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SOLVING FUZZY TRANSPORTATION PROBLEM USING ZERO POINT MAXIMUM ALLOCATION METHOD

L. Sujatha[1], P. Vinothini[2] and R. Jothilakshmi[3]

[1&2]Department of Mathematics, Auxilium College (Autonomous), Vellore
[3]Assistant Professor, Department of Mathematics, Mazharul Uloom College, Ambur

A B S T R A C T **RESEARCH ARTICLE**

The objective of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying the fuzzy supply and the fuzzy demand limits. As the economic growth of a country depends on the increase of the capacity of transport, the study of fuzzy transportation problem is essential. In this paper, mathematical formulation, theoretical background and the procedure are proposed for fuzzy transportation problem using zero point maximum allocation method. The procedure presented is independent of the conventional method. Numerical example is illustrated for the same. The result obtained is compared with the existing result to point out the conclusion.

Keywords:

Trapezoidal fuzzy numbers, Fuzzy transportation problem, Decision making, Non-negative optimal fuzzy solution, Positive optimal fuzzy objective value.

2010Mathematics Subject Classification: 90B50, 90C08, 03E72

1. Introduction

In recent years, Fuzzy transportation problem (FTP) has received much concentration from the researchers which is an important optimization and decision making problems in fuzzy operations research. The basic Transportation problem (TP) was developed by Hitchcock [4]. In order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the model were tried at crisp level. But in real life, supply, demand and transportation cost parameters are uncertain due to several factors such as unavailability of resources or undue delay behind the schedule in the arrival of materials etc. These types of imprecise data may be represented by fuzzy numbers introduced by Zadeh [18]. Zimmerman [19] obtained an optimal solution to a FTP by fuzzy linear programming approach. Many researchers have proposed different techniques to obtain the optimal solution for FTP [2,6,9,12,13,15]. The papers of Pandian and Natarajan [13,14] motivated us present a different form of procedure for FTP under fuzzy environment, where the final optimal objective value is a fuzzy number.

The paper is organized as follows: In section 2, the basic definitions of trapezoidal fuzzy numbers and the Mathematical formulation of FTP are reviewed. In section 3, the Mathematical formulation of FTP is proposed which is equivalent to the Mathematical formulation given in section 2. The theoretical background and the procedure are proposed for solving FTP using zero point maximum allocation method with a suitable numerical example. The result obtained in the proposed method is compared with the existing result under results and discussions. It is followed by the shortcomings of the existing methods. Section 4 concludes the paper.

2. Preliminaries

Definition 2.1: A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$is a trapezoidal fuzzy number where $a_1, a_2, a_3, a_4 \in R$ with $a_1 < a_2 < a_3 < a_4$. Its membership function $\mu_{\tilde{A}}(x)$ is given below.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{for } x \le a_1 \\ (x-a_1)/(a_2-a_1) & \text{for } a_1 \le x \le a_2 \\ 1, & \text{for } a_2 \le x \le a_3 \\ (a_4-x)/(a_4-a_3) & \text{for } a_3 \le x \le a_4 \\ 0, & \text{for } x \ge a_4 \end{cases}$$

If $a_2 = a_3$, then the trapezoidal fuzzy number$\tilde{A} = (a_1, a_2, a_3, a_4)$ becomes the triangular fuzzy number $\tilde{A} = (a_1, a_2, a_4)$.

Definition 2.2: Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number. $\tilde{A}$ is said to be non-negative if $a_i \ge 0$, i = 1,2,3,4 and $\tilde{A}$ is said to be non-positive if $a_i \le 0$, i = 1,2,3,4.

Definition 2.3: Operations on Trapezoidal Fuzzy Numbers
Let $\tilde{A} = (a_1, a_2, a_3, a_4), \tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers then

(i) Addition:
$\tilde{A} \oplus \tilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$

(ii) Subtraction:
$\tilde{A} \ominus \tilde{B} = (a_1-b_4, a_2-b_3, a_3-b_2, a_4-b_1)$

(iii) Scalar Multiplication:
k$\tilde{A} = (ka_1, ka_2, ka_3, ka_4)$if k>0 and
k$\tilde{A} = (ka_4, ka_3, ka_2, ka_1)$ if k<0

Mathematical Formulation for FTP having fuzzy cost, fuzzy supply and fuzzy demand

(P)Minimize $\tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$(2.1)

Subject to

$\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i$, i = 1 to m (2.2)

$\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j \ j = 1 \ to \ n$ (2.3)

$\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j$ (2.4)

$\tilde{x}_{ij} \geq 0, i = 1 \ to \ m \ and$
$j = 1 \ to \ n$ (2.5)

where m = the number of supply points;
n = the number of demand points;
$\tilde{x}_{ij} \approx (x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4)$ is the uncertain number of units shipped from the supply point i to the demand point j;
$\tilde{c}_{ij} = (c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4)$ is the uncertain cost of shipping one unit from the supply point i to the demand point j;
$\tilde{a}_i = (a_i^1, a_i^2, a_i^3, a_i^4)$ is the uncertain supply at the supply point i and
$\tilde{b}_j = (b_j^1, b_j^2, b_j^3, b_j^4)$ is the uncertain demand at the demand point j.

3. Zero Point Maximum Allocation Method

The Mathematical formulation for the FTP (P*) which is equivalent tothe FTP (P) is

(P*) Minimize
$\tilde{z}^* = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}^* \otimes \tilde{x}_{ij}^*$ (3.1)

Subject to $\sum_{j=1}^{n} \tilde{x}_{ij}^* \approx \tilde{a}_i^*$,
i = 1 to m (3.2)

$\sum_{i=1}^{m} \tilde{x}_{ij}^* \approx \tilde{b}_j^*, \ j = 1 \ to \ n$ (3.3)

$\sum_{i=1}^{m} \tilde{a}_i^* \approx \sum_{j=1}^{n} \tilde{b}_j^*$ (3.4)

$\tilde{x}_{ij}^* \leq 0, i = 1 \ to \ m \ and$
$j = 1 \ to \ n$ (3.5)

Here $\tilde{c}_{ij}^* = -\tilde{c}_{ij}$, $\tilde{x}_{ij}^* = -\tilde{x}_{ij}$, $\tilde{a}_i^* = -\tilde{a}_i$, $\tilde{b}_j^* = -\tilde{b}_j$

Theorem 3.1. Let $[x_{ij}^{*01}]$ = $\{x_{ij}^{*01}, i = 1 \ to \ m$, j = 1 to n} be an optimal solution of (P_1^*), $[x_{ij}^{*02}]$ = $\{x_{ij}^{*02}, i = 1 \ to \ m$, j = 1 to n} be an optimal solution of (P_2^*), $[x_{ij}^{*03}]$ = $\{x_{ij}^{*03}, i = 1 \ to \ m$, j = 1 to n} be an optimal solution of (P_3^*) and$[x_{ij}^{*04}]$ = $\{x_{ij}^{*04}, i = 1 \ to \ m$, j = 1 to n} be an optimal solution of (P_4^*), where

(P_1^*) Minimize $z_1^* = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{*1} x_{ij}^{*1}$; $c_{ij}^{*1} = -c_{ij}^4, x_{ij}^{*1} = -x_{ij}^4$ (3.6)

Subject to$\sum_{j=1}^{n} x_{ij}^{*1} = a_i^{*1}$,
i = 1 to m, $a_i^{*1} = -a_i^4$

$\sum_{i=1}^{m} x_{ij}^{*1} = b_j^{*1}$, $j = 1 \ to \ n, \ b_j^{*1} = -b_j^4$ (3.8)

$\sum_{i=1}^{m} a_i^{*1} = \sum_{j=1}^{n} b_j^{*1}$ (3.9)

$x_{ij}^{*1} \leq 0$, for i = 1 to m and j = 1 to n. (3.10)

For l = 1,2,3

(P_{l+1}^*)
Minimize $z_{l+1}^* = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{*l+1} x_{ij}^{*l+1}$

Subject
to$\sum_{j=1}^{n} x_{ij}^{*l+1} = a_i^{*l+1}$, i = 1 to m

$\sum_{i=1}^{m} x_{ij}^{*l+1} = b_j^{*l+1}$, $j = 1 \ to \ n$

$\sum_{i=1}^{m} a_i^{*l+1} = \sum_{j=1}^{n} b_j^{*l+1}$

$x_{ij}^{*l+1} \leq 0$, for i = 1 to m and j = 1 to n.

$x_{ij}^{*0l+1} \geq x_{ij}^{*0l}, i = 1 \ to \ m \ and \ j = 1 \ to \ n.$

Then $[\tilde{x}_{ij}^{*0}]$ = $\{\tilde{x}_{ij}^{*0} = (x_{ij}^{*01}, x_{ij}^{*02}, x_{ij}^{*03}, x_{ij}^{*04}), i = 1 \ to \ m \ and \ j = 1 \ to \ n\}$ is an optimal solution of the FTP (P*) and $[\tilde{x}_{ij}^{0}]$ = $\{\tilde{x}_{ij}^{0} = (x_{ij}^{01}, x_{ij}^{02}, x_{ij}^{03}, x_{ij}^{04}), i = 1 \ to \ m \ and \ j = 1 \ to \ n\}$ is an optimal solution of the given FTP (P).

Proof: Let $[\tilde{y}_{ij}^*]$ = $\{\tilde{y}_{ij}^* = (y_{ij}^{*1}, y_{ij}^{*2}, y_{ij}^{*3}, y_{ij}^{*4}), i = 1 \ to \ m \ and \ j = 1 \ to \ n\}$ be a feasible solutionof the FTP (P*).Clearly, $[y_{ij}^{*1}]$, $[y_{ij}^{*2}]$, $[y_{ij}^{*3}]$ and $[y_{ij}^{*4}]$ are feasible solutions of (P_1^*), (P_2^*), (P_3^*) and (P_4^*) respectively.
Since $[x_{ij}^{*01}]$, $[x_{ij}^{*02}]$, $[x_{ij}^{*03}]$ and $[x_{ij}^{*04}]$ are optimal solutions of (P_1^*), (P_2^*), (P_3^*) and (P_4^*) respectively, we have

$z_1^*([x_{ij}^{*01}]) \leq z_1^*([y_{ij}^{*1}])$,
$z_2^*([x_{ij}^{*02}]) \leq z_2^*([y_{ij}^{*2}])$,
$z_3^*([x_{ij}^{*03}]) \leq z_3^*([y_{ij}^{*3}])$ *and*
$z_4^*([x_{ij}^{*04}]) \leq z_4^*([y_{ij}^{*4}])$,

(i.e) $z^*([\tilde{x}_{ij}^{*0}]) \leq z^*([\tilde{y}_{ij}^*])$, for all feasible solution of the FTP (P*).

Therefore, $[\tilde{x}_{ij}^{*0}]$ = $\{\tilde{x}_{ij}^{*0} = (x_{ij}^{*01}, x_{ij}^{*02}, x_{ij}^{*03}, x_{ij}^{*04}), i = 1 \ to \ m \ and \ j = 1 \ to \ n\}$ is an optimal solution of the FTP (P*).
As $\tilde{x}_{ij}^{*0} = -\tilde{x}_{ij}^{0}$,$[\tilde{x}_{ij}^{0}]$ = $\{\tilde{x}_{ij}^{0} = (x_{ij}^{01}, x_{ij}^{02}, x_{ij}^{03}, x_{ij}^{04}), i = 1 \ to \ m \ and \ j = 1 \ to \ n\}$ is an optimal solution of the given FTP (P), where$x_{ij}^{*01} = -x_{ij}^{04}$, $x_{ij}^{*02} = -x_{ij}^{03}$, $x_{ij}^{*03} = -x_{ij}^{02}$, $x_{ij}^{*04} = -x_{ij}^{01}$.
Hence the theorem.

Remark:
The optimal fuzzy solution $[\tilde{x}_{ij}^{*0}]$ = $\{\tilde{x}_{ij}^{*0} = (x_{ij}^{*01}, x_{ij}^{*02}, x_{ij}^{*03}, x_{ij}^{*04}), i = 1 \ to \ m \ and \ j = 1 \ to \ n\}$ is non-positive and the total minimum fuzzy transportation cost
$\tilde{z}^* = \tilde{z}^*([\tilde{x}_{ij}^{*0}]) = (z_4^*([x_{ij}^{*04}]), z_3^*([x_{ij}^{*03}]), z_2^*([x_{ij}^{*02}]), z_1^*([x_{ij}^{*01}])) =$
$(z_4^*, z_3^*, z_2^*, z_1^*)$is positive because fuzzy supply at each origin and fuzzy demand at each destination, fuzzy transportation cost and fuzzy decision variables are non-positive with respect to the FTP (P*).
(3.7)
The optimal fuzzy solution $[\tilde{x}_{ij}^{0}]$ = $\{\tilde{x}_{ij}^{0} = (x_{ij}^{01}, x_{ij}^{02}, x_{ij}^{03}, x_{ij}^{04}), i = 1 \ to \ m \ and \ j = 1 \ to \ n\}$ is non-negative and the total minimum fuzzy transportation cost
$\tilde{z} = \tilde{z}([\tilde{x}_{ij}^{0}]) = (z_1([x_{ij}^{01}]), z_2([x_{ij}^{02}]), z_3([x_{ij}^{03}]), z_4([x_{ij}^{04}])) =$
$(z_1, z_2, z_3, z_4)$is positive because fuzzy supply at each origin and fuzzy demand at each destination, fuzzy transportation cost and fuzzy decision variables are non-negative with respect to the given FTP (P).

Here $\tilde{z}^* = \tilde{z} \Rightarrow z_1^* = z_4, z_2^* = z_3, z_3^* = z_2, z_4^* = z_1$.

3.1 Procedure for FTP using Zero Point Maximum Allocation Method

Step1: Construct a FTP (P) where fuzzy transportation cost ($\tilde{c}_{ij}$), fuzzy supply ($\tilde{a}_i$)and fuzzy demand ($\tilde{b}_j$)are interms of trapezoidal fuzzy numbers, where
$\tilde{c}_{ij} \geq 0, \tilde{a}_i \geq 0, \tilde{b}_j \geq 0, for\ i = 1\ to\ m\ and\ j = 1\ to\ n.$

Step2: Convert $\tilde{c}_{ij}$ to $\tilde{c}_{ij}^*$, $\tilde{a}_i$ to $\tilde{a}_i^*$ and $\tilde{b}_j$ to $\tilde{b}_j^*$, where$\tilde{c}_{ij}^* \leq 0, \tilde{a}_i^* \leq 0, \tilde{b}_j^* \leq 0$, for i = 1 to m and j = 1 to n.

Step3: The FTP (P^*) is now divided into four stages. The transportation cost c_{ij}^{*k} , supply $a_i^{*k}$and demand $b_j^{*k}$are considered for the four stages k = 1,2,3,4; i =1 to m and j = 1 to n.

Procedure for the first stage of FTP (It is in the form of crisp problem).

Step 4: Check whether the given problem (P_1^*) is a balanced one. If not convert it into a balanced one by introducing a dummy column / dummy row with cost entry as 0.

Step 5: Subtract each row entries of the transportation table $[c_{ij}^{*1}]$ by row maximum that is if u_i^{*1} is the maximum of the i^{th} row of the table $[c_{ij}^{*1}]$ then subtract the i^{th} row entries byu_i^{*1} , so that the resulting table is $[c_{ij}^{*1}$ - $u_i^{*1}]$.

Step 6: Subtract each column entries of the resulting transportation table after applying the step 5 by the column maximum that is if v_j^{*1} is the maximum of the j^{th} column of the resulting table $[c_{ij}^{*1}$ - $u_i^{*1}]$ then subtract j^{th} column entries by v_j^{*1} so that the resulting table is $[(c_{ij}^{*1}$ - $u_i^{*1})$ - $v_j^{*1}]$. It may be noted that $[(c_{ij}^{*1}$ - $u_i^{*1})$ - $v_j^{*1}] \leq 0$ for all i, j. Each row and each column of the resulting table $[(c_{ij}^{*1}$ - $u_i^{*1})$ - $v_j^{*1}]$ has atleast one '0' entry.

Step 7: Choose the row or column with only one '0' and allot the maximum of source and demand corresponding to that cell. Check whether the supply points are fully used and all the demand points are fully received. If so go to step 9. If not, go to step 8.

Step 8: Draw minimum number of lines horizontally and vertically to cover all the zeros. Then choose the largest uncovered element and subtract it from all the uncovered elements and add at the intersection of lines, leaving other elements unchanged. Now, check whether each row and each column has atleast one '0' entry. If so, go to step 7, else go the step 5,
step 6 and then to step 7.

Step 9: This allotment yields an optimal solution $x_{ij}^{*01}$to the first stage ofFTP (P_1^*)with the objective function given in the equation(3.6), subject to the constraints given in the equations (3.7) to (3.10).Now repeat step 4 to step 9 for the second, third and the fourth stages of FTP.

Finally, a) $\{\tilde{x}_{ij}^{*0} = \left(x_{ij}^{*01}, x_{ij}^{*02}, x_{ij}^{*03}, x_{ij}^{*04}\right), i = 1\ to\ m\ and\ j = 1\ to\ n\ \}$ is an optimal solution to the FTP (P^*) by the theorem 3.1. The total minimum fuzzy transportation cost is
$\tilde{z}^* = (z_4^*, z_3^*, z_2^*, z_1^*)$ with respect to the FTP (P^*).

b) Since$\tilde{x}_{ij}^0 = -\tilde{x}_{ij}^{*0}, \{\tilde{x}_{ij}^0 = \left(x_{ij}^{01}, x_{ij}^{02}, x_{ij}^{03}, x_{ij}^{04}\right), i = 1\ to\ m\ and\ j = 1\ to\ n\ \}$ is an optimal solution to the FTP (P) by the theorem 3.1. The total minimum fuzzy transportation cost is $\tilde{z} = (z_1, z_2, z_3, z_4)$ with respect to the given FTP (P).

Remark: An unbalanced fuzzy transportation problem can also be solved by the proposed method.

4. Numerical Example

Step 1: Consider the following fully fuzzy transportation problem.

Table 1: Fuzzy Transportation problem

	D_1	D_2	D_3	D_4	FS
S_1	(1,2,3,4)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(1,6,7,12)
S_2	(0,1,2,4)	(0,0,1,1)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
S_3	(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,17)
FD	(4,7,8,11)	(0,5,6,11)	(1,3,4,6)	(1,2,3,4)	(6,7,21,32)

Here S_1, S_2, S_3 are the sourcesand D_1, D_2, D_3, D_4 are the destinations. FS and FD are the fuzzy supply and the fuzzy demand points respectively.

Step 2:

Table 2

	D_1	D_2	D_3	D_4	FS
S_1	(-4,-3,-2,-1)	(-6,-4,-3,-1)	(-14,-12,-11,-9)	(-11,-8,-7,-5)	(-12,-7,-6,-1)
S_2	(-4,-2,-1,0)	(-1,-1,0,0)	(-8,-7,-6,-5)	(-3,-2,-1,0)	(-3,-2,-1,0)
S_3	(-8,-6,-5,-3)	(-12,-9,-8,-5)	(-19,-16,-15,-12)	(-12,-10,-9,-7)	(-17,-12,-10,-5)
FD	(-11,-8,-7,-4)	(-11,-6,-5,0)	(-6,-4,-3,-1)	(-4,-3,-2,-1)	

Step 3: Now, from the FTP (P^*), the problem (P_1^*), (P_2^*), (P_3^*), (P_4^*) are as follows:

Table 3

	D_1	D_2	D_3	D_4	Supply
S_1	-4	-6	-14	-11	-12
S_2	-4	-1	-8	-3	-3
S_3	-8	-12	-19	-12	-17
Demand	-11	-11	-6	-4	

Table 4

	D_1	D_2	D_3	D_4	Supply
S_1	-3	-4	-12	-8	-7
S_2	-2	-1	-7	-2	-2
S_3	-6	-9	-16	-10	-12
Demand	-8	-6	-4	-3	

Table 5

	D_1	D_2	D_3	D_4	Supply
S_1	-2	-3	-11	-7	-6
S_2	-1	0	-6	-1	-1
S_3	-5	-8	-15	-9	-10
Demand	-7	-5	-3	-2	

Table 6

	D_1	D_2	D_3	D_4	Supply
S_1	-1	-1	-9	-5	-1
S_2	0	0	-5	0	0
S_3	-3	-5	-12	-7	-5
Demand	-4	0	-1	-1	

First stage of FTP:
Step 4:

Table 7

	D_1	D_2	D_3	D_4	Supply
S_1	-4	-6	-14	-11	-12
S_2	-4	-1	-8	-3	-3
S_3	-8	-12	-19	-12	-17
Demand	-11	-11	-6	-4	-32

Here $\sum_{i=1}^{3} a_i^{*1} = \sum_{j=1}^{4} b_j^{*1} = -32$, it is a balanced TP in the first stage.

By applying step 5 and step 6 to table 7, we obtained table 8.

Table 8

	D_1	D_2	D_3	D_4	Supply
S_1	0	-2	-3	-5	-12
S_2	-3	0	0	0	-3
S_3	0	-4	-4	-2	-17
Demand	-11	-11	-6	-4	

By applying step 7 to table 8, we obtained table 9.

Table 9

	D_1	D_2	D_3	D_4	Supply
S_1	-11^*				-12
S_2		-3^*	*	*	-3
S_3	*				-17
Demand	-11	-11	-6	-4	

* denotes the places of 0's.
Here supply points are not fully used and the demand points are not fully received.

Applying step 8 to table 9, we obtained table 10.

Table 10

	D_1	D_2	D_3	D_4	Supply
S_1	-11^*	-1^*			-12
S_2		*	-3^*	*	-3
S_3	*			-4^*	-17

Demand	-11	-11	-6	-4	

Here also the supply points are not fully used and the demand points are not fully received. Therefore repeat the process until all the supply points are fully used and the demand points are fully received.

Table 11: The final optimal table for (P_1^*)

	D_1	D_2	D_3	D_4	Supply
S_1		-11*	-1*		-12
S_2			-3*		-3
S_3	-11*		-2*	-4*	-17
Demand	-11	-11	-6	-4	

(i) The optimal solution for (P_1^*) is $x_{12}^{*01} = -11, x_{13}^{*01} = -1, x_{23}^{*01} = -3, x_{31}^{*01} = -11,$
$x_{33}^{*01} = -2, x_{34}^{*01} = -4$ and the minimum transportation cost = 278.
From the second, third and the fourth stages, we obtain the following result:
(ii) The optimal solution of (P_2^*) is $x_{12}^{*02} = -6, x_{13}^{*02} = -1, x_{23}^{*02} = -2, x_{31}^{*02} = -8,$
$x_{33}^{*02} = -1, x_{34}^{*02} = -3$ and the minimum transportation cost = 144.
(iii) The optimal solution of (P_3^*) is $x_{12}^{*03} = -5, x_{13}^{*03} = -1, x_{23}^{*03} = -1, x_{31}^{*03} = -7,$
$x_{33}^{*03} = -1, x_{34}^{*03} = -2$ and the minimum transportation cost = 100.
(iv) The optimal solution of (P_4^*) is $x_{12}^{*04} = 0, x_{13}^{*04} = -1, x_{23}^{*04} = 0, x_{31}^{*04} = -4,$
$x_{33}^{*04} = 0, x_{34}^{*04} = -1$ and the minimum transportation cost = 28.
The optimal solutionto the FTP (P^*) is
$\tilde{x}_{12}^* = (-11, -6, -5, 0)$; $\tilde{x}_{13}^* = (-1, -1, -1, -1)$; $\tilde{x}_{23}^* = (-3, -2, -1, 0)$;
$\tilde{x}_{31}^* = (-11, -8, -7, -4)$; $\tilde{x}_{33}^* = (-2, -1, -1, 0)$; $\tilde{x}_{34}^* = (-4, -3, -2, -1)$ and the total minimum fuzzy transportation cost is (28,100,144,278).
Thus the optimal solution to the given FTP (P) is $\tilde{x}_{12} = (0,5,6,11)$; $\tilde{x}_{13} = (1,1,1,1)$;
$\tilde{x}_{23} = (0,1,2,3)$; $\tilde{x}_{31} = (4,7,8,11)$; $\tilde{x}_{33} = (0,1,1,2)$; $\tilde{x}_{34} = (1,2,3,4)$ and the total minimum fuzzy transportation cost is (28,100,144,278).

Results and Discussions

The optimal fuzzy solution and the optimal fuzzy objective value obtained under the proposed method coincide with the existing result of Pandian and Natarajan [14].

Shortcomings of Existing Methods

- Some problems provide crisp solutions for FTP and so fuzziness is violated [1,3,5,7,8,17].
- In some problems, the optimal solution of some of the fuzzy decision variables and the optimal objective fuzzy value of a FTP have negative part which depicts that quantity of the product and transportation cost may be negative. But the negative quantity of the product and negative transportation cost has no physical meaning [2,10,11,13,16].

Conclusions

One of the most important and successful applications of quantitative analysis to solving business problems has been in the physical distribution of products. Thus the study of FTP is essential. The procedure developed in this paper provides the optimal fuzzy solution and the optimal fuzzy objective value which are non-negative fuzzy numbers. Hence the method developed in this paper, serve as an important tool for the decision maker while handling the transportation problem under fuzzy environment.

References

[1] Darunee Hunwisai, and Poom Kumam,A method for solving a fuzzy transportation problem via Robust ranking technique and ATM, CogentMathematics, Vol.4, pp.1-11, 2017.

[2] Dinagar, D.S., and Palanivel, K., The transportation problem in fuzzy environment, International Journal of Algorithms, Computing and Mathematics, Vol.2, pp.65-71, 2009.

[3] Elizabeth, S., and Sujatha, L., k-Stage fuzzy transportation problem based on interval valued fuzzy numbers, International Journal of Scientific and Engineering Research, Vol.3, Issue 11, pp.155-162, 2012.

[4] Hitchcock, F.L., The distribution of a product from several sources to numerous localities, MIT Journal of Mathematics and Physics, Vol.20, pp.224-230, 1941.

[5] Jaikumar, K., New approach to solve fully fuzzy transportation problem, International Journal of Mathematics and its Applications, Vol.4, Issue 2, pp. 155-162, 2016.

[6] Liu, S.T., and Kao, C., Solving fuzzy transportation problems based on extension principle, European Journal of Operations Research, Vol.153, pp.661-674, 2004.

[7] Malini, P., and Ananthanarayanan, M., Solving fuzzy transportation problem using ranking of trapezoidal fuzzy numbers, International Journal of Mathematics Research, Vol.8, No.2, pp.127-132, 2016.

[8] Muruganandam, S., and Srinivasan, R., A new algorithm for solving fuzzy transportation problems with trapezoidal fuzzy numbers, International Journal of Recent Trends in Engineering and Research", Vol.2, Issue 03, pp.428-437, 2016.

[9] Nagoor Gani, A., and Abdul Razak, K., Two stage fuzzy transportation problem, Journal of Physical Sciences, Vol.10, pp.63-69, 2006.

[10] Narayanamoorthy, S., Saranya, S., and Maheswari, S., A method for solving fuzzy transportation problem (FTP) using fuzzy russell's method, International Journal of Intelligent Systems and Applications, Vol.02, pp.71-75, 2013.

[11] Nizam Uddin Ahmed, Aminur Rahman Khan, and Sharif Uddin, M.D., Solution of mixed type transportation problem:A fuzzy approach, Buletinul Institutului Politehnic Din Iasi, Publicat de Universitatea Tehnică, Gheorghe Asachi Din Iasi, Tomul LXI(LXV), pp.20-32, 2015.

[12] Oheigeartaigh, M., A fuzzy transportation algorithm, Fuzzy Sets and Systems, Vol.8, pp.235-243, 1982.

[13] Pandian, P., and Natarajan, G., A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems, Applied Mathematical Sciences, Vol.4, pp.79-90, 2010.

[14] Pandian. P., and Natarajan, G., An appropriate method for real life fuzzy transportation problems, International Journal of Information Sciences and Application, Vol.3, No.2, pp.127-134, 2011.

[15] Saad, O.M., and Abbas, S.A., A parametric study on transportation problem under fuzzy environment, The Journal of Fuzzy Mathematics, Vol.11, pp.115-124, 2003.

[16] Surjeet Singh Chauhan (Gonder), and Nidhi Joshi,Solution of fuzzy transportation problem using improved VAM with roubast ranking technique, International Journal of Computer Applications, Vol.82, No.15, pp.6-8, 2013.

[17] Vimala, S., and Krishna Prabha, S., Fuzzy transportation problem through Monalisha's approximation method, British Journal of Mathematics and Computer Science, Vol.17, No.2, pp.1-11, 2016.

[18] Zadeh, L.A., Fuzzy sets, Information and Control, Vol.8, pp.338-353, 1965.

[19] Zimmermann, H.J., Fuzzy programming and linear programming with several objective functions, Fuzzy Sets and Systems, Vol.1, pp.45-55, 1978.

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