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TRIPOLAR FUZZY GRAPHS

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A B S T R A C T **RESEARCH ARTICLE**

In this Paper, we introduce the idea of Tripolar fuzzy graph, expand various method of the signification, dispute the concept of isomorphisms of these graphs and investigate some of their important properties. We then introduce the notation of strong Tripolar fuzzy graph and study some properties. We also discuss some propositions on self complementary and strong Tripolar fuzzy graph.

Keywords:

Tripolar fuzzy graph, Strong Tripolar fuzzy graph, Self complementary, morphisms.

1. Introduction

In 1975, Rosenfeld [46] introduced the concept of fuzzy graphs. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs.The complement of a fuzzy graph was defined by Mordeson and Nair [35] and further studied by Sunitha and Vijayakumar [48].In 1965, Zadeh [52] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty. In 1994, Zhang [57,58] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is [-1,1]. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree (0,1] of an element indicates that the element somewhat satisfies the property, and the membership degree [1,0) of an element indicates that the element somewhat satisfies the implicit counter-property. Although bipolar fuzzy sets and intuitionistic fuzzy sets look similar to each other, they are essentially different sets [28]. In many domains, it is important to be able to deal with bipolar information. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. This domain has recently motivated new research in several directions. The complement of a fuzzy graph was defined by Mordeson and Nair [35] and further studied by Sunitha and Vijayakumar [48]. Bhutani and Rosenfeld introduced the concept of M-strong fuzzy graphs in [10] and studied some of their properties. The concept of strong arcs in fuzzy graphs was discussed in [12]. Recently, Akram [2] has introduced the notion of cofuzzy graphs and investigated several of their properties. Shannon and Atanassov [48] introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs, and investigated some of their properties in [49].

In this paper, Mr.J.Jon Arockiaraj and N.ObedIssac introduce the notion of TPFG describe various methods of their construction, discuss the concept of isomorphism of these graphs, and investigate some of their important properties. We then introduce the notion of strong TPFG and study some of their properties.

2. Preliminaries

Definition 2.1. A graph is an ordered pair G*= (V,E), where V is the set of vertices of G*and E is the set of edges of G* Two vertices x and y in an undirected graphG*are said to be adjacent in G*if {x,y} is an edge of G*. A simple graph is an undirected graph that has no loops and no more than one edge between any two different vertices.

Definition 2.2: Consider the Cartesian product $G^* = G_1^* \times G_2^* = (V, E)$ of graphs G_1^* and G_2^*. Then $V = V_1 \times V_2$ and $E = \{(x,x_2)(x,y_2) | x_1 \in V_1, x_2 y_2 \in E_2\}$ □ $\{(x_1,z)(y_1,z) | \in V_2, x_1 y_1 \in E_1\}$.

Definition 2.3: Let $G_1^* = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple graphs. Then, the composition of graph G_1^* with G_2^* is denoted by $G_1^*[G_2^*] = (V_1 \times V_2, E^0)$, where $E^0 = E \cup \{(x_1,x_2)(y_1,y_2) | x_1 y_1 \in E_1, x_2 \neq y_2\}$ and E is defined in $G_1^* \times G_2^*$. Note that $G_1^*[G_2^*] \neq G_2^*[G_1^*]$.

Definition 2.4: The union of two simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ is the simple graph with the vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2$. The union of G_1^* and G_2^* is denoted by $G^* = G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$.

Definition 2.5: The join of two simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ is the simple graph with the vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2 \cup E'$, where E' is the set of all edges joining the nodes of V_1 and V_2 and assume that $V_1 \cap V_2 \neq \emptyset$,. The join of G_1 and G_2 is denoted by $G = G_1) G_2 = (V_1 \cup V_2, E_1 \cup E_2 \cup E')$.

Definition 2.6: An isomorphism of the graphs G_1^* and G_2^* is a bijection between the vertex sets of G_1^* and G_2^* such that any two vertices v_1 and v_2 of G_1^* are adjacent in G_1^* if and only if $f(v_1)$ and $f(v_2)$ are adjacent in G_2^*. If an isomorphism exists between two graphs, then the graphs are called isomorphic and we write $G_1^* \approx G_2^*$. An automorphism of a graph is a graph isomorphism with itself, i.e., a mapping from the vertices of the given graph G^* back to vertices of G^* such that the resulting graph G^* is isomorphic with G^*.

Definition 2.7: The complementary graph $\bar{G}^*$ of a simple graph has the same vertices as G^*. Two vertices are adjacent in $\bar{G}^*$if and only if they are not adjacent in G^*.

Definition 2.8: A fuzzy subset μ on a set X is a map $\mu : X \rightarrow [0,1]$. A map $v : X \times X \rightarrow [0,1]$ is called a fuzzy relation on X if $v(x,y) \leq \min(\mu(x),\mu(y))$ for all x, y ϵX. A fuzzy relation m is symmetric if $v(x,y) = v(y,x)$ for all x, y ϵX.

Definition 2.9: Let X be a nonempty set. A TPF set B in X is an object having the form
$B = \{(x, \mu^P(x), \mu^N(x), \mu^{\square}(x)) \mid x \in X / \mu^{\square}(x)= \mu^P(x)+\mu^N(x)$, where □is P or N},
where $\mu^P : X \rightarrow [0,1]$ and $\mu^N : X \rightarrow [-1,0]$ and $\mu^{\square}: X \rightarrow [-1,1]$ are mappings.

We use the positive membership degree $\mu^P(x)$ to denote the satisfaction degree of an element x to the property corresponding to a Tripolar fuzzy set B, and the negative membership degree $\mu^N(x)$ to denote the satisfaction degree of an element x to some implicit counter-property corresponding to a Tripolar fuzzy set B and the positive or negative degree $\mu^{\square}(x)$ to denote the satisfaction degree of an element x to some properties corresponding to a Tripolar fuzzy set B.If $\mu^P(x) \neq 0$ and $\mu^N(x) = 0$ and $\mu^{\square}(x) = 0$,it is the situation that x is regarded as having only positive satisfaction for B. If $\mu^P(x) =0$ and $\mu^N(x) \neq 0$ and $\mu^{\square}(x) = 0$,it is the situation that x does not satisfy the property of B but somewhat satisfies the counter properties of B. If $\mu^P(x) =0$ and $\mu^N(x) = 0$ and $\mu^{\square}(x) \neq 0$, it is the situation that x is satisfy the some properties of B.

It is possible for an element x to be such that $\mu^P(x) \neq 0$ and $\mu^N(x) \neq 0$ and $\mu^{\square}(x) \neq 0$, when the membership function of the property overlaps that of its counter property over some portion of X. For the sake of simplicity, we shall use the symbol $B = (\mu^P, \mu^N, \mu^{\square})$for the Tripolar fuzzy set, $B = \{(x, \mu^P(x), \mu^N(x), \mu^{\square}(x)) \mid x \in X \}$, where $\mu^{\square}(x)=\mu^P(x)+\mu^N(x)$ and□is P or N.

Definition 2.10: For every two TPF sets $A =(\mu^P_A, \mu^N_A, \mu^{\square})$and $B =(\mu^P_B,\mu^N_B \mu^{\square})$ in X, we define

1. $(A \cap B)(x) = (\min(\mu^P_A(x),\mu^P_B(x)), \max(\mu^N_A(x),\mu^N_B(x)), \text{minmax}(\mu^P_A(x),\mu^N_B(x)))$.
2. $(A \cup B)(x) = (\max(\mu^P_A(x),\mu^P_B(x)), \min(\mu^N_A(x),\mu^N_B(x)), \text{maxmin}(\mu^N_A(x),\mu^P_B(x)))$.

Definition 2.11: Let $A=(\mu^P_A ,\mu^N_A ,\mu^{\square}_A)$ and $B=(\mu^P_B,\mu^N_B,\mu^{\square}_B)$ be Tripolar fuzzy sets on a set X.
If $A=(\mu^P_A , \mu^N_A ,\mu^{\square})$is a TPF relation on a set X, then $A=(\mu^P_A ,\mu^N_A ,\mu^{\square}_B)$is called a TPF relation on
$B =(\mu^P_B,\mu^N_B \mu^{\square}_B)$ if

$\mu^P_A(x, y) \leq \min(\mu^P_B(x), \mu^P_B(y))$ and --------(1)

$\mu^N_A(x, y) \geq \max(\mu^N_B(x), \mu^N_B(y))$and --------(2)

$\mu^{\square}_A(x, y) \gtreqless \text{minmax}(\mu^P_B(x), \mu^N_B(y))$--------(3)for all x, y ϵX.
{Since, $\mu^P_A(x, y)$ and $\mu^N_A(x, y)$ is Zero.

From (1) and (2) We have,
$\Rightarrow \min(\mu^P_B(x), \mu^P_B(y)) \geq 0 \geq \max(\mu^N_B(x), \mu^N_B(y))$,
$\Rightarrow \mu^P_A(y) \geq \text{minmax}(\mu^P_B(x), \mu^N_B(y)) \geq \mu^N_A(x)$,
$\Rightarrow \mu^P_A(y), \mu^N_A(x) \geq \text{minmax}(\mu^P_B(x), \mu^N_B(y))$,
$\Rightarrow \mu_A^{\square}(x, y) \gtreqless \text{minmax}(\mu^P_B(x), \mu^N_B(y))$-----(3)}
A Tripolar fuzzy relation A on X is called symmetric if
$\mu^P_A(x, y) = \mu^P_A(y, x)$ and
$\mu^N_A(x, y) \gtreqless \mu^N_A(y, x)$ andfor all x, y X.

Throughout this paper, G^* will be a crisp graph, and G a TPFG.

3.Tripolar Fuzzy Graphs

Definition 3.1. A TPFG with a underlying set V is defined to be a pair G = (A,B) where $A = (\mu^P_A,\mu^N_A,\mu^{\square}_A)$is a TPF set in V and $B = (\mu^P_B,\mu^N_B, \mu^{\square}_B)$is a TPF set in E ⊆ V xV such that

$\mu^P_B(\{x,y\}) \leq \min(\mu^P_A(x), \mu_A^P(y))$ and $\mu^N_B(\{x,y\}) \geq \max(\mu^N_A(x), \mu_A^N(y))$

$\mu^{\square}_B(\{x,y\}) \gtreqless \text{minmax}(\mu^P_A(x),\mu^N_A(y))$ for all {x,y} ϵE.
We call A the TPF vertex set of V, B the TPFedge set of E, respectively. Note that B is a symmetric TPFrelation on A. We use the notation xy for an element of E. Thus, G = (A,B) is a TPFG of $G^* = (V,E)$ if
$\mu^P_B(xy) \leq \min(\mu^P_A(x), \mu^P_A(y))$ and
$\mu^N_B(xy) \geq \max(\mu^N_A(x), \mu^N_A(y))$ and
$\mu^{\square}_B(xy) \gtreqless \text{minmax}(\mu^P_B(x), \mu^N_B(y))$ for all xy ϵE.

Example 3.1: Suppose a graph G^*=(V,E)such that V={x,y,z},E={xy,yz,zx}. Let $A=(\mu^P_A,\mu^N_A,\mu^{\square}_A)$ be a TPF subset of V and let $B = (\mu^P_B,\mu^N_B, \mu^{\square}_B)$ be a TPF subset of E ⊆ V xV defined by

	x	y	z
μ^P_A	0.6	0.3	0.5
μ^N_A	-0.4	-0.6	-0.7
$\mu^{\square}_A$	0.2	- 0.3	-0.2

	xy	yz	zx
μ^P_A	0.2	0.2	0.3
μ^N_A	-0.3	-0.4	-0.2
$\mu^{\square}_A$	-0.1	- 0.2	0.1

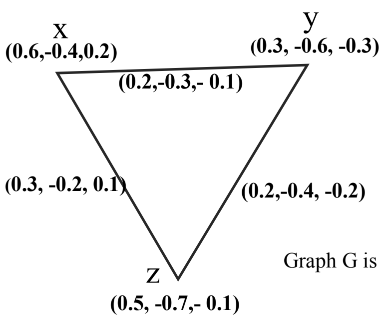


Fig:3.1
Graph G is a Tripolar fuzzy graph

Definition 3.2 Let $A_1 = (\mu^P_{A1}, \mu^N_{A1}, \mu^{\square}_{A1})$ and $A_2 = (\mu^P_{A2}, \mu^N_{A2}, \mu^{\square}_{A2})$ be Tripolar fuzzy subsets of V_1 and V_2 and let $B_1 = (\mu^P_{B1}, \mu^N_{B1}, \mu^{\square}_{B1})$ and $B_2 = (\mu^P_{B2}, \mu^N_{B2}, \mu^{\square}_{B2})$ be Tripolar fuzzy subsets of E_1 and E_2, respectively. Then, we denote the Cartesian product of two Tripolar fuzzy graphs G_1 and G_2 of the graphs G_1 and G_2 by $G_1 \times G_2$ $(A_1 \times A_2, B_1 \times B_2)$, and define as follows:

(i) $(\mu^P_{A1} \times \mu^P_{A2})(x_1,x_2) = \min(\mu^P_{A1}(x_1),\mu^P_{A2}(x_2))$
$(\mu^N_{A1} \times \mu^N_{A2})(x_1,x_2) = \max(\mu^N_{A1}(x_1),\mu^N_{A2}(x_2))$
$(\mu^{\square}_{A1} \times \mu^{\square}_{A2})(x_1,x_2) = \text{minmax}(\mu^{\square}_{A1}(x_1),\mu^{\square}_{A2}(x_2))$ for all $(x_1,x_2) \in V$,
(ii) $(\mu^P_{B1} \times \mu^P_{B2})(x,x_2)(x,y_2) = \min(\mu^P_{A1}(x),\mu^P_{B2}(x_2y_2))$
$(\mu^N_{B1} \times \mu^N_{B2})(x,x_2)(x,y_2) = \max(\mu^N_{A1}(x),\mu^N_{B2}(x_2y_2))$
$(\mu^{\square}_{B1} \times \mu^{\square}_{B2})(x,x_2)(x,y_2) = \text{minmax}(\mu^{\square}_{A1}(x),\mu^{\square}_{B2}(x_2y_2))$ for all $x \in V1$, for all $(x_2,y_2,) \in E_2$
(iii) $(\mu^P_{A1} \times \mu^P_{A2})(x_1,z)(y_1,z) = \min(\mu^P_{B1}(x_1,y_1),\mu^P_{A2}(z))$
$(\mu^N_{A1} \times \mu^N_{A2})(x_1,z)(y_1,z) = \max(\mu^N_{B1}(x_1,y_1),\mu^N_{A2}(z))$
$(\mu^{\square}_{B1} \times \mu^{\square}_{B2})(x_1,z)(y_1,z) = \text{minmax}(\mu^{\square}_{B1}(x_1,y_1),\mu^{\square}_{A2}(z))$ for all $(x_1,y_1) \in E_1$

Proposition 3.1: If G_1 and G_2 are the Tripolar fuzzy graphs,then $G_1 \times G_2$ is a Tripolar fuzzy graph.

Proof: Let $x \in V_1$, $x_2y_2 \in E_2$. Then we have

$$(\mu^P_{B1}\times\mu^P_{B2})((x,x_2)(x,y_2)) = \min(\mu_A{}^P{}_1(x),\mu_B{}^P{}_2(x_2y_2)$$
$$\leq \min(\mu^P_{A1}(x),\min(\mu^P_{A2}(x_2),\mu^P_{A2}(y_2))$$
$$= \min(\min(\mu_A{}^P{}_1(x),\mu^P_{A2}(x_2)), \min(\mu_A{}^P{}_1(x),\mu_A{}^P{}_2(y_2)))$$
$$= \min((\mu_A{}^P{}_1\times\mu_A{}^P{}_1)(x,x_2),(\mu_A{}^P{}_1\times\mu_A{}^P{}_1)(x,y_2))$$
$$(\mu^N_{B1}\times\mu^N_{B2})((x,x_2)(x,y_2)) = \max(\mu^N_{A1}(x),\mu^N_{B2}(x_2y_2)$$
$$\geq \max(\mu^N_{A1}(x),\max(\mu^N_{A2}(x_2),\mu^N_{A2}(y_2)))$$
$$= \max(\max(\mu^N_{A1}(x),\mu^N_{A2}(x_2)), \max(\mu^N_{A1}(x),\mu^N_{A2}(y_2)))$$
$$= \max((\mu^N_{A1}\times\mu^N_{A1})(x,x_2),(\mu^N_{A1}\times\mu_A{}^N{}_1)(x,y_2)))$$
$$(\mu^{\square}_{B1}\times\mu^{\square}_{B2})((x,x_2)(x,y_2)) = \text{minmax}((\mu_A{}^{\square}{}_1(x),\mu_B{}^{\square}{}_2(x_2y_2))$$
$$\gtrless \text{minmax}(\mu^{\square}_{A1}(x),\text{minmax}(\mu^{\square}_{A2}(x_2),\mu^{\square}_{A2}(y_2)))$$
$$=\text{minmax}(\max(\mu_A{}^{\square}{}_1(x),\mu^{\square}_{A2}(x_2)),\text{minmax}(\mu_A{}^{\square}{}_1(x),\mu_A{}^{\square}{}_2(y_2)))$$
$$=\text{minmax}((\mu_A{}^{\square}{}_1\times\mu_A{}^{\square}{}_1)(x,x_2),(\mu_A{}^{\square}{}_1\times\mu_A{}^{\square}{}_1)(x,y_2))$$

Let $z \in V_2$, $x_1y_1 \in E_1$. Then, we have

$$(\mu^P_{B1}\times\mu^P_{B2})((x_1,z)(y_1,z)) = \min(\mu_B{}^P{}_1(x_1y_1),\mu_A{}^P{}_2(z))$$
$$\leq \min(\min(\mu^P_{A1}(x_1),\mu^P_{A1}(y_1)),\mu_A{}^P{}_2(z))$$
$$= \min(\min(\mu^P_{A1}(x_1),\mu^P_{A2}(z)), \min(\mu_A{}^P{}_1(y_1),\mu_A{}^P{}_2(z))$$
$$= \min((\mu^P_{A1}\times\mu_A{}^P{}_2)(x_1,z),(\mu_A{}^P{}_1\times\mu_A{}^P{}_2)(y_1,z))$$
$$(\mu^N_{B1}\times\mu^N_{B2})((x_1,z)(y_1,z)) = \max(\mu_B{}^N{}_1(x_1y_1),\mu_A{}^N{}_2(z))$$
$$\geq \max(\max(\mu^N_{A1}(x_1),\mu^N_{A1}(y_1)), \mu_A{}^N{}_2(z))$$
$$= \max(\max(\mu^N_{A1}(x_1),\mu^N_{A2}(z)), \max(\mu_A{}^N 1(y_1),\mu_A{}^N{}_2(z)))$$
$$= \max((\mu^N_{A1}(x_1)\times\mu_A{}^N{}_2)(x_1,z),(\mu_A{}^N{}_1\times\mu_A{}^N{}_2)(y_1,z))$$
$$(\mu^{\square}_{B1}\times\mu^{\square}_{B2})((x_1,z)(y_1,z)) = \text{minmax}(\mu_B{}^{\square}{}_1(x_1y_1),\mu_A{}^{\square}{}_2(z))$$
$$\gtrless \text{minmax}(\text{minmax}(\mu^{\square}_{A1}(x_1),\mu^{\square}_{A1}(y_1)),\mu^{\square}_{A2}(z))$$
$$=\text{minmax}(\text{minmax}(\mu^{\square}_{A1}(x_1),\mu^{\square}_{A2}(z)),\text{minmax}(\mu^{\square}_{A1}(y_1),\mu^{\square}_{A2}(z)))$$
$$= \text{minmax}((\mu^{\square}_{A1}\times\mu^{\square}_{A2})(x_1,z),(\mu^{\square}_{A1}\times\mu^{\square}_{A2})(y_2,z))$$

This completes the proof. □

Definition 3.3

$A_{1=}(\mu^P_{A1}\circ\mu^N_{A1}\circ\mu^{\square}_{A1})$ and $A_{2=}(\mu^P_{A2}\circ \mu^N_{A2}\circ\mu^{\square}_{A2})$ be Tripolar fuzzy subsets of V1 and V_2 and let $B_{1=}(\mu^P_{B1}\circ \mu^N_{B1}\circ\mu^{\square}_{A1})$and $B_{2=}(\mu^P_{B2}\circ\mu^N_{B2}\circ\mu^{\square}_{A1})$ be Tripolar fuzzy subsets of E_1 and E_2, respectively. Then, we denote the composition of two TPFG G_1 and G_2 of the graphs G_1 and G_2 by $G_1[G_2] = (A_1 \circ A_2, B_1 \circ B_2)$ and define as follows:

(i) $(\mu^P_{A1} \circ \mu^P_{A2})(x_1,x_2) = \min(\mu^P_{A1}(x_1),\mu^P_{A2}(x_2))$
$(\mu^N_{A1} \circ \mu^N_{A2})(x_1,x_2) = \max(\mu^N_{A1}(x_1),\mu^N_{A2}(x_2))$
$(\mu^{\square}_{A1} \circ \mu^{\square}_{A2})(x_1,x_2) = \text{minmax}(\mu^{\square}_{A1}(x_1),\mu^{\square}_{A2}(x_2))$for all $(x_1,x_2) \in V$,
(ii) $(\mu^P_{B1} \circ \mu^P_{B2})(x,x_2)(x,y_2) = \min(\mu^P_{A1}(x),\mu^P_{B2}(x_2y_2))$
$(\mu^N_{B1} \circ \mu^N_{B2})(x,x_2)(x,y_2) = \max(\mu^N_{A1}(x),\mu^N_{B2}(x_2y_2))$
$(\mu^{\square}_{B1} \circ \mu^{\square}_{B2})(x,x_2)(x,y_2) = \text{minmax}(\mu^{\square}_{A1}(x),\mu^{\square}_{B2}(x_2y_2))$for all $x \in V_1$, for all $(x_2,y_2,) \in E_2$,
(iii) $(\mu^P_{A1} \circ \mu^P_{A2})(x_1,z)(y_1,z) = \min(\mu^P_{B1}(x_1,y_1),\mu^P_{A2}(z))$
$(\mu^N_{A1} \circ \mu^N_{A2})(x_1,z)(y_1,z) = \max(\mu^N_{B1}(x_1,y_1),\mu^N_{A2}(z))$
$(\mu^{\square}_{B1} \circ \mu^{\square}_{B2})(x_1,z)(y_1,z) = \text{minmax}(\mu^{\square}_{B1}(x_1,y_1),\mu^{\square}_{A2}(z))$for all $z \in V_2$,for all $(x_1,y_1) \in E_1$,
(iv) $(\mu^P_{A1} \circ \mu^P_{A2})(x_1,x_2)(y_1,y_2) = \min(\mu^P_{A2}(x_2), \mu^P_{A2}(y_2), \mu^P_{A2}(z_2),\mu^P_{B1}(x_1y_1))$
$(\mu^N_{A1} \circ \mu^N_{A2})(x_1,x_2)(y_1,y_2) = \max(\mu^P_{A2}(x_2), \mu^P_{A2}(y_2), \mu^P_{A2}(z_2),\mu^P_{B1}(x_1y_1))$
$(\mu^{\square}_{B1} \circ \mu^{\square}_{B2})(x_1,x_2)(y_1,y_2) = \text{minmax}(\mu^P_{A2}(x_2),\mu^P_{A2}(y_2),\mu^P_{A2}(z_2),\mu^P_{B1}(x_1y_1))$ forall $(x_1,x_2)(y_1,y_2) \in E^{\circ} - E$.

Proposition 3.2 If G_1 and G_2 are Tripolar Fuzzy Graphs, Then $G_1[G_2]$ is a Tripolar graph.

Proof: Let $x \in V_1$, $x_2y_2 \in E_2$. Then we have

$$(\mu^P_{B1}\circ\mu^P_{B2})((x,x_2)(x,y_2)) = \min(\mu_A{}^P{}_1(x),\mu_B{}^P{}_2(x_2y_2))$$
$$\le \min(\mu^P_{A1}(x),\min(\mu^P_{A2}(x_2),\mu^P_{A2}(y_2))$$
$$= \min(\min(\mu_A{}^P{}_1(x),\mu^P_{A2}(x_2)), \min(\mu_A{}^P{}_1(x),\mu_A{}^P{}_2(y_2)))$$
$$= \min((\mu_A{}^P{}_1\circ\mu_A{}^P{}_1)(x,x_2),(\mu_A{}^P{}_1\circ\mu_A{}^P{}_1)(x,y_2))$$
$$(\mu^N_{B1}\circ\mu^N_{B2})((x,x_2)(x,y_2)) = \max(\mu^N_{A1}(x),\mu^N_{B2}(x_2y_2)$$
$$\ge \max(\mu^N_{A1}(x),\max(\mu^N_{A2}(x_2),\mu^N_{A2}(y_2)))$$
$$= \max(\max(\mu^N_{A1}(x),\mu^N_{A2}(x_2)), \max(\mu^N_{A1}(x),\mu^N_{A2}(y_2)))$$
$$= \max((\mu^N_{A1}\circ\mu^N_{A1})(x,x_2),(\mu^N_{A1}\circ\mu_A{}^N{}_1)(x,y_2)))$$
$$(\mu^{\square}_{B1}\circ\mu^{\square}_{B2})((x,x_2)(x,y_2)) = \text{minmax}((\mu_A{}^{\square}{}_1(x),\mu_B{}^{\square}{}_2(x_2y_2))$$
$$\gtrless \text{minmax}(\mu^{\square}_{A1}(x),\text{minmax}(\mu^{\square}_{A2}(x_2),\mu^{\square}_{A2}(y_2)))$$
$$= \text{minmax}(\max(\mu_A{}^{\square}{}_1(x),\mu^{\square}_{A2}(x_2)), \text{minmax}(\mu_A{}^{\square}{}_1(x),\mu_A{}^{\square}{}_2(y_2)))$$
$$= \text{minmax}((\mu_{A1}{}^{\square}\circ\mu_{A1}{}^{\square})(x,x_2),(\mu_{A1}{}^{\square}\circ\mu_{A1}{}^{\square})(x,y_2))$$

Let $z \in V_2$, $x_1y_1 \in E_1$. Then, we have

$$(\mu^P_{B1}\circ\mu^P_{B2})((x_1,z)(y_1,z)) = \min(\mu_B{}^P{}_1(x_1y_1),\mu_A{}^P{}_2(z))$$
$$\le \min(\min(\mu^P_{A1}(x_1),\mu^P_{A1}(y_1)),\mu_A{}^P2(z))$$
$$= \min(\min(\mu^P_{A1}(x_1),\mu^P_{A2}(z)), \min(\mu_A{}^P{}_1(y_1),\mu_A{}^P{}_2(z))$$
$$= \min((\mu^P_{A1}\circ\mu_A{}^P{}_2)(x_1,z),(\mu_A{}^P{}_1\circ\mu_A{}^P{}_2)(y_1,z))$$
$$(\mu^N_{B1}\circ\mu^N_{B2})((x_1,z)(y_1,z)) = \max(\mu_B{}^N{}_1(x_1y_1),\mu_A{}^N{}_2(z))$$
$$\ge \max(\max(\mu^N_{A1}(x_1),\mu^N_{A1}(y_1)), \mu_A{}^N{}_2(z))$$
$$= \max(\max(\mu^N{}_A1(x_1),\mu^N{}_A2(z)), \max(\mu_A{}^N1(y_1),\mu_A{}^N2(z)))$$
$$= \max((\mu^N{}_A1(x_1)\circ\mu_A{}^N2)(x_1,z),(\mu_A{}^N1\circ\mu_A{}^N2)(y_1,z))$$
$$(\mu^{\square}_{B1}\circ\mu^{\square}_{B2})((x_1,z)(y_1,z)) = \text{minmax}(\mu_B{}^{\square}{}_1(x_1y_1),\mu_A{}^{\square}{}_2(z))$$
$$\gtrless \text{minmax}(\text{minmax}(\mu^{\square}_{A1}(x_1),\mu^{\square}_{A1}(y_1)),\mu^{\square}_{A2}(z))$$
$$= \text{minmax}(\text{minmax}(\mu^{\square}_{A1}(x_1),\mu^{\square}_{A2}(z)), \text{minmax}(\mu^{\square}_{A1}(y_1),\mu^{\square}_{A2}(z)))$$
$$= \text{minmax}((\mu^{\square}_{A1}\circ\mu^{\square}_{A2})(x_1,z),(\mu^{\square}_{A1}\circ\mu^{\square}_{A2})(y_2,z))$$

This completes the proof. □

Definition 3.4

$A_{1=}(\mu^P_{A1},\mu^N_{A1},\mu^{\square}_{A1})$ and $A_{2=}(\mu^P_{A2},\mu^N_{A2},\mu^{\square}_{A2})$ be Tripolar fuzzy subsets of V1 and V_2 and Let $B_{1=}(\mu^P_{B1},\mu^N_{B1},\mu^{\square}_{B1})$and $B_{2=}(\mu^P_{B2},\mu^N_{B2},\mu^{\square}_{B1})$ be TPF subsets of $E_1$and E_2, respectively. Then, we denote the composition of two TPFG G_1 and G_2 of the graphs $G^*{}_1$ and $G^*{}_2$ by $G_1UG_2 = (A_1UA_2, B_1UB_2)$ and define as follows:

A) (i)
$$(\mu^P_{A1}U\mu^P_{A2})(x) = \mu^P_{A1}(x) \quad \text{if } x \in V_1\cap \overline{V}_2,$$
$$(\mu^P_{A1}U\,\mu^P_{A2})(x) = \mu^P_{A2}(x) \quad \text{if } x \in V_1\cap \overline{V}_2,$$
$$(\mu^P_{A1}\,U\,\mu^P_{A2}(x) = \max(\mu^P_{A1}(x),\mu^P_{A2}(x))\text{if } x \in V_1\cap V_2.$$

(ii)
$$(\mu^N_{A1}\cap\mu^N_{A2})(x) = \mu^N_{A1}(x) \quad \text{if } x \in V_1\cap \overline{V}_2,$$
$$(\mu^N_{A1}\cap\mu^N_{A2})(x) = \mu^N_{A2}(x) \quad \text{if } x \in V_1\cap \overline{V}_2,$$
$$(\mu^N_{A1}\cap\mu^N_{A2}(x) = \min(\mu^N_{A1}(x),\mu^N_{A2}(x)) \quad \text{if } x \in V_1\cap V_2.$$

(iii)
$$(\mu^{\square}_{A1}\,U\,\mu^{\square}_{A2})(x) = \mu^{\square}_{A1}(x) \quad \text{if } x \in V_1\cap \overline{V}_2,$$
$$(\mu^{\square}_{A1}\,U\,\mu^{\square}_{A2})(x) = \mu^{\square}_{A2}(x) \quad \text{if } x \in V_2\cap\overline{V}_1,$$
$$(\mu^{\square}_{A1}\,U\,\mu^{\square}_{A2}(x) = \text{maxmin}(\mu^N_{A1}(x),\mu^N_{A2}(x)) \quad \text{if } x\in V_1\cap V_2.$$

(iv)
$$(\mu^{\square}_{A1}\cap\mu^{\square}_{A2})(x) = \mu^{\square}_{A1}(x) \quad \text{if} x \in V_1\cap \overline{V}_2,$$
$$(\mu^{\square}_{A1}\cap\mu^{\square}_{A2})(x) = \mu^{\square}_{A2}(x) \quad \text{if } x \in \overline{V_1\cap} V_2,$$
$$(\mu^{\square}_{A1}\cap\mu^{\square}_{A2}(x) = \text{minmax}(\mu^N_{A1}(x),\mu^N_{A2}(x)) \quad \text{if } x\in V_1\cap V_2.$$

B) (i)
$$(\mu^P_{B1}U\mu^P_{B2})(xy) = \mu^P_{B1}(xy) \quad \text{if } xy \in E_1\cap\overline{E}_2,$$
$$(\mu^P_{B1}U\,\mu^P_{B2})(xy) = \mu^P_{B2}(xy) \quad \text{if } xy \in E_1\cap\overline{E}_2,$$
$$(\mu^P_{B1}\,U\,\mu^P_{B2}(xy) = \max(\mu^P_{B1}(xy),\mu^P_{B2}(xy)) \quad \text{if } xy \in E_1\cap E_2.$$

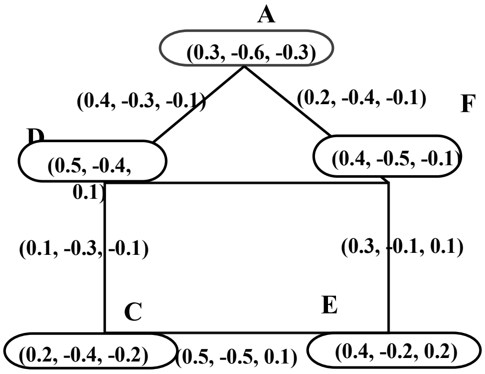
(ii)
$$(\mu^N_{B1}U\mu^N_{B2})(xy) = \mu^N_{B1}(xy) \quad \text{if } xy \in E_1\cap\overline{E}_2,$$
$$(\mu^N_{B1}U\,\mu^N_{B2})(xy) = \mu^N_{B2}(xy) \quad \text{if } xy \in E_1\cap\overline{E}_2,$$
$$(\mu^N_{B1}\,U\,\mu^N_{B2}(xy) = \min(\mu^N_{B1}(xy),\mu^N_{B2}(xy)) \quad \text{if } xy \in E_1\cap E_2.$$

(iii)
$$(\mu^{\square}_{A1}\,U\,\mu^{\square}_{A2})(xy) = \mu^{\square}_{A1}(xy) \quad \text{if } xy \in E_1\cap\overline{E}_2,$$
$$(\mu^{\square}_{A1}\,U\,\mu^{\square}_{A2})(xy) = \mu^{\square}_{A2}(xy) \quad \text{if } xy \in E_2\cap\overline{E}_1,$$
$$(\mu^{\square}_{A1}\,U\,\mu^{\square}_{A2}(xy) = \text{maxmin}(\mu^N_{A1}(xy),\mu^N_{A2}(xy)) \quad \text{if } xy \in E_1\cap E_2.$$

(iv)
$$(\mu^{\square}_{A1}\cap\mu^{\square}_{A2})(xy) = \mu^{\square}_{A1}(xy) \quad \text{if } xy\in E_1\cap\overline{E}_2,$$

$(\mu^{\square}_{A1}\cap\mu^{\square}_{A2})(xy) = \mu^{\square}_{A2}(xy)$ if $xy \in E_2 \cap \overline{E}_1$,
$(\mu^{\square}_{A1}\cap\mu^{\square}_{A2})(xy) = \text{minmax}(\mu^{N}_{A1}(xy),\mu^{N}_{A2}(xy))$ if $xy \in E_1 \cap E_2$.

Example 3.4 Consider the TPFG.



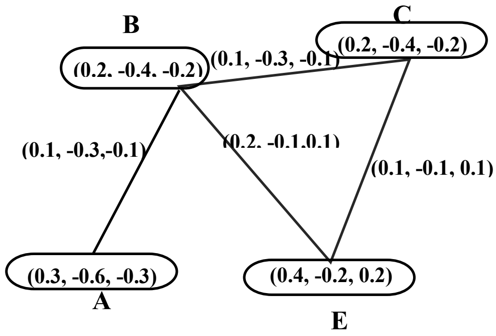
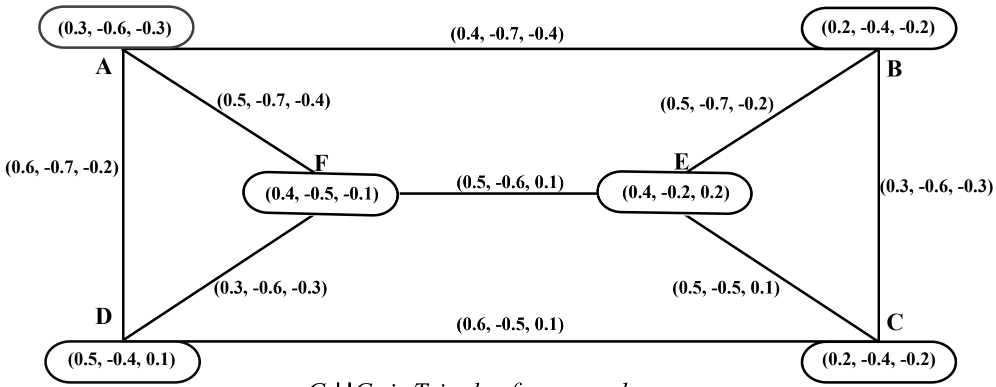


Fig:3.2: G_1 is Tripolar fuzzy graph

Fig:3.3: G_2 is Tripolar fuzzy graph



$G_1 \cup G_2$ is Tripolar fuzzy graph

Fig:3.4

Proposition 3.3. If G_1 and G_2 are TPFG, then $G_1 \cup G_2$ is a TPFG.

Proof. Let $xy \in E_1 \cap E_2$. Then

$(\mu_B{}^P{}_1 \cup \mu^P{}_{B2})(xy) = \max(\mu^P{}_{B1}(xy),\mu_B{}^P{}_2(xy))$
$\leq \max(\min(\mu^P{}_{A1}(x),\mu_A{}^P{}_1(y)),\min(\mu^P{}_{A2}(x),\mu^P{}_{A2}(y)))$
$= \min(\max(\mu^P{}_{A1}(x),\mu^P{}_{A2}(x),\max(\mu^P{}_{A1}(y),\mu^P{}_{A2}(y)))$
$= \min((\mu_A{}^P{}_1 \cup \mu_A{}^P{}_2(x),(\mu_A{}^P{}_1 \cup \mu_A{}^P{}_2)(y))$
$(\mu^N{}_{B1} \cup \mu^N{}_{B2})(xy) = \min(\mu^N{}_{B1}(xy),\mu_B{}^N{}_2(xy))$
$\geq \min(\max(\mu_A{}^N{}_1(x),\mu_A{}^N{}_1(y),\max(\mu^N{}_{A2}(x),\mu^N{}_{A2}(y)))$
$= \max(\min(\mu^N{}_1(x),\mu_A{}^N{}_2(x),\min(\mu^N{}_{A1}(y),\mu^N{}_{A2}(y)))$
$= \max((\mu_A{}^N{}_1 \cup \mu_A{}^N{}_2(x),(\mu_A{}^N{}_1 \cup \mu_A{}^N{}_2)(y))$
$(\mu_B{}^P{}_1 \cup \mu^N{}_{B2})(xy) = \text{maxmin}(\mu^P{}_{B1}(xy),\mu_B{}^N{}_2(xy))$
$= \text{maxmin}(\min(\mu^P{}_{A1}(x),\mu_A{}^P{}_1(y)),\max(\mu^N{}_{A2}(x),\mu^N{}_{A2}(y)))$
$= \text{maxmin}(\min(\mu^P{}_{A1}(x), \min(\mu_A{}^P{}_1(y),\max(\mu^N{}_{A2}(x), \max(\mu^N{}_{A2}(y))$
$= \text{maxmin}(\min(\mu^P{}_{A1}(x), \max(\mu^N{}_{A2}(x), \min(\mu_A{}^P{}_1(y), \max(\mu^N{}_{A2}(y))$
$= \text{minmax}(\text{maxmin}(\mu^P{}_{A1}(x),\mu^N{}_{A2}(x)),\text{maxmin}(\mu^P{}_{A1})\mu^N{}_{A2}(y)))$
$= \text{minmax}((\mu_A{}^P{}_1 \cup \mu_A{}^N{}_2(x),(\mu_A{}^P{}_1 \cup \mu_A{}^N{}_2)(y))$

Similarly, we can show that if $xy\ E_1 \cap \overline{E_2}$, then

$(\mu_B{}^P{}_1 \cup \mu^P{}_{B2})(xy) \leq \min((\mu_A{}^P{}_1 \cup \mu_A{}^P{}_2(x),(\mu_A{}^P{}_1 \cup \mu_A{}^P{}_2)(y))$
$(\mu^N{}_{B1} \cup \mu^N{}_{B2})(xy) \geq \max((\mu_A{}^N{}_1 \cup \mu_A{}^N{}_2(x),(\mu_A{}^N{}_1 \cup \mu_A{}^N{}_2)(y))$
$(\mu_B{}^P{}_1 \cup \mu^N{}_{B2})(xy) \geq \text{minmax}((\mu_A{}^P{}_1 \cup \mu_A{}^N{}_2(x),(\mu_A{}^P{}_1 \cup \mu_A{}^N{}_2)(y))$

If xy $\overline{E_1}\cap E_2$, then

$(\mu_B{}^P{}_1 \cup \mu^P{}_{B2})(xy) \leq \min((\mu_A{}^P{}_1 \cup \mu_A{}^P{}_2(x),(\mu_A{}^P{}_1 \cup \mu_A{}^P{}_2)(y))$

$(\mu^N{}_{B1} \cup \mu^N{}_{B2})(xy) \geq \max((\mu_A{}^N{}_1 \cup \mu_A{}^N{}_2(x),(\mu_A{}^N{}_1 \cup \mu_A{}^N{}_2)(y))$

$(\mu_B{}^P{}_1 \cup \mu^N{}_{B2})(xy) \gtrless \mathrm{minmax}((\mu_A{}^P{}_1 \cup \mu_A{}^N{}_2(x),(\mu_A{}^P{}_1 \cup \mu_A{}^N{}_2)(y))$

This completes the proof.

Proposition 3.4. Let $\{G_i:i\in A\}$ be a family of Tripolar fuzzy graph with the underlying set V. Then $\cap\mu_B{}^{P+N}(xy) = \mathrm{minmax}(\cap\mu_B{}^{P+N}(x), \cap\mu^{P+N}{}_B)(y))$ is a Tripolar fuzzy graph.

Proof. For any x, y $\in$V, we have

Consider, $\cap\mu_B{}^{P+N}(xy) = \{\cap\mu_B{}^P . \cap\mu_B{}^N\}(xy)$

$= \{\cap\mu_B{}^P(xy).\cap\mu_B{}^N(xy)\}$

$= \inf_{i\in A} \mu_B{}^P(xy). \sup_{i\in A} \mu_B{}^N(xy)$

$= \inf_{i\in A} \min\{\mu_{Ai}{}^P(x) , \mu_{Ai}{}^P(y)\} . \sup_{i\in A} \max\{\mu_{Ai}{}^N(x) , \mu_{Ai}{}^N(y)\}$

$= \min\{\inf_{i\in A} \mu_{Ai}{}^P(x) , \inf_{i\in A} \mu_{Ai}{}^P(y)\}. \max\{\sup_{i\in A} \mu_{Ai}{}^N(x) , \sup_{i\in A} \mu_{Ai}{}^N(y)\}$

$= \min \inf_{i\in A} \mu_{Ai}{}^P(x) ,\min \inf_{i\in A} \mu_{Ai}{}^P(y). \max \sup_{i\in A} \mu_{Ai}{}^N(x) , \max \sup_{i\in A} \mu_{Ai}{}^N(y)$

$= \min \inf_{i\in A} \mu_{Ai}{}^P(x) .\max \sup_{i\in A} \mu_{Ai}{}^N(x) ,\min \inf_{i\in A} \mu_{Ai}{}^P(y). \max \sup_{i\in A} \mu_{Ai}{}^N(y)$

$= \mathrm{minmax}\{\inf_{i\in A}\mu_{Ai}{}^P(x) . \inf_{i\in A} \mu_{Ai}{}^N(x)\}, \mathrm{minmax}\{\sup_{i\in A} \mu_{Ai}{}^P(y). \sup_{i\in A} \mu_{Ai}{}^N(y)\}$

$= \mathrm{minmax}\{\inf_{i\in A}\mu_{Ai}{}^{P+N}(x)\}, \mathrm{minmax}\{\sup_{i\in A} \mu_{Ai}{}^{P+N}(y)\}$

$= \mathrm{minmax}\{\inf_{i\in A}\mu_{Ai}{}^{P+N}(x) , \sup_{i\in A} \mu_{Ai}{}^{P+N}(y)\}$

$= \mathrm{minmax}\{\cap\mu_{Ai}{}^{P+N}(x) , \cap \mu_{Ai}{}^{P+N}(y)\}$

or

$= \mathrm{minmax}\{\cap\mu_{Ai}{}^{P+N}(x,y)\}$

Then $\cap G_i$ is a Tripolar Fuzzy graph.

Definition 3.5 Let be $A_1=(\mu^P{}_{A1},\mu^N{}_{A1},\mu^{\square}{}_{A1})$ and $A_2=(\mu^P{}_{A2},\mu^N{}_{A2},\mu^{\square}{}_{A2})$ be TPFsubsets of V_1 and V_2, and let $B_1=(\mu^P{}_{B1},\mu^N{}_{B1},\mu^{\square}{}_{B1})$ and $A_2=(\mu^P{}_{B2},\mu^N{}_{B2},\mu^{\square}{}_{B2})$ be TPF subsets of E_1 and E_2 respectively. Then, we denote the join of two TPFG $G^*{}_1$ and $G^*{}_2$ of the graphs G_1 and G_2 by $G_1 + G_2 = (A_1 + A_2,B_1+ B_2)$ and define as follows:

(i) $(\mu^P{}_{A1}+\mu^P{}_{A2})(x) = (\mu^P{}_{A1}\cup\mu^P{}_{A2})(x)$,

$(\mu^N{}_{A1}+\mu^N{}_{A2})(x) = (\mu^N{}_{A1}\cap\mu^N{}_{A2})(x)$,

$(\mu^P{}_{A1}+\mu^N{}_{A2})(x)= (\mu^P{}_{A1}\cup\mu^N{}_{A2})(x)$ if $x\in V_1\cap V_2$

(ii) $(\mu^P{}_{B1}+\mu^P{}_{B2})(xy) = (\mu^P{}_{B1}\cup\mu^P{}_{B2})(xy)$,

$(\mu^N{}_{B1}+\mu^N{}_{B2})(xy) = (\mu^N{}_{B1}\cap\mu^N{}_{B2})(xy)$

$(\mu^P{}_{B1}+\mu^N{}_{B2})(xy)= (\mu^P{}_{B1}\cup\mu^N{}_{B2})(xy)$, if xy $\in E_1\cap E_2$

(iii) $(\mu^P{}_{B1}+\mu^P{}_{B2})(xy) = \max(\mu^P{}_{A1}(x),\mu^P{}_{A2}(y))$

$(\mu^N{}_{B1}+\mu^N{}_{B2})(xy) = \min(\mu^N{}_{A1}(x),\mu^N{}_{A2}(y))$

$(\mu^P{}_{B1}+\mu^N{}_{B2})(xy) = \mathrm{maxmin}(\mu^P{}_{A1}(x),\mu^N{}_{A2}(y))$,

If xy $\in E'$, where E' is the set of all edges joining the nodes of V_1 and V_2

Proposition 3.5. If G_1 and G_2 are the Tripolar fuzzy graphs, then $G_1 + G_2$ is a TPFG.

Proof: Let xy $\in E'$. Then

$(\mu^P{}_{B1}+\mu^P{}_{B2})(xy)= \max(\mu^P{}_{A1}(x),\mu^P{}_{A2}(y))$

$\leq \max((\mu^P{}_{A1}\cup\mu^P{}_{A2})(x), (\mu^P{}_{A1}\cup\mu^P{}_{A2})(y))$

$= \max((\mu^P{}_{A1}+\mu^P{}_{A2})(x), (\mu^P{}_{A1}+\mu^P{}_{A2})(y))$.

$(\mu^N{}_{B1}+\mu^N{}_{B2})(xy) = \min(\mu^N{}_{A1}(x),\mu^N{}_{A2}(y))$

$\geq \min((\mu^N{}_{A1}\cup\mu^N{}_{A2})(x), (\mu^N{}_{A1}\cup\mu^N{}_{A2})(y))$

$= \min((\mu^N{}_{A1}+\mu^N{}_{A2})(x), (\mu^N{}_{A1}+\mu^N{}_{A2})(y))$.

$(\mu^P{}_{B1}+\mu^N{}_{B2})(xy) = \mathrm{maxmin}(\mu^P{}_{A1}(x),\mu^N{}_{A2}(y))$

$\gtrless \mathrm{maxmin}((\mu^P{}_{A1}\cup\mu^N{}_{A2})(x), (\mu^P{}_{A1}\cup\mu^N{}_{A2})(y))$

$= \mathrm{maxmin}((\mu^P{}_{A1}+\mu^N{}_{A2})(x), (\mu^P{}_{A1}+\mu^N{}_{A2})(y))$.

Let xy $\in E_1 \cup E_2$. Then the result follows from Proposition3.3.Thiscompletes the proof.

Proposition 3.6: Prove that $(\mu^{P+N}{}_{B1})(xy) = \mathrm{minmax}((\mu^{P+N}{}_{A1})(x,y))$ is aTPFG.

Proof: Let xy $\in E'$. Then

$(\mu^{P+N}{}_{B1})(xy) = (\mu^P{}_{B1}.\mu^N{}_{B1})(xy)$

$= \mu^P{}_{B1}(xy).\mu^N{}_{B1}(xy)$

$=\min((\mu^P{}_{A1}\cup\mu^P{}_{A2})(x),(\mu^P{}_{A1}\cup\mu^P{}_{A2})(y)).\max((\mu^N{}_{A1}\cup\mu^N{}_{A2})(x),(\mu^N{}_{A1}\cup\mu^N{}_{A2})(y))$

$= \min((\mu^P{}_{A1})(x), (\mu^P{}_{A1})(y)) . \max((\mu^N{}_{A1})(x), (\mu^N{}_{A1})(y))$

$= \mathrm{minmax}((\mu^{P+N}{}_{A1})(x)), \mathrm{minmax}((\mu^{P+N}{}_{A1})(y))$

$= \text{minmax}((\mu^{P+N}{}_{A1})(x,y))$

4. Strong Tripolar fuzzy graphs

Definition 4.1. A Tripolar fuzzy graph G = (A,B) is called strong if $\mu^P{}_B(xy) = \min(\mu^P{}_A(x),\mu_A{}^P(y))$ and $\mu^N{}_B(xy) = \max(\mu^N{}_A(x),\mu^N{}_A(y))$ and $\mu^{\square}{}_B(xy) = \text{minmax}(\mu^{\square}{}_A(x),\mu^{\square}{}_A(y))$ for all xy ϵE.

	x	y	z
$\mu^P{}_A$	0.8	0.4	0.5
$\mu^N{}_A$	-0.4	-0.7	-0.3
$\mu^{\square}{}_A$	0.4	0.3	0.2

Example 4.1 Consider a graph G^* s FG subset of V and letB be a TPF subset of E defined by

	xy	yz	zx
$\mu^P{}_A$	0.4	0.3	0.5
$\mu^N{}_A$	-0.3	-0.1	-0.2
$\mu^{\square}{}_A$	-0.1	0.2	0.1

Proposition 4.1 If G_1 and G_2 are the strong TPFG, then $G_1\ G_2, G_1[G_2]$ and $G_1 + G_2$ are STPFG.

Proof. The proof follows from Propositions 3.1, 3.2 and 3.5

Remark.

1. The union of two strong TPFG is not necessary a strong TPFG
2. If $G_1 \times G_2$ is strong TPFG, then at least G_1 or G_2 must be strong.

$$\mu^P{}_{B1}(x_1y_1) < \min\{\mu^P{}_{A1}(x),\mu^P{}_{A1}(y)\}, \quad \mu^P{}_{B2}(x_1y_1) < \min\{\mu^P{}_{A2}(x),\mu^P{}_{A2}(y)\},$$

$$\mu^N{}_{B1}(x_1y_1) > \max\{\mu^N{}_{A1}(x),\mu^N{}_{A1}(y)\}, \quad \mu^N{}_{B2}(x_1y_1) > \max\{\mu^N{}_{A2}(x),\mu^N{}_{A2}(y)\},$$

$$\mu^{\square}{}_{B1}(x_1y_1) \quad \text{minmax}\{\mu^{\square}{}_{A1}(x),\mu^{\square}{}_{A1}(y)\}, \quad \mu^{\square}{}_{B2}(x_1y_1) \quad \text{minmax}\{\mu^{\square}{}_{A2}(x),\mu^{\square}{}_{A2}(y)\},$$

Hence

$$(\mu^P{}_{B1}\times\mu^P{}_{B2})((x, x_2)(x, y_2))<\min((\mu^P{}_{B1}\times\mu^P{}_{B2})((x, x_2)),(\mu^P{}_{B1}\times\mu^P{}_{B2})((x, y_2))$$

$$(\mu^N{}_{B1}\times\mu^N{}_{B2})((x, x_2)(x, y_2))>\max((\mu^N{}_{B1}\times\mu^N{}_{B2})((x, x_2)),(\mu^N{}_{B1}\times\mu^N{}_{B2})((x, y_2))$$

$$(\mu^{\square}{}_{B1}\times\mu^{\square}{}_{B2})((x, x_2)(x, y_2)) \quad \text{minmax}((\mu^{\square}{}_{B1}\times\mu^{\square}{}_{B2})((x, x_2)),(\mu^{\square}{}_{B1}\times\mu^{\square}{}_{B2})((x, y_2))$$

3.If $G_1[G_2]$ is strong TPFG, then at least G_1 or G_2 must be strong.

Definition 4.2: A strong Tripolar fuzzy graph G is called self complementary if $G \approx \overline{\overline{G}}$.

Proposition 4.2: Let G be a self complementary strong Tripolar fuzzy graph. Then

$$\sum_{x\neq y}\mu^P{}_B(xy) = \sum_{x\neq y}\min(\mu^P{}_A(x),\mu^P{}_A(y)),$$

$$\sum_{x\neq y}\mu^N{}_B(xy) = \sum_{x\neq y}\max(\mu^N{}_A(x),\mu^N{}_A(y)),$$

$$\sum_{x\neq y}\mu^{\square}{}_B(xy) = \sum_{x\neq y}\text{minmax}(\mu^{\square}{}_A(x),\mu^{\square}{}_A(y)).$$

Proof: Let G be a self complementary strong TPFG. Then there exists an automorphism $f: V \to V$ such that $\mu_A{}^P(f(x)) = \mu^P{}_A(x)$ and $\mu_A{}^N(f(x)) = \mu^N{}_A(x)$ and $\overline{\mu_A{}^{\square}}(f(x)) = \mu^{\square}{}_A(x)$ for all x ϵV and $\mu_B{}^P(f(x)f(y)) = \mu_B{}^P(xy)$ and $\mu_B{}^N(f(x)f(y)) = \mu_B{}^N(xy)$ and $\mu_B{}^{\square}(f(x)f(y)) = \mu_B{}^{\square}(xy)$ for all x, y ϵV.

By definition of $\overline{G}$, we have

$$\overline{\mu_B}{}^P(f(x)f(y))=\min(\mu_A{}^P(\overline{f(x)}),\mu_A{}^P(\overline{f(y)})),\mu^P{}_B(xy)=\min(\mu_A{}^P(x),\mu^P{}_A(y)),$$

$$\sum_{x\neq y}\mu^P{}_B(xy)=\sum_{x\neq y}\min(\mu^P{}_A(x),\mu_A{}^P(y)),$$

$$\mu_B{}^N(f(x)\overline{f(y)})=\max(\mu_A{}^N(f(x),\mu_A{}^N(\overline{f(y)})),\mu^N{}_B(\overline{xy})=\max(\mu^N{}_A(x),\mu^N{}_A(y)),$$

$$\sum_{x\neq y}\mu^N{}_B(xy)=\sum_{x\neq y}\max(\mu^N{}_A(x),\mu_A{}^N(y)),$$

$$\mu_B{}^{\square}(f(x)\overline{f(y)})=\text{minmax}(\mu_A{}^{\square}(f(x),\mu_A{}^{\square}(\overline{f(y)})),\mu^{\square}{}_B(\overline{xy})=\text{minmax}(\mu^{\square}{}_A(x),\mu^{\square}{}_A(y)),$$

$$\sum_{x\neq y}\mu^{\square}{}_B(xy)=\sum_{x\neq y}\text{minmax}(\mu^{\square}{}_A(x),\mu_A{}^{\square}(y)),$$

This completes the proof.

Remark.

1.Let G be a strong TPFG. If $\mu^P{}_B(xy) = \min \mu^P{}_A(x),\mu^P{}_A(y)$ and $\mu^N{}_B(xy) = \max \mu_A{}^N(x),\mu^N{}_A(y)$,and $\mu^{\square}{}_B(xy) = \max \mu_A{}^{\square}(x),\mu^{\square}{}_A(y)$ for allx, y ϵV, then G is self complementary.

2.Let G_1 and G_2 be strong TPFG. Then $G_1 \cong G_2$ if and only if $\overline{G_1} \cong \overline{G_2}$.

5. Automorphic Tripolar fuzzy graphs

Definition 5.1: Let G_1 and G_2 be the Tripolar fuzzy graphs. A homomorphism $f: G_1 \to G_2$ is a mapping $f: V_1 \to V_2$ which satisfies the following conditions:

(a) $\mu^P_{A1}(x_1) \leq \mu^P_{A2}(f(x_1))$, $\mu^N_{A1}(x_1) \geq \mu^N_{A2}(f(x_1))$,
$\mu^P_{A1}(x_1) \geq \mu^N_{A2}(f(x_1))$,$\mu^N_{A1}(x_1) \leq \mu^P_{A2}(f(x_1))$,
(b) $\mu^P_{B1}(x_1y_1) \leq \mu^P_{B2}(f(x_1)f(y_1))$,$\mu^N_{B1}(x_1y_1) \geq \mu^N_{B2}(f(x_1)f(y_1))$,
$\mu^P_{A1}(x_1y_1) \geq \mu^N_{A2}(f(x_1)f(y_1))$,$\mu^N_{A1}(x_1y_1) \leq \mu^P_{A2}(f(x_1)f(y_1))$,for all $x_1 \in V_1$, $x_1y_1 \in E_1$.

Definition 5.2. Let G_1 and G_2 be TPFG. An isomorphism $f : G_1 \rightarrow G_2$ is a bijective mapping $f : V_1 \rightarrow V_2$ which satisfies the following conditions:

(c) $\mu^P_{A1}(x_1) = \mu^P_{A2}(f(x_1))$, $\mu^N_{A1}(x_1) = \mu^N_{A2}(f(x_1))$, $(\mu^P_{A1}+\mu^N_{A1})(x_1) = (\mu^P_{A2}+\mu^N_{A2})(f(x_1))$,
(d) $\mu^P_{B1}(x_1y_1)=\mu^P_{B2}(f(x_1)f(y_1))$,$\mu^N_{B1}(f(x_1)f(y_1))=\mu^N_{B2}(f(x_1)f(y_1))$, $(\mu^P_{B1}+\mu^N_{B1})(f(x_1)f(y_1))=(\mu^P_{B1}+\mu^N_{B2})(f(x_1)f(y_1))$,for all $x_1 \in V_1$, $x_1y_1 \in E_1$.

Definition 5.3. Let G_1 and G_2 be TPFG. Then, a weak isomorphism $f : G_1 \rightarrow G_2$ is a bijective mapping $f : V_1 \rightarrow V_2$ which satisfies the following conditions:

(e) f is homomorphism,
(f) $\mu^P_{A1}(x_1) = \mu^P_{A2}(f(x_1))$, $\mu^N_{A1}(x_1) = \mu^N_{A2}(f(x_1))$, $(\mu^P_{A1}+\mu^N_{A1})(x_1) = (\mu^P_{A2}+\mu^N_{A2})(f(x_1))$,
for all $x_1 \in V_1$. Thus, a weak isomorphism preserves the weights of the nodes but not necessarily the weightsof the arcs.

Example 5.3. Consider TPFG $G_1$and G_2 of G_1 and G_2, respectively.
A map f: $V_1 \rightarrow V_2$ defined by $f(a_1) = b_2$ and $f(b_1) = a_2$. Then we see that:
(g) $\mu^P_{A1}(a_1) = \mu^P_{A2}(b_2))$, $\mu^N_{A1}(a_2) = \mu^N_{A2}(b_1))$, $(\mu^P_{A1}+\mu^N_{A1})(a_1) = (\mu^P_{A2}+\mu^N_{A2})(b_2))$,
$\mu^P_{A1}(b_1) = \mu^P_{A2}(a_2))$, $\mu^N_{A1}(b_2) = \mu^N_{A2}(a_1))$, $(\mu^P_{A1}+\mu^N_{A1})(b_1) = (\mu^P_{A2}+\mu^N_{A2})(a_2))$,
(h) $\mu^P_{B1}(a_1b_1) \neq \mu^P_{B2}(f(a_1)f(b_1)) = \mu^P_{B2}(a_2b_2)$, $\mu^N_{B1}(f(a_1)f(b_1)) \neq \mu^N_{B2}(f(a_1)f(b_1))=\mu^N_{B2}(a_2b_2)$,
$(\mu^P_{B1}+\mu^N_{B1})(f(a_1)f(b_1)) \neq (\mu^P_{B1}+\mu^N_{B2})(f(a_2)f(b_2)) = (\mu^P_{B1}+\mu^N_{B2})(a_2b_2)$
Hence the map is a weak isomorphism but not isomorphism.

Definition 5.4: A Tripolar fuzzy set $A = (\mu^P_A,\mu^N_A,\mu^\square_A(x))$in a semigroup S is called a Tripolar subsemigroup of S if it satisfies:

$\mu^P_A(xy) \geq \min\{\mu^P_A(x),\mu^P_A(y)\}$,$\mu^N_A(xy) \leq \max\{\mu^N_A(x),\mu^N_A(y)\}$, $\mu^\square_A(xy) \gtreqless \text{minmax}\{\mu^\square_A(x),\mu^\square_A(y)\}$,
where$\mu^\square_A(x)= \mu^P(x)+\mu^N(x)$ for all $x,y \in S$:

A Tripolar fuzzy set $A = \mu^P_A,\mu^N_A,\mu^\square_A$ in a group G is called a Tripolar fuzzy subgroup of a group G if it is a Tripolar fuzzy subsemigroup of G and satisfies:
$\mu^P_A(x^{-1}) = \mu^P_A(x)$, $\mu^N_A(x^{-1}) = \mu_A^N(x)$,$\mu^\square_A(x^{-1}) =\mu^\square_A(x)$
where$\mu^\square_A(x^{-1})= \mu^P(x^{-1})+\mu^N(x^{-1})$ for all $x \in G$:
We now show how to associate a TPF group with a TPFG in a natural way.

Proposition 5.1. Let G =(A,B) be a TPFG and Let Aut(G) be the set of all Automorphisms of G. Then (Aut(G),) forms a group.

Proof. Let ϕ,Aut(G) and Let x, y $\in$V. Then
$\mu^P_B((\phi\circ\psi)(x)(\phi\circ\psi)(y)) = \mu_B^P((\phi(\psi(x))(\phi(\psi))(y)) \geq \mu^P_B(\psi(x)\psi(y)) \geq \mu^P_B(xy)$,
$\mu^N_B((\phi\circ\psi)(x)(\phi\circ\psi)(y)) = \mu^N_B((\phi(\psi(x))(\phi(\psi))(y))) \leq \mu^N_B(\psi(x)\psi(y)) \leq \mu_B^N(xy)$,
$\mu^\square_B((\phi\circ\psi)(x)(\phi\circ\psi)(y)) =\mu^\square_B((\phi(\psi(x))(\phi(\psi))(y))) \lesseqgtr \mu^\square_B(\psi(x)\psi(y)) \lesseqgtr \mu_B^\square(xy)$,
$\mu^P_B((\phi\circ\psi)(x)) = \mu^P_B((\phi(\psi(x)))) \geq \mu^P_B(\psi(x)) \geq \mu_B^P(x)$,
$\mu^N_B((\phi\circ\psi)(x)) = \mu^N_B((\phi(\psi(x)))) \leq \mu^N_B(\psi(x)) \leq \mu^N_B(x)$.
$\mu^\square_B((\phi\circ\psi)(x)) \lesseqgtr \mu^\square_B((\phi(\psi(x)))) \lesseqgtr \mu^\square_B(\psi(x)) \lesseqgtr \mu^\square_B(x)$.
Thus $\phi\circ\psi \in$Aut(G). Clearly, Aut(G) satisfies associativity under the operation $\circ$, $\phi\circ e = \phi = e\circ\phi$, $\mu^P_A(\phi^{-1}) = \mu^P_A(\phi)$, $\mu^N_A(\phi^{-1}) = \mu^N_A(\phi)$,$\mu^\square_A(\phi^{-1}) = \mu^\square_A(\phi)$ for all $\phi \in$Aut(G). Hence (Aut(G),$\circ$) forms a group.

Proposition 5.2: Let G = (A,B) be a TPFG and let Aut(G) be the set of all automorphisms of G.
Let $g = (\mu^P_g,\mu^N_g,\mu^\square_g)$ be a Tripolar fuzzy set in Aut(G) defined by
$\mu^P_g(\phi) = \sup\{\mu^P_B(\phi(x),\phi(y)) : (x,y) \in V \times V\}$,
$\mu^N_g(\phi) = \inf\{\mu^N_B(\phi(x),(y)) : (x,y) \in V \times V\}$,
$\mu^\square_g(\phi) = \mu^P_g(\phi)+\mu^N_g(\phi)$for all $\phi \in$Aut(G).
Then $g = (\mu_g^P,\mu^N_g\mu^\square_g)$is a Tripolar fuzzy group on Aut(G).

Conclusions

We have introduced the concept of Tripolar fuzzy graphs (TPFG) in this paper. The Tripolar fuzzy models give more precision, flexibility and compatibility to the system as compared to the classical and fuzzy models.
The concept of Tripolar fuzzy graphs can be applied in various areas of engineering, computer science: database theory, expert systems, neural networks, artificial intelligence, signal processing, pattern recognition, robotics, computer networks, and medical diagnosis.

We plan to extend our research of fuzzification to

- TPFG appliction
- TPFG hypergraphs, intuitionistic fuzzy hypergraphs, ext
- Regular and Irregular TPFG

- Operation on TPFG
- Metric in TPFG
- Balanced Tripolar intutionistc fuzzy graphs

References

[1] Akram M (2011) Bipolar fuzzy graphs. Inf Sci 181(24):5548-5564, DOI:10.1016/j.ins.2011. 07.037.

[2] M. Akram, Cofuzzy graphs, *journal of Fuzzy Mathematics* 19 (4) (2011).

[3] M. Akram, Intuitionistic(S,T)-fuzzy lie ideals of lie algebras, Quasigroups and Related Systems15 (2007)201-218.

[4] M. Akram, K.H. Dar, Generalized Fuzzy K-Algebras, VDM Verlag, 2010.

[5] Alaoui, on fuzzification of some concepts of graphs, Fuzzy Sets and Systems 101 (1999) 363-389.

[6] K.T. Atanassov, Intuitionistic Fuzzy Fets: Theory and Applications, Studies in Fuzziness and Soft Computing, Physica-Verl., Heidelberg, New York, 1999.

[7] K.T. Atanassov, Index matrix representation of the intuitionistic fuzzy graphs, Preprint MRl-MFAIS-10-94, Sofia, 1994, pp. 36 -41.

[8] K.T. Atanassov, G. Pasi, R. Yager, V. Atanassova, Intuitionistic fuzzy graph interpretations of multi-person multi-criteria decision making, in, EUSFlAT Conference 2003, pp. 177-182.

[9] P. Bhattacharya, Some remarks on fuzzy graphs, Pattern Recognition letter 6 (1987) 297-302.

[10] K.R. Bhutani, On automorphism of fuzzy graphs, Pattern Recognition letter 9 (1989) 159-16 2.

[11] K.R. Bhutani, A. Rosenfeld, Strong arcs in fuzzy graphs, Information Sciences 152 (2003) 319322.

[12] K.R. Bhutani, A. Rosenfeld, Fuzzy end nodes in fuzzy graphs, Information Science 152 (2003) 323-326.

[13] K.R. Bhutani, A. Battou, On M-strong fuzzy graphs, Information Sciences 155 (2003) 103-109.

[14] Bloch, Dilation and erosion of spatial bipolar fuzzy sets, lecture Notes in Artificial Intelligence (2007) 385-393.

[15] Bloch, Geometry of spatial bipolar fuzzy sets based on bipolar fuzzy numbers and mathematical morphology, Fuzzy logic and Applications, lecture Notes in Computer Science 5571 (2009) 237-245.

[16] Boulmakoul, Fuzzy graphs modelling for HazMat telegeomonitoring, European *journal of Operational Research* 175 (3) (2006) 1514-1525.

[17] J.C. Chen, C.H. Tsai, Conditional edge-fault-tolerant Hamiltonicity of dual-cubes, Information Sciences 181 (2011) 6 20-6 27.

[18] C.T. Cheng, C.P. Ou, K.W. Chau, Combining a fuzzy optimal model with a genetic algorithm to solve multiojective rainfall-runoff model calibration, *journal of Hydrology* 26 8 (1-4) (2002) 72-86.

[19] D. Dubois, S. Kaci, H. Prade, bipolarity in reasoning and decision, an introduction, in: Int. Con. on Inf. Pro. Man. Unc. IPMU'04, 2004, pp. 959-966.

[20] F. Fang, The bipancycle-connectivity of the hypercube, Information Science 178 (2008) 46 79-46 87.

[21] F. Harary, Graph Theory, third ed., Addison-Wesley, Reading, MA, 1972.

[22] W. Homenda, W. Pedrycz, Balanced fuzzy gates, RSCTC (2006) 107-116.

[23] Hossein Rashmanlou, Sovan Samanta, Madhumangal Paland Rajab Ali Borzooei (2015), A study on bipolar fuzzy graphs. *Journal of Intelligent & Fuzzy Systems* 28 (2015) 571-580DOI:10.3233/IFS-141333

[24] K.P. Huber, M.R. Berthold, Application of fuzzy graphs for metamodeling, in: Proceedings of the 2002 IEEE Conference, pp. 6 40-6 44.

[25] M.G. Karunambigai, P. Rangasamy, K.T. Atanassov, N. Palaniappan, An intuitionistic fuzzy graph method for finding the shortest paths in networks, in: O. Castillo *et al*. (Eds.), Theor. Adv. and Appl. of Fuzzy logic, vol. 42, ASC, 2007, pp. 3-10.

[26] Kiss, An application of fuzzy graphs in database theory, Pure Mathematics and Applications 1 (3-4) (1991) 337-342.

[27] L.T. Koczy, Fuzzy graphs in the evaluation and optimization of networks, Fuzzy Sets and Systems 46 (1992) 307-319.

[28] K.-M. lee, bipolar-valued fuzzy sets and their basic operations, in: Proceedings of the International Conference, Bangkok, Thailand, 2000, pp. 307-317.

[29] K.-M. lee, Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets, and bipolar-valued fuzzy sets, *journal of Fuzzy logic Intelligent Systmes* 14 (2004) 125-129.

[30] Y. li, Finite automata theory with membership values in lattices, Information Sciences 181 (2011) 1003-1017.

[31] J -Y lin, C.T. Cheng, K.-W. Chau, Using support vector machines for long-term discharge prediction, Hydrological Sciences journal 51 (4) (2006) 599- 6 12.

[32] S. Mathew, M.S. Sunitha, Types of arcs in a fuzzy graph, Information Sciences 179 (11) (2009) 176 0-176 8.

[33] S. Mathew, M.S. Sunitha, Node connectivity and arc connectivity of a fuzzy graph, Information Sciences 180 (4) (2010) 519-531.

[34] J.N. Mordeson, C.S. Peng, Operations on fuzzy graphs, Information Sciences 79 (1994) 159-170.

[35] J.N. Mordeson, Fuzzy line graphs, Pattern Recognition letter 14 (1993) 381-384.

[36] J.N. Mordeson, P.S. Nair, Fuzzy Graphs and Fuzzy Hypergraphs, second ed., Physica Verlag, Heidelberg, 1998, 2001.

[37] J.N. Mordeson, P.S. Nair, Cycles and cocyles of fuzzy graphs, Information Science 90 (1996) 39-49.

[38] N. Muttil1, K.W. Chau, Neural network and genetic programming for modelling coastal algal blooms, *International journal of Environment and Pollution* 28 (3-4) (2006) 223-238.

[39] A.Nagoorgani, K. Radha, Isomorphism on fuzzy graphs, *International |ournal of Computational and Mathematical Sciences* 2 (2008) 190-196.

[40] Pang, R. Zhang, Q. Zhang, |. Wang, Dominating sets in directed graphs, Information Sciences 180 (2010) 36 47-36 52.

[41] Perchant, I. Bloch, Fuzzy morphisms between graphs, Fuzzy Sets and Systems 128 (2002) 149-16 8.

[42] R. Parvathi, M.G. Karunambigai, K.T. Atanassov, Operations on intuitionistic fuzzy graphs, in: Fuzzy Systems, 2009. FUZZ-IEEE 2009. IEEE International Conference, pp. 1396 -1401.

[43] G. Pasi, R. Yager, K.T. Atanassov, Intuitionistic fuzzy graph interpretations of multi-person multi-criteria decision making: generalized net approach, in: Intelligent Systems, Proceedings of the 2004 2nd International IEEE Conference, vol. 2, 2004, pp. 434-439.

[44] W. Pedrycz, Fuzzy Sets Engineering, CRC Press, Boca Raton, Fl, 1995.

[45] W. Pedrycz, Human centricity in computing with fuzzy sets: an interpretability quest for higher order granular constructs, j. Ambient Intelligence and Humanized Computing 1 (1) (2010) 6 5-74.

[46] W. Pedrycz, A. Bargiela, Fuzzy clustering with semantically distinct families of variables: descriptive and predictive aspects, Pattern Recognition letters 31 (13) (2010) 1952-1958.

[47] Rosenfeld, A. Fuzzy graphs, in: l.A. Zadeh, K.S. Fu, M. Shimura (Eds.), Fuzzy Sets and their Applications, Academic Press, New York, 1975, pp. 77-95. [46] M.S. Sunitha, A. Vijayakumar, Complement of a fuzzy graph, *Indian journal of Pure and Applied Mathematics* 33(9), pp. 1451-146 4.

[48] Shannon,A K.T. Atanassov, A first step to a theory of the intuitionistic fuzzy graphs, in: D. lakov (Ed.), Proceeding of FUBEST, Sofia, Sept. 28-30 1994 pp. 59-6 1.

[49] M.S. Sunitha, A. Vijayakumar, Complement of a fuzzy graph, *Indian Journal of Pure and Applied Mathematics* 33(9), pp. 1451-1464.

[50] Shannon, A K.T. Atanassov, Intuitionistic fuzzy graphs from a-, b-, and (a,b)-levels, Notes on Intuitionistic Fuzzy Sets 1 (1) (1995) 32-35.

[51] L. Wu, E. Shan, Z. liu, on the k-tuple domination of generalized de Brujin and Kautz digraphs, Information Sciences 180 (2010) 4430-4435.

[52] J.X. Xie, C.T. Cheng, K.W. Chau, Y.Z. Pei, A hybrid adaptive time-delay neural network model for multi-step-ahead prediction of sunspot activity, *International journal of Environment and Pollution* 28 (3-4) (2006) 36 4-381.

[53] L.A. Zadeh, Fuzzy sets, Information and Control 8 (196 5) 338-353.

[54] L.A. Zadeh, Similarity relations and fuzzy orderings, Information Sciences 3 (2) (1971) 177-200.

[55] L.A. Zadeh, Toward a generalized theory of uncertainty (GTU) an outline, Information Sciences 172 (2005)1-40.

[56] L.A. Zadeh, From imprecise to granular probabilities, Fuzzy Sets and Systems 154 (2005) 370-374.

[57] L.A. Zadeh, Is there a need for fuzzy logic→, Information Sciences 178 (2008) 2751-2779

[58] W.-R. Zhang, bipolar fuzzy sets and relations: a computational framework forcognitive modeling and multiagent decision analysis, Proceedings of IEEE Conf., 1994, pp. 305-309.

[59] W.-R. Zhang, bipolar fuzzy sets, Proceedings of FUZZ-IEEE (1998) 835-840.

[60] J. Zhang, X. Yang, Some properties of fuzzy reasoning in propositional fuzzy logic systems, Information Sciences 180 (2010) 466 1-46 71.

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