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PARTIAL PRODUCT PROCESSES AND REPLACEMENT PROBLEM

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A B S T R A C T **RESEARCH ARTICLE**

In this paper, we introduce a partial product process which is a sequence of non-negative random variables $X_1, X_2, ..., X_n$ such that the distribution function of X_1 is $F(x)$ and the distribution function of X_{i+1} is $F(\beta_i x)$ $(i = 1, 2, 3, ..., n-1)$ where $\beta_i > 0$ are constants and $\beta_i = \beta_0 \beta_1 \beta_2 ... \beta_{i-1}$. Then based on this process maintenance model for a deteriorating system under policy N is discussed. An explicit expression for the long run average cost per unit time is derived and numerical example is given.

1. Introduction

In the maintenance problem of repairable system, initially a common assumption that after repair, the system is "as good as new" is used. Then minimal repair model was introduced by Barlow in which a system after repair would function again but with the same failure rate and the same effective age as at the time of failure. In practical, many systems are deteriorating due to ageing effect and accumulated wear. Accordingly, the consecutive operating times after repair will be decreasing while the corresponding repair times after failure will be increasing. Consequently, a monotone process model would be a most appropriate model for a deteriorating system.

In this paper we introduce a partial product process and prove that it is a monotone process and study its application to maintenance model for a deteriorating system under policy N.

2. Partial Product Process

At first, we recall some definitions.

Definition 2.1.

Given two random variables X and Y , X is said to be stochastically larger than Y (or Y is stochastically less than X) if

$P(X > \alpha) \geq P(Y > \alpha)$ for all real α.

This is written as $X \geq_{st} Y \; or \; Y \leq_{st} X.$

Definition 2.2.

A stochastic process $\{X_n, n = 1, 2, 3, ...\}$ is said to be stochastically increasing (decreasing) if

$X_n \leq_{st} (\geq_{st}) X_{n+1}$ for all $n = 1, 2, 3, ...$

We now give the definition of partial product process.

Definition 2.3.

Let $\{X_n, n = 1, 2, 3, ...\}$ be a sequence of non-negative independent random variables and let $F(x)$ be the distribution function of X_1. Then $\{X_n, n = 1, 2, 3, ...\}$ is called a partial product process if the distribution function of X_{i+1} is $F(\beta_i x)$ $(i = 1, 2, 3, ..., n-1)$ where $\beta_i > 0$ are constants and $\beta_i = \beta_0 \beta_1 \beta_2 ... \beta_{i-1}$.

Lemma 2.1.

For real $\beta_i (i = 1, 2, 3, ...)$, $\beta_i = \beta_0^{2^{i-1}}$.

Proof. The proof is by induction on i.

When $i = 1$, $\beta_1 = \beta_0 = \beta_0^{2^{i-1}}$.

Thus, the result is true for $i = 1$.
Assume that the result is true for $i = n$.

$i.e \; \beta_n = \beta_0^{2^{n-1}}$ is true.

Then we have to prove that the result is true for $i = n + 1$.

$$\begin{aligned}
\beta_{n+1} &= (\beta_0 \beta_1 \beta_2 ... \beta_{n-1}) \beta_n \\
&= \beta_n \beta_n \\
&= \beta_n^2 \\
&= \left(\beta_0^{2^{n-1}}\right)^2 \qquad \left(\because \beta_n = \beta_0^{2^{n-1}}\right) \\
&= \beta_0^{2\left(2^{n-1}\right)} \\
&= \beta_0^{2^{(n+1)-1}}
\end{aligned}$$

Thus, the result is true for $i = n+1$ also.

By Lemma 1, the distribution function of X_{i+1} is $F\left(\beta_0^{2^{i-1}} x\right)$ for $i = 1,2,3,...,n-1$.

Lemma 2.2.

Givena partial product process $\{X_n, n = 1,2,3,...\}$,

(i)If $\beta_0 > 1$, then $\{X_n, n = 1,2,3,...\}$ is stochastically decreasing.

(ii) If $0 < \beta_0 < 1$, then $\{X_n, n = 1,2,3,...\}$ is stochastically increasing.

Proof. Let $\beta_0 > 1$.

Note that for any $\alpha \geq 0$,

$$F(\alpha) \leq F(\beta_0 \alpha) \leq F\left(\beta_0^2 \alpha\right) \leq ...F\left(\beta_0^{2^{n-1}} \alpha\right)$$

$$\Rightarrow P(X_1 > \alpha) \geq P(X_2 > \alpha) \geq P(X_3 > \alpha) \geq ... \geq P(X_n > \alpha)$$

This implies $\{X_n, n = 1,2,3,...\}$ is stochastically decreasing if $\beta_0 > 1$.

Similarly, $\{X_n, n = 1,2,3,...\}$ is stochastically increasing if $0 < \beta_0 < 1$.

Now, we can define the monotone partial product process.

Definition 2.4

A partial product process is called a decreasing partial product process if $\beta_0 > 1$, and is called increasing partial product process if $0 < \beta_0 < 1$.

It is clear that if $\beta_0 = 1$ then the partial product process is a renewal process.

Lemma 2.3

Let $E(X_1) = \lambda, Var(X_1) = \sigma^2$. Then for $i = 1,2,3,...,n-1$,

$$E(X_{i+1}) = \frac{\lambda}{\beta_0^{2^{i-1}}} \text{ and } Var(X_{i+1}) = \frac{\sigma^2}{\beta_0^{2^i}}.$$

Proof. By Lemma 1, the density function of X_{i+1} is $\beta_0^{2^{i-1}} f\left(\beta_0^{2^{i-1}} x\right)$ for $i = 1,2,3,...,n-1$.

Then,

$$E(X_{i+1}) = \int x\left[\beta_0^{2^{i-1}} f\left(\beta_0^{2^{i-1}} x\right)\right] dx$$

$$= \beta_0^{2^{i-1}} \int x f\left(\beta_0^{2^{i-1}} x\right) dx$$

$$Put\ y = \beta_0^{2^{i-1}} x$$

$$\Rightarrow dy = \beta_0^{2^{i-1}} dx$$

Then,

$$E(X_{i+1}) = \beta_0^{2^{i-1}} \int \left(\frac{y}{\beta_0^{2^{i-1}}}\right) f(y) \left(\frac{dy}{\beta_0^{2^{i-1}}}\right)$$

$$= \frac{1}{\beta_0^{2^{i-1}}} \int y f(y) dy$$

$$= \frac{1}{\beta_0^{2^{i-1}}} E(X_1)$$

$$\Rightarrow E(X_{i+1}) = \frac{\lambda}{\beta_0^{2^{i-1}}}, \quad (i = 1,2,3,...,n-1)$$

Similarly, $E\left(X_{i+1}^2\right) = \dfrac{E\left(X_1^2\right)}{\beta_0^{2^i}}$.

$$\therefore Var(X_{i+1}) = E\left(X_{i+1}^2\right) - E(X_{i+1})^2$$

$$= \frac{E\left(X_1^2\right)}{\beta_0^{2^i}} - \left(\frac{E(X_1)}{\beta_0^{2^{i-1}}}\right)^2$$

$$= \frac{E\left(X_1^2\right)}{\beta_0^{2^i}} - \frac{E(X_1)^2}{\beta_0^{2^i}}$$

$$= \frac{Var(X_1)}{\beta_0^{2^i}} = \frac{\sigma^2}{\beta_0^{2^i}}$$

3. The Model

We consider a maintenance model for a deteriorating system under the following assumptions.

(1) Initially a new system is installed. Whenever the system fails, it may be repaired or replaced by a new and identical one.

(2) Let X_1 be the operating time before the 1st failure and let $F(x)$ be the distribution function of X_1. Let X_{i+1} be the operating time after the i^{th} repair for $i = 1,2,3,...,n-1$. Then the distribution function of X_{i+1} is assumed to be $F\left(\beta_0^{2^{i-1}} x\right)$ where $\beta_0 \geq 1$ is a constant. That is the successive operating times $\{X_n, n = 1,2,3,...\}$ after repair constitute a decreasing partial product process or a renewal process. Also, assume that $E(X_1) = \lambda > 0$.

(3) Let Y_1 be the repair time after the 1st failure and let $G(y)$ be the distribution function of Y_1. For $i = 1,2,3,...,n-1$, let Y_{i+1} be the repair time after the

$(i+1)^{th}$ failure. Then the distribution function of Y_{i+1} is assumed to be $G\left(\gamma_0^{2^{i-1}} x\right)$ where $0 < \gamma_0 \le 1$ is a constant. That is the consecutive repair times $\{Y_n, n = 1,2,3,...\}$ form an increasing partial product process or a renewal process. Moreover, assume that $E(Y_1) = \mu \ge 0$. $\mu = 0$ means that the repair time is negligible.

(4) Let Z be the replacement time with $E(Z) = \tau$.

(5) The repair cost rate is c, the reward rate is r and the replacement cost is R.

4. The replacement policy N

Definition 4.1.

A replacement policy N is a policy in which we replace the system at the N[th] failure of the system.

Our aim is to find an optimal replacement T^* such that the long-run average cost per unit time is minimized.

By the renewal reward theorem, the long-run average cost per unit time under the replacement policy T is given by

$$C(N) = \frac{\text{the expected cost incurred in a cycle}}{\text{the expected length of a cycle}}$$

$$= \frac{E\left(c\sum_{k=1}^{N-1} Y_k + R - r\sum_{k=1}^{N} X_k\right)}{E\left(\sum_{k=1}^{N} X_k + \sum_{k=1}^{N-1} Y_k + Z\right)}$$

$$= \frac{c\sum_{k=1}^{N-1} E(Y_k) + R - r\sum_{k=1}^{N} E(X_k)}{\sum_{k=1}^{N} E(X_k) + \sum_{k=1}^{N-1} E(Y_k) + E(Z)}$$

$$= \frac{c\left[E(Y_1) + \sum_{k=2}^{N-1} E(Y_k)\right] + R - r\left[E(X_1) + \sum_{k=2}^{N} E(X_k)\right]}{E(X_1) + \sum_{k=2}^{N} E(X_k) + E(Y_1) + \sum_{k=2}^{N-1} E(Y_k) + \tau}$$

$$= \frac{c\left[\mu + \sum_{k=2}^{N-1} \frac{\mu}{\gamma_0^{2^{k-1}}}\right] + R - r\left[\lambda + \sum_{k=2}^{N} \frac{\lambda}{\beta_0^{2^{k-1}}}\right]}{\lambda + \sum_{k=2}^{N} \frac{\lambda}{\beta_0^{2^{k-1}}} + \mu + \sum_{k=2}^{N-1} \frac{\mu}{\gamma_0^{2^{k-1}}} + \tau}$$

$$= \frac{\left(c\mu\left[1 + \sum_{k=2}^{N-1} \frac{1}{\gamma_0^{2^{k-1}}}\right] + R - \lambda r\left[1 + \sum_{k=2}^{N} \frac{1}{\beta_0^{2^{k-1}}}\right]\right)}{\left(\lambda\left[1 + \sum_{k=2}^{N} \frac{1}{\beta_0^{2^{k-1}}}\right] + \mu\left[1 + \sum_{k=2}^{N-1} \frac{1}{\gamma_0^{2^{k-1}}}\right] + \tau\right)} + r - r$$

$$= \frac{\left((c+r)\mu\left[1 + \sum_{k=2}^{N-1} \frac{1}{\gamma_0^{2^{k-1}}}\right] + R + r\tau\right)}{\left(\lambda\left[1 + \sum_{k=2}^{N} \frac{1}{\beta_0^{2^{k-1}}}\right] + \mu\left[1 + \sum_{k=2}^{N-1} \frac{1}{\gamma_0^{2^{k-1}}}\right] + \tau\right)} - r$$

Thus, $C(N) = D(N) - r$ where

$$D(N) = \frac{\left((c+r)\mu\left[1 + \sum_{k=2}^{N-1} \frac{1}{\gamma_0^{2^{k-1}}}\right] + R + r\tau\right)}{\left(\lambda\left[1 + \sum_{k=2}^{N} \frac{1}{\beta_0^{2^{k-1}}}\right] + \mu\left[1 + \sum_{k=2}^{N-1} \frac{1}{\gamma_0^{2^{k-1}}}\right] + \tau\right)}$$

We can determine the optimal replacement policy N* by minimizing $C(N)$ or $D(N)$.

5. Numerical Example

Let $\beta_0 = 1.05, \gamma_0 = 0.95, R = 5000, \lambda = 40,$
$\mu = 15, c = 10, r = 50 \; and \; \tau = 10$

Then

$$C(N) = \frac{\left(900\left[1+\sum_{k=2}^{N-1}\frac{1}{0.95^{2^{k-1}}}\right] + 5000 + 500\right)}{\left(40\left[1+\sum_{k=2}^{N}\frac{1}{1.05^{2^{k-1}}}\right] + 15\left[1+\sum_{k=2}^{N-1}\frac{1}{0.95^{2^{k-1}}}\right] + 10\right)} - 50$$

The numerical results are presented in Table 5.1 and Figure 5.1 respectively.

Table 5.1 Values of N and $C(N)$

N	$C(N)$	N	$C(N)$
1	60.8021390	7	-3.88837109
2	13.19041758	8	3.42414105
3	-0.94993124	9	9.56279092
4	-6.68762557	10	9.99934153
5	**-8.44271604**	11	10.00000001
6	-7.44278906	12	10.00000000

It is clear from the above table, the minimum value of $C(N)$occurs at N*=5.

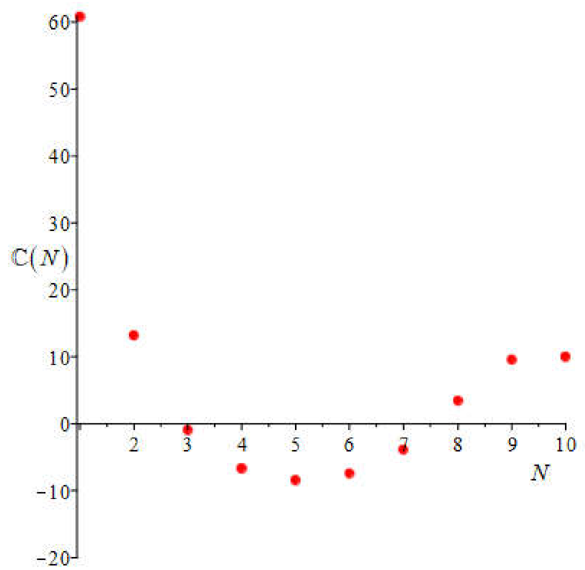

Figure 5.1 The graph of $C(N)$against N

Conclusion

In this paper, we have introduced a partial product process and showed that it is a monotone process. By considering a maintenance model for a deteriorating system, an explicit expression for the long-run average cost per unit time under the replacement policy N is derived. A numerical example is given to explain the methodology used.

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