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A NEW APPROACH FOR SOLVING ASSIGNMENT PROBLEM IN NEUTROSOPHIC ENVIRONMENT BY ZERO SUFFIX METHOD

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A B S T R A C T **RESEARCH ARTICLE**

This paper presents the application of Neutrosophic Set Theory (NST) in solving Generalized Assignment Problem (GAP). GAP has been solved earlier under fuzzy environment. NST is a generalization of the concept of classical set, fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set. Elements of Neutrosophic set are characterized by a truth-membership function, falsity and also indeterminacy which is a more realistic way of expressing the parameters in real life problem. Here the elements of the cost matrix for the GAP are considered as neutrosophic elements which have not been considered earlier by any other author. The problem has been solved by evaluating score function matrix and then solving it by Zero Suffix method to get the optimal assignment. The method has been demonstrated by a suitable numerical example.

Keywords:

NST, GAP, Zero Suffix Method

1. Introduction

The concept of fuzzy sets and the degree of membership/truth (T) was first introduced by Zadeh in 1965 [8]. This concept is very much useful to handle uncertainty in real life situation. After two decades, Turksen [6] introduced the concept of interval valued fuzzy set which was not enough to consider the non-membership function. In the year 1999, Atanassov [1], [2], [3] proposed the degree ofnonmembership/falsehood (F) and intuitionistic fuzzy set (IFS) which is not only more practical in real life but also the generalization of fuzzy set. The paper considers both the degree of membership $\mu_{\tilde{A}}(x) \in [0,1]$Of each element $x \in M$ to a set A and the degree of non-membership $\nu_A(x) \in [0,1]$Such that $\mu_A(x) + \nu_A(x) \leq 1$ IFS deals with incomplete informationboth for membership and non-membership function but not with indeterminacy membership function which is also very natural and obvious part in real life situation. Wang et. Al [7] first considered this indeterminate information which is more practical and useful in real life problems. F.Smarandacheet.Al [4] introduced the degree of indeterminacy/neutrality (I)as independent component in 1995 (published in 1998) and defined the neutrosophic set. He coined the words “neutrosophy” and “neutrosophic”.

2. Preliminaries

Definition:(Fuzzy Set ,see [1]) : Let X be a nonempty set. A fuzzy set $\bar{A} = \{< x, \mu_{\bar{A}}(x) > | x \in X\}$ where $\mu_{\bar{A}}(x)$ is called the membership function which maps each element of X to a valued between 0 and 1

Definition:(Fuzzy Number): A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set on real line ℝ such that

(i) There exit at least one $x \in \mathbb{R}$ with $\mu_{\tilde{A}}(x) = 1$
(ii) $\mu_{\tilde{A}}(x)$ is piecewise continuous

Definition:(Trapezoidal Fuzzy Number):A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is trapezoidal fuzzy number, where $a_1, a_2, a_3, and\ a_4$ are real numbers and its membership function is given as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & for\ \ a_1 \leq x \leq a_2 \\ 1, & for\ \ a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & for\ \ a_3 \leq x \leq a_4 \\ 0, & otherwise \end{cases}$$

Definition:(Intuitionistic Fuzzy Set): Let X be a nonempty set. An intuitionistic fuzzy set $A^I$of X is defined as $\overline{A}^I = \{< x, \mu_{\bar{A}^I}(x), \nu_{\bar{A}^I}(x) > | x \in X\}$ where $\mu_{\bar{A}^I}(x)\ and\ \nu_{\bar{A}^I}(x)$ are membership and non-membership functions such that $\mu_{\bar{A}^I}(x), \nu_{\bar{A}^I}(x)$:X→[0,1] and$0 \leq \mu_{\bar{A}^I}(x) + \nu_{\bar{A}^I}(x) \leq 1$ for all $x \in X$

Definition:(Intuitionistic Fuzzy Number): An intuitionistic fuzzy subset $\tilde{A}^I = \{< x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x) > | x \in \mathbb{R}\}$ of the real line ℝ is called an intuitionistic fuzzy number (IFN) if the following conditions hold:

(i) There exists $m \in \mathbb{R}$ such that $\mu_{\tilde{A}^I}(m) = 1$ and $\nu_{\tilde{A}^I}(x) = 0$
(ii) $\mu_{\tilde{A}^I}$is continuous function from ℝ → [0,1] such that $\mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \leq 1$ for all $x \in X$
(iii) The membership and non-membership functions of $\tilde{A}^I$ are in the following form:

$$\mu_{\tilde{A}^I}(x) = \begin{cases} 0, & for\ -\infty \le x \le a_1 \\ f(x), & for\ a_1 \le x \le a_2 \\ 1, & for\ x = a_2 \\ g(x), & for\ a_2 \le x \le a_3 \\ 0, & for\ a_3 \le x \le \infty \end{cases}$$

$$\nu_{\tilde{A}^I}(x) = \begin{cases} 1, & for\ -\infty \le x \le a_1' \\ f'(x), & for\ a_1' \le x \le a_2 \\ 0, & for\ x = a_2 \\ g'(x), & for\ a_2 \le x \le a_3' \\ 1, & for\ a_3' \le x \le \infty \end{cases}$$

Where f, f', g, g' are function from $\mathbb{R} \to [0,1]$, $f\ and\ g'$ are strictly increasing functions, and $g\ and\ f'$ are strictly decreasing functions with the conditions $0 \le f(x) + f'(x) \le 1$ and $0 \le g(x) + g'(x) \le 1$.

Definition:(Trapezoidal Intuitionistic Fuzzy Number): A trapezoidal intuitionistic fuzzy number is denoted by $\tilde{A}^I = (a_1, a_2, a_3, a_4),\ (a_1', a_2, a_3, a_4')$, where $a_1' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4'$ with membership and non-membership functions are defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & for\ a_1 \le x \le a_2 \\ 1, & for\ a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & for\ a_3 \le x \le a_4 \\ 0, & otherwise \end{cases}$$

$$\nu_{\tilde{A}}(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1'}, & for\ a_1' \le x \le a_2 \\ 0, & for\ a_2 \le x \le a_3 \\ \dfrac{x - a_3}{a_4' - a_3}, & for\ a_3 \le x \le a_4' \\ 1, & otherwise \end{cases}$$

Definition:(Neutrosophic Set):Let X be a nonempty set. Then a neutrosophic set $\overline{A}^N$ of X is defined as

$$\bar{A}^N = \{ < x, T_{\bar{A}^N}(x), I_{\bar{A}^N}(x), F_{\bar{A}^N}(x) >$$

$$|x \in X, T_{\bar{A}^N}(x), I_{\bar{A}^N}(x), F_{\bar{A}^N}(x) \in\ ]^-0,1[\ \}$$

Where $T_{\bar{A}^N}(x), I_{\bar{A}^N}(x), and\ F_{\bar{A}^N}(x)$ are truth membership function, and a falsity-membership function and there is no restriction on the sum of $T_{\bar{A}^N}(x), I_{\bar{A}^N}(x), and\ F_{\bar{A}^N}(x)$ so

$^-0 \le T_{\bar{A}^N}(x) + I_{\bar{A}^N}(x) + F_{\bar{A}^N}(x) \le 3^+$ and $]^-0,1^+[$ is a nonstandard unit interval.

But it is difficult to apply neutrosophic set theories in real life problems directly. So wang introduced single valued neutrosophic set as a subset of neutrosophic set and the definition is as follows.

Definition:(Single Valued Neutrosophic Set):Let X be a nonempty set. Then a single valued neutrosophic set $\overline{A}_s^N$ of X is defined as $\overline{A}_s^N = \{< x, T_{\bar{A}^N}(x), I_{\bar{A}^N}(x), F_{\bar{A}^N}(x) > | x \in X\}$ where $T_{\bar{A}^N}(x), I_{\bar{A}^N}(x), and\ F_{\bar{A}^N}(x) \in [0,1]$ for each $x \in X$ and $0 \le T_{\bar{A}^N}(x) + I_{\bar{A}^N}(x) + F_{\bar{A}^N}(x) \le 3$

Definition:(Single Valued Trapezoidal Neutrosophic Number): Let $w_{\tilde{a}} u_{\tilde{a}} y_{\tilde{a}} \in [0,1]$ and $a_1, a_2, a_3, a_4 \in \mathbb{R}$ such that $a_1 \le a_2 \le a_3 \le a_4$. Then a single valued trapezoidal neutrosophic number, $\tilde{a} =< (a_1, a_2, a_3, a_4); w_{\tilde{a}} u_{\tilde{a}} y_{\tilde{a}} >$ is a special neutrosophic set on the real line set $\mathbb{R}$, whose truth membership, indeterminacy-membership, and falsity-membership function are given as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} w_{\tilde{a}}\left(\dfrac{x - a_1}{a_2 - a_1}\right), & for\ a_1 \le x \le a_2 \\ w_{\tilde{a}}, & for\ a_2 \le x \le a_3 \\ w_{\tilde{a}}\left(\dfrac{a_4 - x}{a_4 - a_3}\right), & for\ a_3 \le x \le a_4 \\ 0, & otherwise \end{cases}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} \dfrac{a_2 - x + u_{\tilde{a}}(x - a_1)}{a_2 - a_1}, & for\ a_1 \le x \le a_2 \\ u_{\tilde{a}}, & for\ a_2 \le x \le a_3 \\ \dfrac{x - a_3 + u_{\tilde{a}}(a_4 - x)}{a_4 - a_3}, & for\ a_3 \le x \le a_4 \\ 1, & otherwise \end{cases}$$

$$\lambda_{\tilde{a}}(x) = \begin{cases} \dfrac{a_2 - x + y_{\tilde{a}}(x - a_1)}{a_2 - a_1}, & for\ a_1 \le x \le a_2 \\ y_{\tilde{a}}, & for\ a_2 \le x \le a_3 \\ \dfrac{x - a_3 + y_{\tilde{a}}(a_4 - x)}{a_4 - a_3}, & for\ a_3 \le x \le a_4 \\ 1, & otherwise \end{cases}$$

Where $w_{\tilde{a}}, u_{\tilde{a}}, and\ y_{\tilde{a}}$ denote the maximum truth-membership degree, minimum –indeterminacy membership degree, and minimum falsity-membership degeree, respectively . A single valued trapezoidal neutrosophic number $\tilde{a} =< (a_1, a_2, a_3, a_4); w_{\tilde{a}} u_{\tilde{a}} y_{\tilde{a}} >$ may express an ill-defined quantity about a, which is approximately equal to $[a_3, a_2]$.

Definition:(Arithmetic Operations on Single valued single valued Trapezoidal Neutrosophic numbers and $k \ne 0$;trapezoidal neutrosophic numbers): Let $\tilde{a} =< (a_1, a_2, a_3, a_4); w_{\tilde{a}} u_{\tilde{a}} y_{\tilde{a}} >$ and $\tilde{b} =< (b_1, b_2, b_3, b_4); w_{\tilde{a}} u_{\tilde{a}} y_{\tilde{a}} >$ then be two $\tilde{a} + \tilde{b} =< (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); w_{\tilde{a}} \wedge w_{\tilde{a}}, u_{\tilde{a}} \vee u_{\tilde{a}}, y_{\tilde{a}} \vee y_{\tilde{a}} >$,

(i) $\tilde{a} - \tilde{b} =< (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4); w_{\tilde{a}} \wedge w_{\tilde{a}}, u_{\tilde{a}} \vee u_{\tilde{a}}, y_{\tilde{a}} \vee y_{\tilde{a}} >$,

(ii) $$\tilde{b} = \begin{cases} < (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4); w_{\tilde{a}} \wedge w_{\tilde{a}}, u_{\tilde{a}} \vee u_{\tilde{a}}, y_{\tilde{a}} \vee y_{\tilde{a}} >, & if\ a_4 > 0, b_4 > 0 \\ < (a_1 b_4, a_2 b_3, a_3 b_2, a_4 b_1); w_{\tilde{a}} \wedge w_{\tilde{a}}, u_{\tilde{a}} \vee u_{\tilde{a}}, y_{\tilde{a}} \vee y_{\tilde{a}} >, & if\ a_4 < 0, b_4 > 0 \\ < (a_4 b_4, a_3 b_3, a_2 b_2, a_1 b_1); w_{\tilde{a}} \wedge w_{\tilde{a}}, u_{\tilde{a}} \vee u_{\tilde{a}}, y_{\tilde{a}} \vee y_{\tilde{a}} >, & if\ a_4 < 0, b_4 < 0 \end{cases}$$

(iii) $\frac{\tilde{a}}{\tilde{b}} = \begin{cases} < \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right); w_{\tilde{a}} \wedge w_{\tilde{a}}, u_{\tilde{a}} \vee u_{\tilde{a}}, y_{\tilde{a}} \vee y_{\tilde{a}} >, & if\ a_4 > 0, b_4 > 0 \\ < \left(\frac{a_4}{b_4}, \frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); w_{\tilde{a}} \wedge w_{\tilde{a}}, u_{\tilde{a}} \vee u_{\tilde{a}}, y_{\tilde{a}} \vee y_{\tilde{a}} >, & if\ a_4 < 0, b_4 > 0 \\ < \left(\frac{a_4}{b_1}, \frac{a_3}{b_2}, \frac{a_2}{b_3}, \frac{a_1}{b_4}\right); w_{\tilde{a}} \wedge w_{\tilde{a}}, u_{\tilde{a}} \vee u_{\tilde{a}}, y_{\tilde{a}} \vee y_{\tilde{a}} >, & if\ a_4 < 0, b_4 < 0 \end{cases}$

(iv) $k\tilde{a} = \begin{cases} < (ka_1, ka_2, ka_3, ka_4); w_{\tilde{a}} u_{\tilde{a}} y_{\tilde{a}} > & if\ k > 0 \\ < (ka_4, ka_3, ka_2, ka_1); w_{\tilde{a}} u_{\tilde{a}} y_{\tilde{a}} > & if\ k < 0, \end{cases}$

(v) $\tilde{a}^I = < \left(\frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right); w_{\tilde{a}} u_{\tilde{a}} y_{\tilde{a}} > \ where\ \tilde{a} \neq 0.$

Definition:(Score and Accuracy functions of single valued Trapezoidal Neutrosophic Numbers).One can compare any two single valued trapezoidal neutrosophic numbers based on the score and accuracy functions. Let $\tilde{a} = \langle (a_1, a_2, a_3, a_4);\ w_{\tilde{a}} u_{\tilde{a}} y_{\tilde{a}} \rangle$be a single valued trapezoidal neutrosophic number; then

(i) Score function

$$S(\tilde{a}) = \left(\frac{1}{16}\right)[a_1 + a_2 + a_3 + a_4] \times [\mu_{\tilde{a}} + (1 - \nu_{\tilde{a}}) + (1 - \lambda_{\tilde{a}})];$$

(ii) accuracy functions

$$A(\tilde{a}) = \left(\frac{1}{16}\right)[a_1 + a_2 + a_3 + a_4] \times [\mu_{\tilde{a}} + (1 - \nu_{\tilde{a}}) + (1 + \lambda_{\tilde{a}})];$$

Definition:(Comparison of Single Valued Trapezoidal Neutrosophic Numbers): Let $\tilde{a}\ and\ \tilde{b}$ be any two single valued trapezoidal neutrosophic numbers; then one has the following :

(1).If S($\tilde{a}$)< S($\tilde{b}$) then $\tilde{a} < \tilde{b}$

(2).If S($\tilde{a}$)= S($\tilde{b}$) and if

(i) If A($\tilde{a}$) < A($\tilde{b}$) then $\tilde{a} < \tilde{b}$
(ii) If A($\tilde{a}$) > A($\tilde{b}$) then $\tilde{a} > \tilde{b}$
(iii) If A($\tilde{a}$) = A($\tilde{b}$) then $\tilde{a} = \tilde{b}$.

Example:

Let $\tilde{a} =< (4,8,10,16); 0.5,0.3,0.6 >$ and $\tilde{b} =< (3,7,11,14); 0.4,0.5,0.6 >$ be two single valued trapezoidal neutrosophic numbers; then

(i) $\tilde{a} + \tilde{b} = < (7,15,21,30); 0.4,0.5,0.6 >,$
(ii) $\tilde{a} - \tilde{b} =< (-10, -3, 3, 13); 0.4,0.5,0.6 >$
(iii) $\tilde{a}\tilde{b} =< (12,56,110,224); 0.4,0.5,0.6 >$
(iv) $\frac{\tilde{a}}{\tilde{b}} =< \left(\frac{4}{14}, \frac{8}{11}, \frac{10}{7}, \frac{16}{3}\right); 0.4,0.5,0.6 >$
(v) $3\tilde{a} =< (12,24,30,48); 0.4,0.5,0.6 >$
(vi) $\tilde{a}^I =< \left(\frac{1}{16}, \frac{1}{10}, \frac{1}{8}, \frac{1}{4}\right); 0.4,0.5,0.6 >$
(vii) $S(\tilde{a}) = \left(\frac{1}{16}\right)[4 + 8 + 10 + 16] \times \left[0.5 + \left(1 - 0.3 + (1 - 0.6)\right)\right] = \mathbf{3.8},$
(viii) $A(\tilde{a}) = \left(\frac{1}{16}\right)[4 + 8 + 10 + 16] \times [0.5 + (1 - 0.3) + (1 + 0.6)] = \mathbf{6.65}$

3. Generalized Assignment Problem using Neutrosophic Set Theory

In this section, we have formulated the GAP using NST. GAP has been solved earlier in different ways by different mathematicians. David B. Shymos and Eva Tardos [April, 1991] considered the GAP as the problem of scheduling parallel machines and solved it by polynomial time algorithm. Dr. ZeevNutov [June, 2005] solved GAP considering it as a Maxprofit scheduling problems. Supriya Kar, Dr. Kajla Basu, Dr. Sathi Mukherjee They solved FGAP also under Hesitant Fuzzy Environment [Springer India, Opsearch, 29th October 2014, ISSN 0030-3887. Here, we have used NST to solve GAP because in neutrosophy, every object has not only a certain degree of truth, but also a falsity degree and anindeterminacy degree that have to be considered independently.

3.1 Mathematical model of Assignment Problem In Neutrosophic Environment

In this model, Assignment problem is introduced in a single valued neutrosophic environment, consider assignment problem with “m” jobs and “n” personsand the mathematical model for NGAP will be as follows

Now the mathematical formulation of the problem is given by

$$\text{Minimize } \tilde{Z}^N = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{i,j}\, \tilde{c}_{i,j}^N$$

Subject to

$$\sum_{j=1}^{n} x_{i,j} = 1\,, i = 1,2,3 \ldots . m$$

$$\sum_{i=1}^{m} \tilde{c}_{i,j}^N \leq a_j\,, j = 1,2,3 \ldots . n$$

$x_{i,j} \geq 0\ for\ all\ i,j$

4. Procedure for Proposed Algorithm based on Neutrosophic Numbers

Input Assumption. The parameters of the problem will be represented by either crisp or trapezoidal neutrosophic numbers.

4.1 Zero Suffix Method

We, now introduce a new method called the zero suffix method for finding an optimal solution to the assignment problem.

The zero suffix method proceeds as follows.

Step 1: Calculate the score value of each neutrosophic cost $\tilde{c}_{i,j}^{N}$ and replace all the neutrosophic costs by its score value to obtain the Generalized assignment problem.

Step 2: Subtract each row entries of the NGAP table from the correspondingrow minimum after that subtract each column entries of the NGAP table from the corresponding column minimum.

Step 3: In the reduced cost matrix there will be at least one zero in each row and column, then find the suffix value of all the zeros in the reduced cost matrix by following simplification, the suffix value is denoted by S,

$$\text{Therefore } S=\left\{\frac{\text{Add the costs of nearest adjacent sides of zero which are greater than zero}}{\text{No. of costs added}}\right\}$$

Step 4: Choose the maximum of S, if it has one maximum value encircle the corresponding zero first then, if it has more equal values then choose arbitrarily.

Step 5: After encircling omit the corresponding row or column, the resultant matrix must possess at least one zero is each row and column, else repeat 3.

Step 6: Repeat 3 to 3 until each row possess atleastone encircled zero.

5. Illustrative Example

Table 1: Input data for neutrosophic assignment problem.

	J_1	J_2	J_3	J_4
M_1	(3, 5, 6, 8); 0.6, 0.5, 0.4	(5, 8, 10, 14); 0.3, 0.6, 0.6	(12, 15, 19, 22); 0.6, 0.4, 0.5	(14, 17, 21, 28); 0.8, 0.2.0.6
M_2	(0, 1, 3, 6); 0.7, 0.5, 0.3	(5, 7, 9, 11); 0.9, 0.7, 0.5	(15, 17, 19, 22); 0.4, 0.8, 0.4	(9, 11, 14, 16); 0.5, 0.4, 0.7
M_3	(4,8,11,15); 0.6, 0.3, 0.2	(1, 3, 4, 6); 0.6, 0.3, 0.5	(5, 7, 8, 10); 0.5, 0.4, 0.7	(5, 9, 14, 19); 0.3, 0.7, 0.6

Table 2: Crisp assignment problem.

	J_1	J_2	J_3	J_4
M_1	2	3	7	10
M_2	1	3	6	4
M_3	5	2	3	3

Table 3: Zero suffix method.

	J_1	J_2	J_3	J_4
M_1	0	1	0	0
M_2	0	0	0	1
M_3	1	0	1	0

Therefore optimal assignment is,

$M_1 \rightarrow J_2$, $M_2 \rightarrow J_4$, $M_3 \rightarrow J_1$, $M_3 \rightarrow J_3$

Conclusion

Neutrosophic set theory is a generalization of classical set, fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set because it not only considers the truth membership T_A and falsity membership F_A, but also an indeterminacy function I_A which is very obvious in real life situation. In this paper, we have considered the cost matrix as neutrosophic elements considering the restrictions on the available costs. By calculation Score function matrix, the problem is solved by ZSM which is very simple yet efficient method to solve GAP. Now to verify the solution the problem has been transformed to LPP form and solved by standard software LINGO 9.0.

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