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HESITANCY DOUBLE LAYERED AND TRIPLE LAYERED FUZZY GRAPH

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A B S T R A C T **RESEARCH ARTICLE**

In this paper we have introduced the hesitancy double layered fuzzy graph which has its basis in double layered fuzzy graph. We have also discussed about the degree, order and vertex of the hesitancy double layered fuzzy graph. We have extended the concept to hesitancy triple layered fuzzy graph.

Keywords:

Intuitionistic fuzzy graph, double layered fuzzy graph, triple layered fuzzy graph, hesitancy fuzzy graph, hesitancy double layered fuzzy graph, hesitancy triple layered fuzzy graph.

1. Introduction

Zadeh published his seminal paper on fuzzy sets in the year 1965 [13]. In 1975 Azriel Rosenfeld introduced fuzzy graph and several fuzzy similarities of graph theoretical concepts such as connectedness, cycles, paths, etc. [14]. Fuzzy line graphs and its properties where introduced by Mordeson in 1993 [11]. As a generalization of fuzzy sets Atanassov introduced the intuitionistics fuzzy set in 1986 [16]. In 2010 V. Tora developed hesitant fuzzy set [9]. T. Pathinathan and J. Jesintha Roseline introduced Double layered fuzzy graph in 2014 [1]. In 2015, T. Pathinathan and J. Jesintha Roseline developed the concept of intuitionistic double layered fuzzy graph and also they have discussed various properties of intuitionistic fuzzy graph such as its cartesian product and vertex degree [6]. In the year 2015 T. Pathinathan, and J. Jesintha Roseline extended the double layered fuzzy concept to triple layered fuzzy graph and studied its properties [7]. T. Pathinathan, J. Jon Arockiaraj and J. Jesintha Roseline derived hesitancy fuzzy graph and discussed its principle properties [8]. In this paper we have introduced hesitancy double layered fuzzy graph and give the necessary conditions to construct a hesitancy double layered fuzzy graph. Also we have discussed its properties such as order, vertex and degree. We have also introduced HTLFG and given an example. Further studies could be carried out in future.

2. Preliminaries

Definition 2.1 Let V be non-empty set. A fuzzy graph is a pair of functions $G:(\mu_1,\mu_2)$ where μ_1 is a fuzzy subset of V, μ_2 is a symmetric fuzzy relation on μ_1. i.e. $\mu_1 : V \to [0,1]$ and $\mu_2 : V\times V \to [0,1]$ such that $\mu_2(\mathrm{u},\mathrm{v}) \le \mu_1(\mathrm{u}) \wedge \mu_1(\mathrm{v})$ for all u, v in V. The underlying crisp graph $G:(\mu_1,\mu_2)$ is denoted as $G^*:(\mu_1^*,\mu_2^*)$ where μ_1^* is referred to as the non-empty set V of vertices and $\mu_2^* = E \subseteq V\times V$. The crisp graph (V,E) is a special case of the fuzzy graph G with each vertex and edge of (V,E) having degree of membership 1.

Definition 2.2 An intuitionistic fuzzy graph (IFG) is of the form $G:\langle V,E\rangle$ where

(i) $V=\{v_1,v_2,....,v_n\}$ such that $\mu_1 : V\to[0,1]$ and $\gamma_1 : V\to[0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i)+\gamma_1(v_i) \le 1 \quad \cdots(1)$ for every $v_i \in V$, ($i=1,2,..n$),

(ii) $E\subseteq V\times V$ where $\mu_2 : V\times V\to[0,1]$ and $\gamma_2 : V\times V\to[0,1]$ are such that

$$\mu_2(v_i,v_j) \le \mu_1(v_i)\wedge\mu_1(v_j) \qquad \cdots\cdots\cdots\cdots\cdots(2)$$

$$\gamma_2(v_i,v_j) \le \gamma_1(v_i)\vee\gamma_1(v_j) \qquad \cdots\cdots\cdots\cdots\cdots(3)$$

and $0 \le \mu_2(\mathrm{v}_i,\mathrm{v}_j)+\gamma_2(\mathrm{v}_i,\mathrm{v}_j) \le 1 \quad \cdots\cdots\cdots(4)$

for every $(\mathrm{v}_i,\mathrm{v}_j)\in \mathrm{E}$, $(i,j=1,2,....n)$.

2.2 Note

(i) The triple $(\mathrm{v}_i,\mu_{1i},\gamma_{1i})$ denotes the degree of membership and non-membership of the vertex v_i. The triple $(\mathrm{e}_{ij},\mu_{2ij},\gamma_{2ij})$ denotes the degree of membership and non-membership of the edge $e_{ij} = (v_i, v_j)$ on V. That is $\mu_{1i} = \mu_1(v_i), \gamma_{1i} = \gamma_1(v_i)$ and $\mu_{2ij} = \mu_2(v_i, v_j), \gamma_{2ij} = \gamma_2(v_i, v_j)$.

(ii) When $\mu_{2ij} = 0 = \gamma_{2ij}$, for some i and j, then there is no edge between v_i and v_j.

Definition 2.3 Intuitionistic Double Layered Fuzzy Graph (IDLFG):

Let $G: \langle (v_i, \mu_1, \gamma_1), (e_{ij}, \mu_2, \gamma_2) \rangle$ be an intuitionistic fuzzy graph with the underlying crisp graph $G^*: (\sigma^*, \mu^*)$. The pair $IDL(G): \langle (v_i, \mu_{DL1}, \gamma_{DL1}), (e_{ij}, \mu_{DL2}, \gamma_{DL2}) \rangle$ is called the intuitionistic fuzzy graph and is defined as follows. The node set of $IDL(G)$ be $\langle \mu_{DL1}, \gamma_{DL1} \rangle$. The fuzzy subset $\langle \mu_{DL1}(u), \gamma_{DL1}(u) \rangle$ is defined as

$$\langle \mu_{DL1}(u), \gamma_{DL1}(u) \rangle = \begin{cases} \langle \mu_1(u), \gamma_1(u) \rangle & \text{if } u \in \sigma^* \\ \langle \mu_2(uv), \gamma_1(uv) \rangle & \text{if } uv \in \mu^* \end{cases} \quad \text{where } 0 \le \langle \mu_{DL1} + \gamma_{DL1} \rangle \le 1$$

The fuzzy relation $\langle \mu_{DL2}, \gamma_{DL2} \rangle$ on $\sigma^* \cup \mu^*$ is defined as

$$\langle \mu_{DL2}(uv), \gamma_{DL2}(uv) \rangle = \begin{cases} \langle \mu_2(u,v), \gamma_2(u,v) \rangle & \text{if } u, v \in \sigma^* \\ \langle \mu_2(e_i) \wedge \mu_2(e_j), \gamma_2(e_i) \vee \gamma_2(e_j) \rangle & \text{if the edge } e_i \text{ and } e_j \text{ have a node in common between them} \\ \langle \mu_1(u_i) \wedge \mu_2(e_i), \gamma_1(u_i) \vee \gamma_2(e_i) \rangle & \text{if } u_i \in \sigma^* \text{ and } e_i \in \mu^* \text{ and each } e_i \text{ is incident with single } u_i \text{ either clock wise or anticlockwise} \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.4 Triple Layered Fuzzy Graph (TLFG):

Let $G: (\sigma, \mu)$ be a fuzzy graph with the underlying crisp graph $G^*: (\sigma^*, \mu^*)$. The pair $TL(G): (\sigma_{TL}, \mu_{TL})$ is defined as follows. The node set of $TL(G)$ be $\sigma^* \cup \mu^* \cup \mu^*$. The fuzzy subset

$$\sigma_{TL} = \sigma_{TL} = \begin{cases} \sigma(u) & \text{if } u \in \sigma^* \\ 2\mu(uv) & \text{if } uv \in \mu^* \end{cases}$$

The fuzzy relation μ_{TL} on $\sigma^* \cup \mu^*$ is defined as

$$\mu_{TL} = \begin{cases} \mu(uv) & \text{if } u, v \in \sigma^* \\ \mu(e_i) \wedge \mu(e_j) & \text{if the edge } e_i \text{ and } e_j \text{ have a node in common between them.} \\ \sigma(u_i) \wedge \mu(e_i) & \text{if } u_i \in \sigma^* \text{ and } e_i \in \mu^* \text{ and each } e_i \text{ is incident with } u_i \text{ in clockwise direction.} \\ \sigma(u_j) \wedge \mu(e_i) & \text{if } u_j \in \sigma^* \text{ and } e_i \in \mu^* \text{ and each } e_i \text{ is incident with } u_j \text{ in anticlockwise direction.} \\ 0 & \end{cases}$$

By definition, $\mu_{TL}(u,v) \le \sigma_{TL}(u) \wedge \sigma_{TL}(v)$ for all u,v in $\sigma^* \cup \mu^*$. Here μ_{TL} is a fuzzy relation on the fuzzy subset σ_{TL}. Hence the pair $TL(G): (\sigma_{TL}, \mu_{TL})$ is defined as triple layered fuzzy graph (TLFG).

Definition 2.5 A hesitancy fuzzy graph is of the form G = (V, E), where

(i) $V = \{v_1, v_2, \ldots v_n\}$ such that $\mu_1: V \to [0,1], \gamma_1: V \to [0,1]$ and $\beta_1: V \to [0,1]$ denote the degree of membership, non-membership and hesitancy of the element $v_i \in V$ respectively and $\mu_1(v_i) + \gamma_1(v_i) + \beta_1(v_i) = 1$ for every $v_i \in V$, where $\beta_1(v_i) = 1 - [\mu_1(v_i) + \gamma_1(v_i)] \quad \cdots\cdots(1)$ and

(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$, $\gamma_2: V \times V \to [0,1]$ and $\beta_2: V \times V \to [0,1]$ are such that,

$\mu_2\left(v_i, v_j\right) \le \min\left[\mu_1(v_i),\ \mu_1(v_j)\right]$ ……(2)

$\gamma_2\left(v_i, v_j\right) \le \max\left[\gamma_1(v_i),\ \gamma_1(v_j)\right]$ ……(3)

$\beta_2\left(v_i, v_j\right) \le \min\left[\beta_1(v_i),\ \beta_1(v_j)\right]$ ……(4)

and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) + \beta_2(v_i, v_j) \le 1$ for every $(v_i, v_j) \in E$.

Example: 2.3.1

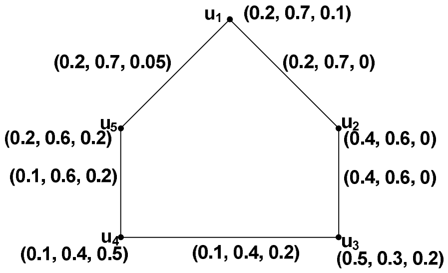


Figure 1: Hesitancy Fuzzy Graph.

3. Hesitancy Double Layered Fuzzy Graph:

Let $G: \left\langle (v_i, \mu_1, \gamma_1, \beta_1), (e_{ij}, \mu_2, \gamma_2, \beta_2) \right\rangle$ be hesitancy fuzzy graph with the underlying crisp graph $G^*: (\sigma^*, \mu^*)$.The pair $HDL(G): \left\langle (v_i, \mu_{DL1}, \gamma_{DL1}, \beta_{DL1}), (e_{ij}, \mu_{DL2}, \gamma_{DL2}, \beta_{DL2}) \right\rangle$ is called the hesitancy doubled layered fuzzy graph and is defined as follows. The vertex set of $HDL(G)$ be $\left\langle \mu_{DL1}, \gamma_{DL1}, \beta_{DL1} \right\rangle$, the fuzzy subset $\left\langle \mu_{DL1}(u), \gamma_{DL1}(u), \beta_{DL1}(u) \right\rangle$ is defined as

$\left\langle \mu_{DL1}(u), \gamma_{DL1}(u), \beta_{DL1}(u) \right\rangle = \begin{cases} \left\langle \mu_1(u), \gamma_1(u), \beta_1(u) \right\rangle & if\ u \in \sigma^* \\ \left\langle \mu_2(uv), \gamma_1(uv), \beta_1(uv) \right\rangle & if\ uv \in \mu^* \end{cases}$. where $\left\langle \mu_{DL1} + \gamma_{DL1} + \beta_{DL1} \right\rangle = 1$ then the fuzzy relation $\left\langle \mu_{DL2}, \gamma_{DL2}, \beta_{DL2} \right\rangle$ on $\sigma^* \cup \mu^*$ is defined as

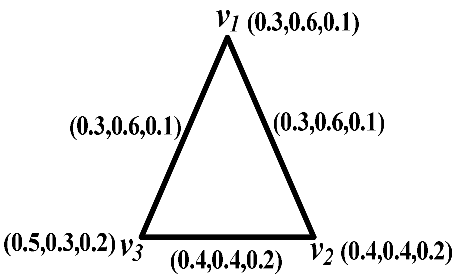
(i) $\left\langle \mu_{DL2}(uv), \gamma_{DL2}(uv), \beta_{DL2}(uv) \right\rangle = \left\langle \mu_2(u,v), \gamma_2(u,v), \beta_2(u,v) \right\rangle$ If $u, v \in \sigma^*$.

(ii) $\left\langle \mu_{DL2}(uv), \gamma_{DL2}(uv), \beta_{DL2}(uv) \right\rangle = \left\langle \mu_2(e_i) \wedge \mu_2(e_j), \gamma_2(e_i) \vee \gamma_2(e_j), \beta_2(e_i) \wedge \beta_2(e_j) \right\rangle$ if the edge e_i and e_j have common node between them.

(iii) $\left\langle \mu_{DL2}(uv), \gamma_{DL2}(uv), \beta_{DL2}(uv) \right\rangle = \left\langle \mu_1(u_i) \wedge \mu_2(e_i), \gamma_1(u_i) \vee \gamma_2(e_i), \beta_1(u_i) \wedge \beta_2(e_i) \right\rangle$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ and each e_i is incident with single u_i either clock wise or anticlockwise

(iv) $\left\langle \mu_{DL2}(uv), \gamma_{DL2}(uv), \beta_{DL2}(uv) \right\rangle = 0$ Otherwise.

Example: 3.1.1



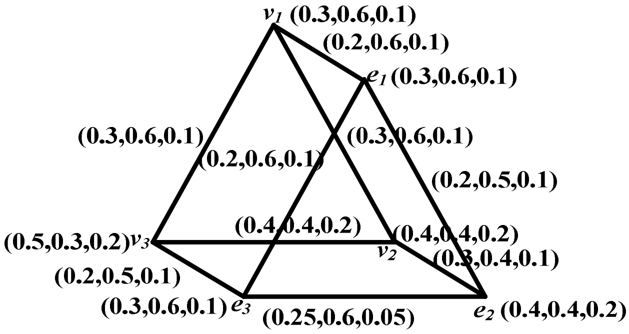
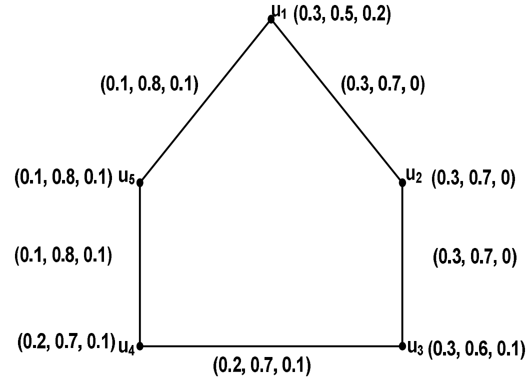


Figure 2: Hesitancy Double Layered Fuzzy Graph

In this example, by using the above mentioned definition we have obtained the new graph called Hesitancy double layered fuzzy graph. The vertex set $\langle \mu_{DL1},\ \gamma_{DL1}, \beta_{DL1} \rangle$ denote the degree of membership, non-membership and hesitancy of the element $v_i \in V$ and the edge set satisfies the condition $\langle \mu_{DL1} + \gamma_{DL1} + \beta_{DL1} \rangle = 1$.

Example: 3.1.2



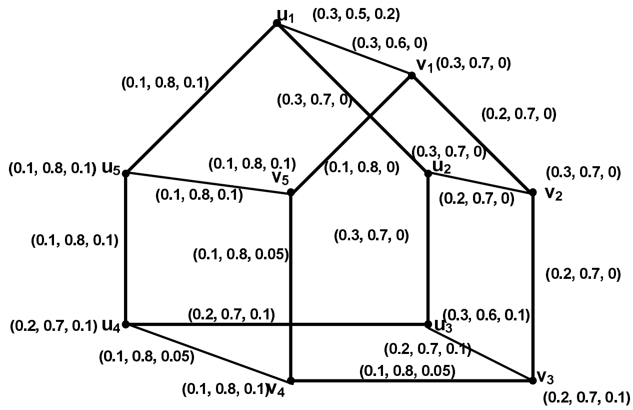


Figure 3: Hesitancy Double Layered Fuzzy Graph

The vertex set $\langle \mu_{DL1},\ \gamma_{DL1}, \beta_{DL1} \rangle$ denote the degree of membership, non-membership and hesitancy of the element $v_i \in V$ and the edge set satisfies the condition $\langle \mu_{DL1} + \gamma_{DL1} + \beta_{DL1} \rangle = 1$.

4. Order, Degree and Vertex of HDLFG:

Theorem 4.1 Order HDL (G) = Order (G) + Size (G), where G is an hesitancy fuzzy graph.

Proof:

As the vertex set of HDL (G) is $\sigma^* \cup \mu^*$ and the fuzzy subset $\langle \mu_{DL1}(\mathrm{u}),\ \gamma_{DL1}(\mathrm{u}),\ \beta_{DL1}(\mathrm{u}) \rangle$ on $\sigma^* \cup \mu^*$ is defined as

$$\langle \mu_{DL1}(\mathrm{u}),\ \gamma_{DL1}(\mathrm{u}),\ \beta_{DL1}(\mathrm{u}) \rangle = \begin{cases} \langle \mu_1(\mathrm{u}),\ \gamma_1(\mathrm{u}), \beta_1(\mathrm{u}) \rangle \ \text{if } \mathrm{u} \in \sigma^* \\ \langle \mu_2(\mathrm{uv}),\ \gamma_1(\mathrm{uv}), \beta_1(\mathrm{uv}) \rangle \ \text{if } \mathrm{uv} \in \mu^* \end{cases}$$

Order HDL (G)

$$= \sum_{u \in V \cup E} \langle \mu_{DL1}(\mathrm{u}),\ \gamma_{DL1}(\mathrm{u}),\ \beta_{DL1}(\mathrm{u}) \rangle$$

i.e. $= \sum_{u \in V} \sigma_{DL}(\mathrm{u}) + \sum_{u \in E} \sigma_{DL}(\mathrm{u}) =$

$$\sum_{u \in V} \langle \mu_1(\mathrm{u}),\ \gamma_1(\mathrm{u}),\ \beta_1(\mathrm{u}) \rangle + \sum_{u \in E} \langle \mu_2(\mathrm{uv}),\ \gamma_2(\mathrm{uv}),\ \beta_2(\mathrm{uv}) \rangle$$

= Order (G) + Size (G).

Example 4.1.1

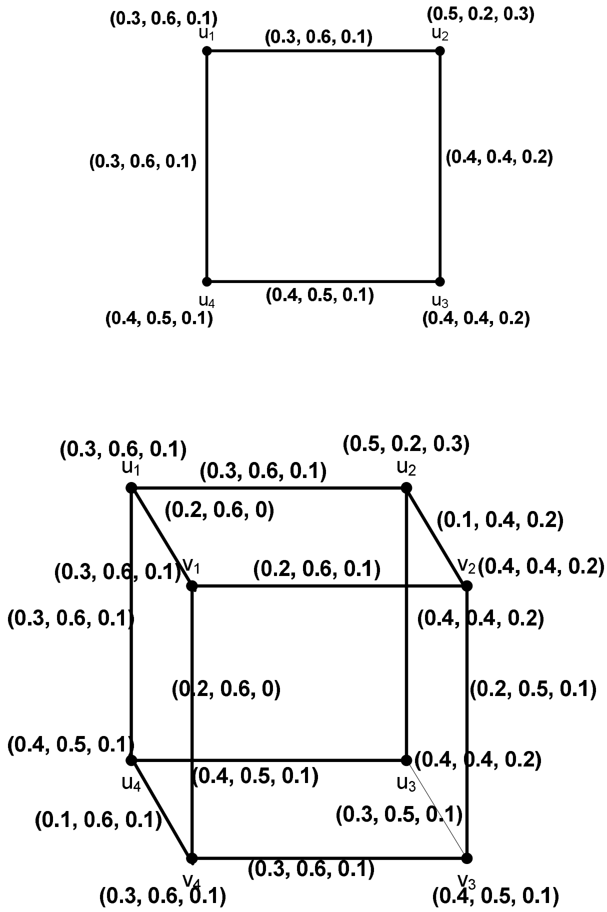


Figure 4: Hesitancy Double Layered Fuzzy Graph

Here in the parental fuzzy graph $G:(\sigma,\ \mu)$

Order (G) = (1.7, 1.7, 0.7)

Size (G) = (1.2, 2.1, 0.7)

Order HDL (G) = (2.9, 3.8, 1.4) = Order (G) + Size (G).

Theorem 4.2 Size HDL (G) = 2 Size (G) + $\sum_{e_i,e_j\in E}\left\langle \mu_2(\mathrm{e}_i)\wedge\mu_2(\mathrm{e}_j),\ \gamma_2(\mathrm{e}_i)\vee\gamma_2(\mathrm{e}_j),\ \beta_2(\mathrm{e}_i)\wedge\beta_2(\mathrm{e}_j)\right\rangle$

where G is a hesitancy fuzzy graph and $i, j \in N$.

Proof:

$$\textit{Size}\ \mathrm{HDL(G)}=\sum_{u,v\in V\cup E}\left\langle \mu_{DL2}(\mathrm{uv}),\gamma_{DL2}(\mathrm{uv}),\beta_{DL2}(\mathrm{uv})\right\rangle$$

$$=\sum_{u,v\in V}\left\langle \mu_2(\mathrm{uv}),\gamma_2(\mathrm{uv}),\beta_2(\mathrm{uv})\right\rangle+$$

$$\sum_{\mathrm{u}\in\mathrm{v},e_i\in E}\left\langle \mu_2(\mathrm{e}_i)\wedge\mu_2(\mathrm{e}_j),\gamma_2(\mathrm{e}_i)\vee\gamma_2(\mathrm{e}_j),\beta_2(\mathrm{e}_i)\wedge\beta_2(\mathrm{e}_j)\right\rangle$$

$$+$$

$$\sum_{\mathrm{u}\in\mathrm{v},e_i\in E}\left\langle \mu_1(\mathrm{u}_i)\wedge\mu_2(\mathrm{e}_j),\gamma_1(\mathrm{u}_i)\vee\gamma_2(\mathrm{e}_j),\beta_1(\mathrm{u}_i)\wedge\beta_2(\mathrm{e}_j)\right\rangle$$

$$=\sum_{u,v\in V}\left\langle \mu_2(\mathrm{uv}),\gamma_2(\mathrm{uv}),\beta_2(\mathrm{uv})\right\rangle+$$

$$\sum_{\mathrm{u}\in\mathrm{v},e_i\in E}\left\langle \mu_2(\mathrm{e}_i)\wedge\mu_2(\mathrm{e}_j),\gamma_2(\mathrm{e}_i)\vee\gamma_2(\mathrm{e}_j),\beta_2(\mathrm{e}_i)\wedge\beta_2(\mathrm{e}_j)\right\rangle$$

$$+$$

$$\sum_{\mathrm{u}\in\mathrm{v},e_i\in E}\left\langle \mu_2(\mathrm{e}_i),\gamma_2(\mathrm{e}_i),\beta_2(\mathrm{e}_i)\right\rangle$$

$\Rightarrow$ The membership value of $\sum_{\mathrm{u}\in\mathrm{v},e_i\in E}\left\langle \mu_2(\mathrm{e}_i),\gamma_2(\mathrm{e}_i),\beta_2(\mathrm{e}_i)\right\rangle$ will be less than the value of the

vertices then $\sum_{\mathrm{u}\in\mathrm{v},e_i\in E}\left\langle \mu_2(\mathrm{e}_i),\gamma_2(\mathrm{e}_i),\beta_2(\mathrm{e}_i)\right\rangle$ = Size (G)

$$=\sum_{u,v\in V}\left\langle \mu_2(\mathrm{uv}),\gamma_2(\mathrm{uv}),\beta_2(\mathrm{uv})\right\rangle$$

i.e. = Size (G) + $\sum_{\mathrm{u}\in\mathrm{v},e_i\in E}\left\langle \mu_2(\mathrm{e}_i)\wedge\mu_2(\mathrm{e}_j),\gamma_2(\mathrm{e}_i)\vee\gamma_2(\mathrm{e}_j),\beta_2(\mathrm{e}_i)\wedge\beta_2(\mathrm{e}_j)\right\rangle$

+ Size (G) = 2 Size (G) + $\sum_{\mathrm{u}\in\mathrm{v},e_i\in E}\left\langle \mu_2(\mathrm{e}_i)\wedge\mu_2(\mathrm{e}_j),\gamma_2(\mathrm{e}_i)\vee\gamma_2(\mathrm{e}_j),\beta_2(\mathrm{e}_i)\wedge\beta_2(\mathrm{e}_j)\right\rangle$

Hence proved.

Example 4.2.1

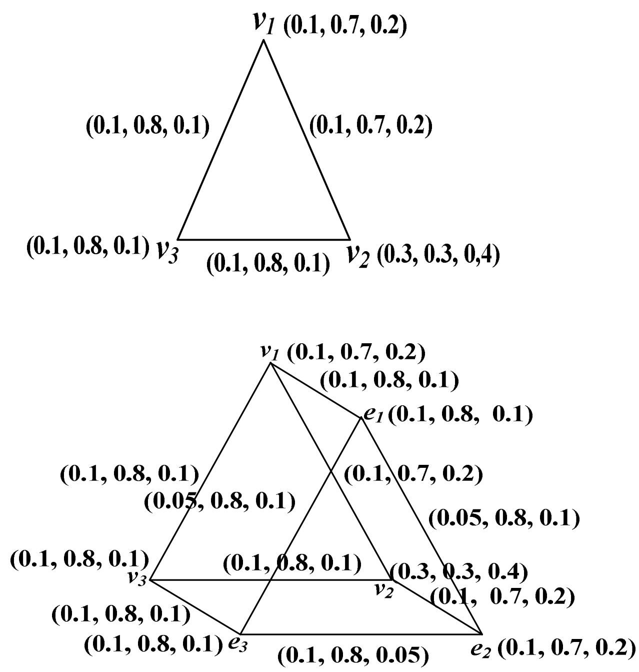


Figure 5: Hesitancy Double Layered Fuzzy Graph

In the parental fuzzy graph $G:(\sigma,\ \mu)$

Size (G) = $(0.3,\ 2.3,\ 0.4)$

$$\sum_{u\in v, e_i\in E}\left\langle \mu_2(e_i)\wedge\mu_2(e_j), \gamma_2(e_i)\vee\gamma_2(e_j), \beta_2(e_i)\wedge\beta_2(e_j)\right\rangle =$$

$(0.2,\ 2.35,\ 0.25)$

And $Size$ HDL(G) = $(0.8,\ 6.95,\ 1.05)$

$$= 2\,\text{Size}(G) + \sum_{u\in v, e_i\in E}\left\langle \mu_2(e_i)\wedge\mu_2(e_j), \gamma_2(e_i)\vee\gamma_2(e_j), \beta_2(e_i)\wedge\beta_2(e_j)\right\rangle$$

Theorem 4.3

Let G be an hesitancy fuzzy graph then

$$d_{HDL(G)}(u) = \begin{cases} d_G(u) + \left\langle \mu_1(u_i)\wedge\mu_2(e_i), \gamma_1(u_i)\vee\gamma_2(e_i), \beta_1(u_i)\wedge\beta_2(e_i)\right\rangle \ \text{if } u\in\sigma^* \\ \sum_{e_i, e_j\in\mu^*}\left\langle \mu_2(e_i)\wedge\mu_2(e_j), \gamma_2(e_i)\vee\gamma_2(e_j), \beta_2(e_i)\wedge\beta_2(e_j)\right\rangle + \left\langle \mu_1(u_i)\wedge\mu_2(e_i), \gamma_1(u_i)\vee\gamma_2(e_i), \beta_1(u_i)\wedge\beta_2(e_i)\right\rangle \end{cases}$$

Proof:

Let $d_G(u) = \left(\sum_{u\notin V}\mu_2(u,v), \sum_{u\notin V}\gamma_2(u,v), \sum_{u\notin V}\beta_2(u,v)\right)$

Case 1: Let $u\in\sigma^*$ then

$$d_{HDL(G)}(u) = \sum_{u\notin V}\left\langle \mu_{DL2}(u,v), \gamma_{DL2}(u,v), \beta_{DL2}(u,v)\right\rangle$$

$$= \sum_{u\notin V}\left\langle \mu_2(u,v), \gamma_2(u,v), \beta_2(u,v)\right\rangle +$$

$$\left\langle \mu_1(u_i)\wedge\mu_2(e_i), \gamma_1(u_i)\vee\gamma_2(e_i), \beta_1(u_i)\wedge\beta_2(e_i)\right\rangle$$

Since in $\sum_{u\notin V}\left\langle \mu_2(u,v), \gamma_2(u,v), \beta_2(u,v)\right\rangle$ the vertices which are adjacent in G are also adjacent in HDLFG.

$\Rightarrow d_{HDL(G)}(u) =$

$$d_G(u) + \left\langle \mu_1(u_i)\wedge\mu_2(e_i), \gamma_1(u_i)\vee\gamma_2(e_i), \beta_1(u_i)\wedge\beta_2(e_i)\right\rangle.$$

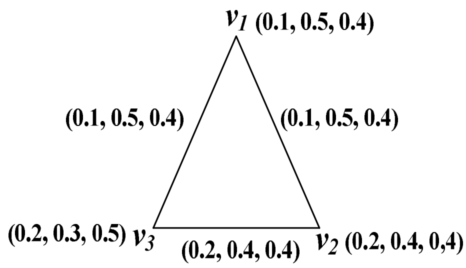
Case 2: Let $u\in\mu^*$ then

$$d_{HDL(G)}(u) = \sum_{u\notin V}\left\langle \mu_{DL2}(u,v), \gamma_{DL2}(u,v), \beta_{DL2}(u,v)\right\rangle$$

$$= \left\langle \mu_2(e_i)\wedge\mu_2(e_j), \gamma_2(e_i)\vee\gamma_2(e_j), \beta_2(e_i)\wedge\beta_2(e_j)\right\rangle + \mu_{DL}(u_i, e_i) =$$

$$\left\langle \mu_2(e_i)\wedge\mu_2(e_j), \gamma_2(e_i)\vee\gamma_2(e_j), \beta_2(e_i)\wedge\beta_2(e_j)\right\rangle + \left\langle \mu_1(u_i)\wedge\mu_2(e_i), \gamma_1(u_i)\vee\gamma_2(e_i), \beta_1(u_i)\wedge\beta_2(e_i)\right\rangle$$

Example 4.3.1



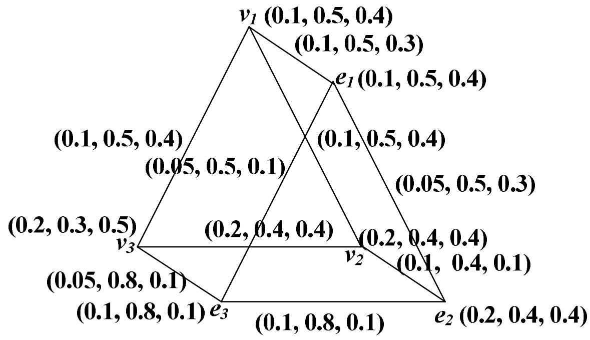


Figure 6: Hesitancy Double Layered Fuzzy Graph

Case 1:

Let $v_1\in\sigma_{HDL}$, since $v_1\in\sigma^*$ that is the node set of the parental graph G.

Then the degree $d_{HDL}(v_1) = (0.3,\ 1.5,\ 1.1)$, where $d(v_1) = (0.2,\ 1.0,\ 0.8)$ and

$$\left\langle \mu_1(u_i)\wedge\mu_2(e_i), \gamma_1(u_i)\vee\gamma_2(e_i), \beta_1(u_i)\wedge\beta_2(e_i)\right\rangle = (0.1,\ 0.5,\ 0.3),$$

$$d_{HDL}(v_1) = d(v_1) + \left\langle \mu_1(u_i)\wedge\mu_2(e_i), \gamma_1(u_i)\vee\gamma_2(e_i), \beta_1(u_i)\wedge\beta_2(e_i)\right\rangle$$

i.e. = $(0.2,\ 1.0,\ 0.8) + (0.1,\ 0.5,\ 0.3) = (0.3,\ 1.5,\ 1.1)$.

Case 2:

Let $e_1\in\mu_{HDL}$, since $e_1\in\mu^*$, the edge set of the parental graph G. Then the degree $d_{HDL}(e_1) = (0.2,\ 1.5,\ 0.7) =$

$$\left\langle \mu_2(e_i)\wedge\mu_2(e_j), \gamma_2(e_i)\vee\gamma_2(e_j), \beta_2(e_i)\wedge\beta_2(e_j)\right\rangle + \left\langle \mu_1(u_i)\wedge\mu_2(e_i), \gamma_1(u_i)\vee\gamma_2(e_i), \beta_1(u_i)\wedge\beta_2(e_i)\right\rangle.$$

5. Hesitancy Triple Layered Fuzzy Graph:

Let $G: \langle (v_i, \mu_1, \gamma_1, \beta_1), (e_{ij}, \mu_2, \gamma_2, \beta_2) \rangle$ be hesitancy fuzzy graph with the underlying crisp graph $G^*: (\sigma^*, \mu^*)$. The pair $HTL(G): \langle (v_i, \mu_{TL1}, \gamma_{TL1}, \beta_{TL1}), (e_{ij}, \mu_{TL2}, \gamma_{TL2}, \beta_{TL2}) \rangle$ is called the hesitancy triple layered fuzzy graph and is defined as follows. The vertex set of $HDL(G)$ be $\langle \mu_{TL1}, \gamma_{TL1}, \beta_{TL1} \rangle$, the fuzzy subset $\langle \mu_{TL1}(u), \gamma_{TL1}(u), \beta_{TL1}(u) \rangle$ is defined as

$$\langle \mu_{TL1}(u), \gamma_{TL1}(u), \beta_{TL1}(u) \rangle = \begin{cases} \langle \mu_1(u), \gamma_1(u), \beta_1(u) \rangle \text{ if } u \in \sigma^* \\ \langle 2\mu_2(uv), 2\gamma_1(uv), 2\beta_1(uv) \rangle \text{ if } uv \in \mu^* \end{cases}.$$

where $\langle \mu_{TL1} + \gamma_{TL1} + \beta_{TL1} \rangle = 1$ then the fuzzy relation $\langle \mu_{TL2}, \gamma_{TL2}, \beta_{TL2} \rangle$ on $\sigma^* \cup \mu^*$ is defined as

(i)
$$\langle \mu_{TL2}(uv), \gamma_{TL2}(uv), \beta_{TL2}(uv) \rangle = \langle \mu_2(u,v), \gamma_2(u,v), \beta_2(u,v) \rangle \text{ if } u, v \in \sigma^*.$$

(ii)
$$\langle \mu_{TL2}(uv), \gamma_{TL2}(uv), \beta_{TL2}(uv) \rangle = \langle \mu_2(e_i) \wedge \mu_2(e_j), \gamma_2(e_i) \vee \gamma_2(e_j), \beta_2(e_i) \wedge \beta_2(e_j) \rangle$$

if the edge e_i and e_j have common node between them.

(iii)
$$\langle \mu_{TL2}(uv), \gamma_{TL2}(uv), \beta_{TL2}(uv) \rangle = \langle \mu_1(u_i) \wedge \mu_2(e_i), \gamma_1(u_i) \vee \gamma_2(e_i), \beta_1(u_i) \wedge \beta_2(e_i) \rangle$$

if $u_i \in \sigma^*$ and $e_i \in \mu^*$ and each e_i is incident with single u_i either clock wise or anticlockwise

(iv)
$$\langle \mu_{TL2}(uv), \gamma_{TL2}(uv), \beta_{TL2}(uv) \rangle = \langle \mu_1(u_j) \wedge \mu_2(e_i), \gamma_1(u_j) \vee \gamma_2(e_i), \beta_1(u_j) \wedge \beta_2(e_i) \rangle$$

if $u_j \in \sigma^*$ and $e_i \in \mu^*$ and each e_i is incident with single u_i either clock wise or anticlockwise

(v) $\langle \mu_{TL2}(uv), \gamma_{TL2}(uv), \beta_{TL2}(uv) \rangle = 0$ Otherwise.

Example 5.1

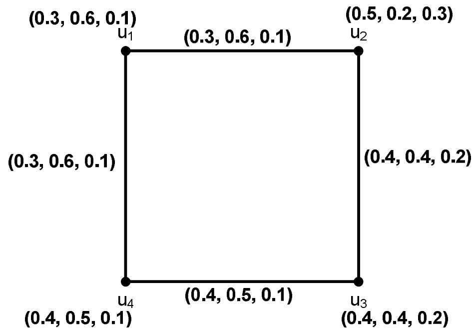


Figure 7: Hesitancy Triple Layered Fuzzy Graph

In this example, by using the above mentioned definition we have obtained the new graph called Hesitancy Triple layered fuzzy graph. The vertex set $\langle \mu_{DL1}, \gamma_{DL1}, \beta_{DL1} \rangle$ denote the degree of membership, on-membership and hesitancy of the element $v_i \in V$ and the edge set satisfies the condition $\langle \mu_{DL1} + \gamma_{DL1} + \beta_{DL1} \rangle = 1$.

Note 5.2

Order, degree and size of hesitancy triple layered fuzzy graph can be derived by using the condition defined for hesitancy double layered fuzzy graph.

Conclusion:

In this paper we have introduced hesitancy double layered fuzzy graph and triple layered fuzzy graph. We have also discussed about its order, degree and size. This can be further extended to study the condition of hesitancy double layered fuzzy graph to be balanced.

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