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A SHORT NOTE ON INTUITIONISTIC FUZZY VOLTERRA SPACES

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A B S T R A C T **RESEARCH ARTICLE**

In this paper we investigate several characterizations of intuitionistic fuzzy Volterra spaces and study the conditions under which an intuitionistic fuzzy topological space becomes an intuitionistic fuzzy Volterra space.

Keywords:

Intuitionistic fuzzy nowhere dense set, intuitionistic fuzzy dense set, intuitionistic fuzzy first category, intuitionistic fuzzy D-Baire and intuitionistic fuzzy Volterra spaces.

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1. Introduction

The fuzzy concept has invaded almost all branches of mathematics ever since the introduction of fuzzy set by Zadeh [19]. The theory of fuzzy topological spaces was introduced and developed by Chang [6]. The idea of "Intuitionistic fuzzy set" was first published by Atanassov [1]. Later, in [5] this concept was generalised to "Intuitionistic L-fuzzy set"by Atanassov and Stoneva. The concept of Volterra spaces have been studied extensively in classical topology [8, 9, 12, 13]. The concept of fuzzy Volterra space is introduced and studied by Thangaraj and Soundararajan [18]. The concept of intuitionistic fuzzy Volterra space was introduced and studied by Soundararajan, Rizwan and Syed Tahir Hussainy [14].In this paper, we study the characterizations of intuitionistic fuzzy Volterra space sand study the conditions underwhich an intuitionistic fuzzy topological space becomes an intuitionistic fuzzy Volterra space. Intuitionistic fuzzy first category space, intuitionistic fuzzy σ-first category space, intuitionistic fuzzy submaximal space, intuitionistic fuzzy D-Baire space and intuitionistic fuzzy *P*-space are considered for this work.

2. Preliminaries

Definition 1. [3] Let X be a non-empty set. An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$ where $\mu_A(x): X\to[0,1]$ and $\nu_A(x): X\to[0,1]$ denote the membership and non-membership functions of A respectively, and $0\le\mu_A(x)+\nu_A(x)\le 1$, for each $x\in X$.

Definition 2. [3] Let A and B be two IFSs of the non-empty set X such that

$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$,
$B=\{\langle x,\mu_B(x),\nu_B(x)\rangle : x\in X\}$.

We define the following basic operations on A and B:

1. $A\subseteq B$ iff $\mu_A(x)\le\mu_B(x)$ and $\nu_A(x)\ge\nu_B(x), \forall x\in X$
2. $A\supseteq B$ iff $\mu_A(x)\ge\mu_B(x)$ and $\nu_A(x)\le\nu_B(x), \forall x\in X$
3. $A=B$ iff $\mu_A(x)=\mu_B(x)$ and $\nu_A(x)=\nu_B(x), \forall x\in X$
4. $A\cup B=\{\langle x,\mu_A(x)\vee\mu_B(x),\nu_A(x)\wedge\nu_B(x)\rangle : x\in X\}$
5. $A\cap B=\{\langle x,\mu_A(x)\wedge\mu_B(x),\nu_A(x)\vee\nu_B(x)\rangle : x\in X\}$
6. $A^C=\{\langle x,\nu_A(x),\mu_A(x)\rangle : x\in X\}$.

Definition 3. [7] An Intuitionistic fuzzy topology (IFT) on X is a family T of IFSs in X satisfying the following axioms.

1. $0,1\in T$
2. $G_1\cap G_2\in T$, for any $G_1,G_2\in T$
3. $\cup G_i\in T$ for any family $\{G_i / i\in J\}\subseteq T$.

In this case the pair (X,T) is called an Intuitionistic fuzzy topological space (IFTS) and any IFS in T is known as Intuitionistic fuzzy open set (IFOS) in X. The complement A^c of an IFOS A in an IFTS (X, T) is called an Intuitionistic fuzzy closed set (IFCS) in X.

Definition 4. [7] Let (X, T) be an IFTS and $A=\langle X,\mu_A,\nu_A\rangle$ be an IFS in X. Then the Intuitionistic fuzzy interior and an Intuitionistic fuzzy closure are defined by

$IFint(A) = \cup\{G / G$ is an IFOS in X and $G\subseteq A\}$,
$IFcl(A) = \cap\{K / K$ is an IFCS in X and $A\subseteq K\}$.

Theorem 1. *Let* (X, T) *be an IFTS. Let A be an IFS in* (X, T). *Then*

1. $1 - IFcl(A) = IFint(1 - A)$
2. $1 - IFint(A) = IFcl(1 - A)$.

Definition 5. [11] An Intuitionistic Fuzzy set A in an Intuitionistic fuzzy topological space (X, T) is called an intuitionistic fuzzy dense if there exists no Intuitionistic fuzzy closed set B in (X, T) such that $A\subset B\subset 1$.

Definition 6. [17] An Intuitionistic Fuzzy set A in an Intuitionistic fuzzy topological space (X, T) is called an intuitionistic fuzzy nowhere dense set if there exists no Intuitionistic fuzzy open set U in (X, T) such that $U \subset IFcl\ A$. That is, if $IFint\ IFcl\ (A) = 0$.

Theorem 2. *[16] If A is an intuitionistic fuzzy nowhere dense set in IFTS (X, T), then* $1 - A$ *is an intuitionistic fuzzy dense set in (X, T).*

Theorem 3. *[16] Let (X, T) be an IFTS. An intuitionistic fuzzy set A is an intuitionistic fuzzy dense and intuitionistic fuzzy open set in (X, T), then* $1 - A$ *is an intuitionistic fuzzy nowhere dense set in (X, T).*

Definition 7. [14] An Intuitionistic fuzzy set A in an Intuitionistic fuzzy topological space an Intuitionistic fuzzy $G_\delta -$ set in (X, T) if $A = \bigcap_{i=1}^{\infty} A_i$ where $A_i \in T, \forall i.$

Definition 8. [14] An Intuitionistic fuzzy set A in an Intuitionistic fuzzy topological space (X, T) is called an Intuitionistic fuzzy $F_\sigma -$ set in (X, T)) if $A = \bigcup_{i=1}^{\infty} A_i$ where $1 - A_i \in T, \forall i.$

Lemma 1. *[14] A is an Intuitionistic fuzzy* $G_\delta -$ *set in an Intuitionistic fuzzy topological space (X, T) if and only if* $1 - A$ *an Intuitionistic fuzzy* $F_\sigma -$ *set in (X, T).*

3. Intuitionistic Fuzzy Volterra Spaces

Definition 9. [14] An Intuitionistic fuzzy topological space (X, T) (IFTS) is called an Intuitionistic fuzzy Volterra space if $IFCl\left(\bigcap_{i=1}^{\infty} A_i\right) = 1$, where A_i's are Intuitionistic fuzzy dense and Intuitionistic fuzzy $G_\delta -$ sets in (X, T).

Definition 10. [14] Let (X, T) be an IFTS. Then (X, T) is called an Intuitionistic fuzzy weakly Volterra space if $IFCl\left(\bigcap_{i=1}^{\infty} A_i\right) \neq 0$, where A_i's are Intuitionistic fuzzy dense and Intuitionistic fuzzy $G_\delta -$ sets in (X, T).

Definition 11. [14] An IFS A is an IFTS (X, T) is called an Intuitionistic fuzzy first category set if $A = \bigcup_{i=1}^{\infty} A_i$, where A_i's are intuitionistic fuzzy nowhere dense sets in (X, T). Any other intuitionistic fuzzy set in (X, T) is said to be of second category.

Definition 12. [14] An IFTS (X, T) is called an Intuitionistic fuzzy first category space if $1 = \bigcup_{i=1}^{\infty} A_i$, where A_i's are intuitionistic fuzzy nowhere dense sets in (X, T) . (X, T) is called an intuitionistic fuzzy second category space if it is not an intuitionistic fuzzy first category space.

Definition 13. [14] Let (X, T) be an IFTS. An intuitionistic fuzzy set A in (X, T) is called an intuitionistic fuzzy $\sigma-$ nowhere dense set if A is an intitionistic fuzzy $F_\sigma -$ set such that $IFint\ (A) = 0$.

Definition 14. [14] An IFS A is an IFTS (X, T) is called an Intuitionistic fuzzy $\sigma-$ first category set if $A = \bigcup_{i=1}^{\infty} A_i$, where A_i's are intuitionistic fuzzy $\sigma-$ nowhere dense sets in (X, T). Any other intuitionistic fuzzy set in (X, T) is said to be of second category.

Theorem 4. *[14] In a IFTS (X, T), an Intuitionistic fuzzy set A is Intuitionistic fuzzy* $\sigma-$ *nowhere dense if and only if* $1 - A$ *is an Intuitionistic fuzzy dense and Intuitionistic fuzzy* $G_\delta -$ *set.*

Definition 15. [10] Let (X, T) be an intuitionistic fuzzy topological space. Then (X, T) is called an intuitionistic fuzzy Baire space if $IFint\ \left(\bigcup_{i=1}^{\infty} A_i\right)$, where A_i's are Intuitionistic fuzzy nowhere dense sets in (X, T).

Theorem 5. *[10] Let (X, T) be an IFTS. Then the following are equivalent.*

1. *(X, T) is an intuitionistic fuzzy Baire space*
2. *If int (A) = 0 for every intuitionistic fuzzy first category category set A in (X, T).*
3. *If cl (A) = 0 for every intuitionistic fuzzy residual set A in (X, T).*

Theorem 6. *[15] If A is an intuitionistic fuzzy dense and intuitionistic fuzzy* $G_\delta -$ *set in an IFTS (X, T), then* $1 - A$ *is an intuitionistic fuzzy first category set in (X, T).*

Proposition 1. If each intutionistic fuzzy first category set is an intutionistic fuzzy closed set in an intutionistic fuzzy Baire space (X, T), then (X, T) is an intutionistic fuzzy Volterra space.

Proof: Let $A_i'^s$ ($i = 1, \ldots, N$) be intuitionistic fuzzy dense and intuitionistic fuzzy $G_\delta -$ set in (X, T). Then by theorem-6 $(1 - A_i)'^s$ are intuitionistic fuzzy first category sets in (X, T). By hypothesis $(1 - A_i)'^s$ are intuitionistic fuzzy closed sets in (X, T). Hence $A_i'^s$ are intuitionistic fuzzy open sets in (X, T). Thus $A_i'^s$ ($i = 1, \ldots, N$) are intuitionistic fuzzy dense and intuitionistic fuzzy open sets in (X, T). Let B_i ($i = 1, \ldots, \infty$) be intuitionistic fuzzy dense and intuitionistic fuzzy open sets in (X, T) in which the first N intuitionistic fuzzy open and intuitionistic fuzzy dense sets be A_i ($i = 1, \ldots, N$). Since B_i's are intuitionistic fuzzy open and intuitionistic fuzzy dense sets in $(X; T)$ by theorem-3 (1 - *Bi*)*'s* intuitionistic fuzzy nowhere dense sets in (X, T)*:* Since (X, T) is an intuitionistic fuzzy Baire space, $\mathrm{IF}int(\bigcup_{i=0}^{\infty}(1 - \mathrm{B}_i)) = 0$ This implies $1\text{-}\mathrm{IF}int(\bigcup_{i=0}^{\infty}(1 - \mathrm{B}_i)) = 1 \Rightarrow \mathrm{IF}cl(\bigcap_{i=0}^{\infty}\mathrm{B}_i) = 1$ But $1 = \mathrm{IF}cl(\bigcap_{i=0}^{\infty}\mathrm{B}_i) \subseteq \mathrm{IF}cl(\bigcap_{i=0}^{N}\mathrm{B}_i)$. Therefore, $\mathrm{IF}cl(\bigcap_{i=0}^{N}\mathrm{B}_i) = 1$ Hence $1 = \mathrm{IF}cl(\bigcap_{i=0}^{N}A_i) = 1$ where $A_i^{'s}$ are intuitionistic fuzzy G_δ and int fuzzy dense sets in (X, T), Therefore (X, T) is an intuitionistic fuzzy Volterra space.

Definition 16. [10] An Intuitionistic Fuzzy set A in an Intuitionistic fuzzy topological space (X, T) is called an intuitionistic fuzzy residual set if 1- A is an intuitionistic fuzzy first category set in (X, T).

Proposition 2. If each intuitionistic fuzzy residual set is an intuitionistic fuzzy open set in an intuitionistic fuzzy Baire space (X, T), then (X, T) is an intuitionistic fuzzy Volterra space.

Proof. Let int fuzzy residual sets $B_i^{'s}$ be int fuzzy open set in an int fuzzy Baire space (X, T), Then $(1 - B_i)^{'s}$ are intuitionistic fuzzy first category sets in (*X, T*) and $(1-B_i)'^s$ are int fuzzy closed in (*X, T*), Then by Propositionosition-1 (*X, T*) is an int fuzzy Volterra space.

Definition 17. [15] An IFTS (*X, T*) is called an intuitionistic fuzzy D-Baire space, if every intuitionistic fuzzy first category set in (*X, T*) is an intuitionistic fuzzy nowhere dense set in (*X, T*).

Proposition 3. If the IFTS (*X, T*) is an intuitionistic fuzzy D-Baire space, then (*X, T*) is an intuitionistic fuzzy Volterra space.

Proof. Let $A_i'^s(i=1 \; to \; N)$ be intuitionistic fuzzy dense and intuitionistic fuzzy G_δ - sets in (*X, T*). Then by theorem-6 $(1-A_i)'^s$ are intuitionistic fuzzy first category sets in (*X, T*). Since (*X, T*) is an intuitionistic fuzzy D-Baire space, the intuitionistic fuzzy first category sets $(1-A_i)'^s$ are intuitionistic fuzzy nowhere dense sets in (*X, T*). Let $B_i'^s(i=1 \; to \; \infty)$ be intuitionistic fuzzy nowhere dense sets in (*X, T*) in which the first *N* intuitionistic fuzzy nowhere dense sets are $(1-A_i)'^s(i=1 \; to \; N)$ Then $\cup_{i=1}^{\infty} B_i$ is an intuitionistic fuzzy first category set in (*X, T*). Since (*X, T*) is intuitionistic fuzzy D-Baire space, the intuitionistic fuzzy first category set $\cup_{i=1}^{\infty} B_i$ is an intuitionistic fuzzy nowhere dense set in (*X, T*). Therefore

$$\mathrm{IF}int \; \mathrm{IF}cl(\bigcup_{i=0}^{\infty} \mathrm{B}_i) = 0$$

$$\mathrm{IF}int(\cup_{i=1}^{\infty} \mathrm{B}_i) \subseteq \mathrm{IF}int\mathrm{IF}cl(\cup_{i=1}^{\infty} \mathrm{B}_i) = 0$$

This implies that

$$\mathrm{IF}int(\bigcup_{i=0}^{\infty} \mathrm{B}_i) = 0$$

$$\Rightarrow 1-\mathrm{IF}int(\bigcup_{i=0}^{\infty} \mathrm{B}_i) = 1$$

$$\Rightarrow \mathrm{IF}cl(\cap_{i=1}^{\infty}(1-\mathrm{B}_i)) = 1$$

But

$$\mathrm{IF}cl(\cap_{i=1}^{\infty}(1-\mathrm{B}_i)) \subseteq \mathrm{IF}cl(\cap_{i=1}^{N}(1-\mathrm{B}_i))$$

Therefore,

$$\mathrm{IF}cl(\cap_{i=1}^{N}(1-\mathrm{B}_i)) = 1$$

Therefore,

$$\mathrm{IF}cl(\cap_{i=1}^{N}(1-(1-\mathrm{A}_i))) = 1$$

$$\mathrm{IF}cl(\cap_{i=1}^{N} \mathrm{A}_i) = 1$$

Hence (*X, T*) is an intuitionistic fuzzy Volterra space.

Definition 18. [14] An IFTS (*X, T*) is called an intuitionistic fuzzy p-space if countable intersection of intuitionistic fuzzy open sets in (*X, T*) is intuitionistic fuzzy open in (*X, T*).

Proposition 4. If the IFTS (*X, T*) is an intutionistic fuzzy Baire and intuitionistic fuzzy p-space, then (*X, T*) is an intuitionistic fuzzy Volterra space.

Proof. Let $(A_i)'^s(i=1 \; to \; N)$ be intuitionistic fuzzy dense and intuitionistic fuzzy G_δ - sets in (*X,T*). Since (*X, T*) is an intuitionistic fuzzy p-space, the intuitionistic fuzzy G_δ - sets $(A_i)'^s$ are intuitionistic fuzzy open sets in (*X, T*). Hence $(A_i)'^s$ are intuitionistic fuzzy dense and intuitionistic fuzzy open sets in (*X, T*). Then by theorem $(1-A_i)'^s$ are intuitionistic fuzzy nowhere dense sets in (*X, T*). Let $(B_i)'^s(i=1 \; to \; \infty)$ be intuitionistic fuzzy nowhere dense sets in (*X, T*) in which the first *N* intuitionistic fuzzy nowhere dense sets are $(1-A_i)'^s(i=1 \; to \; N)$. Since (*X, T*) is an intuitionistic fuzzy Baire space,

$$\mathrm{IF}int(\bigcup_{i=0}^{\infty} \mathrm{B}_i) = 0$$

Now

$$int(\bigcup_{i=1}^{N} \mathrm{B}_i) \subseteq int(\bigcup_{i=1}^{\infty} \mathrm{B}_i) = 0$$

Implies that

$$int(\bigcup_{i=1}^{N} \mathrm{B}_i) = 0$$

$$int(\bigcup_{i=1}^{N}(1-\mathrm{A}_i)) = 0$$

$$1-int(\bigcup_{i=1}^{N}(1-\mathrm{A}_i)) = 1$$

$$cl(\bigcap_{i=1}^{N} \mathrm{A}_i) = 1$$

Hence (*X, T*) is an intuitionistic fuzzy Volterra space.

Definition 19. [11] An Intuitionistic fuzzy topological space (*X, T*) is called Intuitionistic fuzzy submaximal space if for each Intuitionistic fuzzy set *A* in (*X, T*) such that *IFcl*(*A*) = 1, then $A \in T$

Proposition 5. If the IFTS (*X, T*) is in intuitionistic fuzzy Baire and intuitionistic fuzzy submaximal space, then (*X; T*) is an intuitionistic fuzzy Volterra space.

Proof. Let $(A_i)'^s(i=1 \; to \; N)$ be intuitionistic fuzzy dense and intuitionistic fuzzy G_δ-sets in (*X, T*). Since (*X, T*) is an intuitionistic fuzzy submaximal space, the intuitionistic fuzzy dense sets are intuitionistic fuzzy open in (*X, T*). Therefore $(A_i)'^s(i=1 \; to \; N)$ are intuitionistic fuzzy open sets in (*X, T*). Then proceeding as in the proof of the Propositionosition, we've $IFcl(\bigcap_{i=1}^{N} \mathrm{A}_i) = 1$ Hence (*X, T*) is an intuitionistic fuzzy Volterra space.

Proposition 6. If each intuitionistic fuzzy σ-first category set is an intuitionistic fuzzy σ- nowhere dense set in an IFTS (*X, T*), then (*X, T*) is an intuitionistic fuzzy Volterra space

. *Proof.* Let $(A_i)'^s(i=1 \; to \; N)$ be intuitionistic fuzzy dense and intuitionistic fuzzy G_δ-sets in (*X, T*). Then by theorem $(1-A_i)'^s$ are intuitionistic fuzzy σ-nowhere dense set in (*X, T*). Let $(B_i)'^s(i=1 \; to \; \infty)$ be intuitionistic fuzzy σ-nowhere dense sets in (*X, T*) in which the first *N* intuitionistic fuzzy σ-nowhere dense sets are

$(1-A_i)'s\ (i = 1\ to\ N)$ Now $\bigcup_{i=1}^{\infty} B_i$ is an intuitionistic fuzzy *σ*-first category set in (*X, T*).By hypothesis $\bigcup_{i=1}^{\infty} B_i$ is an intuitionistic fuzzy *σ*-nowhere dense set in (*X, T*). Therefore $\bigcup_{i=1}^{\infty} B_i$ is an intuitionistic fuzzy *Fσ*-set such that $IFint(\bigcup_{i=1}^{\infty} B_i) = 0$

Now $IF\,int(\bigcup_{i=1}^{N} B_i) \subseteq IF\,int(\bigcup_{i=1}^{\infty} B_i) = 0$

Implies that

$$IFint(\bigcup_{i=1}^{N} B_i) = 0$$

$$IF\,int(\bigcup_{i=1}^{N} (1\text{-}A_i)) = 0$$

$$IFcl(\bigcap_{i=1}^{N} A_i) = 1$$

Hence (X,T) is an intuitionistic fuzzy Volterra space.

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