)$

$1 - IFintA \subset IFcl(B) = B$

Hence $IFint\,A \neq 0$.

References

[1] Atanassov, K.T., Intuitionistic Fuzzy Sets - Theory and Applications, Springer Verlag, New York. (1999).

[2] Atanassov, K.T., lntuitionistic fuzzy sets, in: V. Sgurev. Ed., VII ITKR's Session, Soa Central Sci. and Techn. Library, Bulg. Academy of Sciences, 1984.

[3] Atanassov, K.T., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1986), 87-96.

[4] Atanassov, K.T. and Stoeva, S. Intuitionistic fuzzy sets, Polish Syrup. on Interval & Fuzzy Mathematics, Poznan, 8(1983), 23-26.

[5] Atanassov, K.T. and Stoeva, S. Intuitionistic L-fuzzy sets, in: R. Trappl, Ed., Cybernetics and System Research, Elsevier, Amsterdam, (1984), 539-540.

[6] Chang, C.L., Fuzzy topological spaces, J. Math. Anal. Appl, 24(1968), 182-185.

[7] Coker, D. An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 88(1997), 81-89.

[8] David Gauld, Sina Greenwood and Zbigniew Piotrowski, On Volterra Spaces-II, Papers on General Topology and Applications, Ann. New York Acad. Sci., 806(1996), 169-173.

[9] David Gauld and Zbigniew Piotrowski, On Volterra Spaces, Far East J. Math. Sci., 1 (1993), No.2., 209-214.

[10] Dhavaseelan, R. Roja, E. and Uma, M.K. Intuitionistic Fuzzy Resolvable and Intuitionistic Fuzzy irresolvable Spaces, Scientia Magna, 7(2), (2011), 59-67.

[11] Gruenhage, G. and Lutzer, D., Baire and Volterra Spaces, Proc. Amer. Soc., 128(16), (2000), 3115-3124.

[12] Jiling Cao and David Gauld, Volterra Spaces Revisited, J. Aust. Math. Soc. 79 (2005), 61-76.

[13] Soundararajan, S. Rizwan, U and Syed Tahir Hussainy, On Intuitionistic Fuzzy Volterra Spaces, International Journal of Science and Humanities, 1(2), (2015)pp 727 - 738.

[14] Thakur, S.S. and Dhavaseelan, R. Nowhere Dense Sets in Intuitionistic Fuzzy Topological Spaces, Proc. of National Seminar on Recent Developments in Topology, Dept. of Math., Sri Saradha College for Women, Salem, Feb 2015, 17-

21.

[15] Thangaraj, G. and Soundararajan, S. On Fuzzy Volterra Spaces, J. Fuzzy Math., 21(4) (2013), pp 895-904.

[16] Zadeh, L.A. Fuzzy sets, Inform and Control, 8(1965), 338-353.

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