International Journal of Current Advanced Research
ISSN: O: 2319-6475, ISSN: P: 2319-6505, Impact Factor: SJIF: 6.614
Available Online at www.journalijcar.org
Volume: 7| Issue: 1| Special Issue January: 2018 | Page No.109-114
DOI: http://dx.doi.org/10.24327/IJCAR

GENERALIZED HYERS-ULAM-RASSIAS STABILITY OF A FUNCTIONAL EQUATION IN INTUITIONISTIC FUZZY NORMED SPACES

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A B S T R A C T **RESEARCH ARTICLE**

In this paper, we investigate the problem of Hyers- Ulam-Rassias stability of the additive functional equation f(3x + y) + f(x + 3y) = 4f(x) + 4f(y) in Intuitionistic fuzzy normed spaces.

1. Introduction and Preliminaries

In 1940, S. M. Ulam[11] raised the following question. Under what conditions does there exists an additive mapping near an approximately addition mapping? The case of approximately additive functions was solved by D. H. Hyers[4] under certain assumption. In 1978, a generalized version of the theorem of Hyers for approximately linear mapping was given by Th. M. Rassias[8]. The stability concept that was introduced and investigated by Rassias is called the Hyers-Ulam-Rassias stability. During the last decades, the stability problems of several functional equations have been extensively investigated by a number of authors[[1, 2, 3, 5, 6, 7, 9]]and references therein.

In the present paper, the authors determine the stability results concerning the following additive functional equation $f(3x+y)+f(x+3y)=4f(x)+4f(y)$ in intuitionistic fuzzy normed spaces(IFNS).

Here we recall some notations and basic definitions.

Definition 1.1 *A binary operation* $*:[0,1]\times[0,1]\to[0,1]$ *is said to be a continuous* t *-norm if it satisfies the following conditions:*

- $*$ is associative and commutative
- $*$ is continuous
- $a*1=a$ for all $a\in[0,1]$
- $a*b\le c*d$ whenever $a\le c$ and $b\le d$ for each $a,b,c,d\in[0,1]$.

Definition 1.2 *A binary operation* $\Diamond:[0,1]\times[0,1]\to[0,1]$ *is said to be a continuous* t *-conorm if it satisfies the following conditions:*

- $\Diamond$ is associative and commutative
- $\Diamond$ is continuous
- $a\Diamond 0=a$ for all $a\in[0,1]$
- $a\Diamond b\le c\Diamond d$ whenever $a\le c$ and $b\le d$ for each $a,b,c,d\in[0,1]$.

Using the above two definitions, Saadati and Park [10] introduced the concept of intuitionistic fuzzy normed spaces as follows:

Definition 1.3 *The five-tuple* $(X,\mu,\nu,*,\Diamond)$ *is said to be an intuitionistic fuzzy normed spaces(IFNS) if* X *is a vector space,* $*$ *is continuous* t *-norm,* $\Diamond$ *is a continuous* t *-conorm and* μ,ν *are fuzzy sets on* $X\times(0,\infty)$ *satisfying the following conditions. For every* $x,y\in X$ *and* $s,t>0$

- $\mu(x,t)+\nu(x,t)\le 1$
- $\mu(x,t)>0$
- $\mu(x,t)=1$ iff $x=0$
- $\mu(\alpha x,t)=\mu\left(x,\frac{t}{|\alpha|}\right)$ for each $\alpha\neq 0$
- $\mu(x,t)*\mu(y,s)\le\mu(x+y,t+s)$
- $\mu(x,.):(0,\infty)\to[0,1]$ is continuous
- $lim_{t\to\infty}\ \mu(x,t)=1$ and $lim_{t\to 0}\ \mu(x,t)=0$
- $\nu(x,t)<1$
- $\nu(x,t)=0$ iff $x=0$
- $\nu(\alpha x,t)=\nu\left(x,\frac{t}{|\alpha|}\right)$ for each $\alpha\neq 0$
- $\nu(x,t)\Diamond\nu(y,s)\ge\nu(x+y,t+s)$

- $\nu(x,.):(0,\infty)\to[0,1]$ is continuous
- $lim_{t\to\infty}\ \nu(x,t)=0$ and $lim_{t\to 0}\nu(x,t)=1$.

In this case (μ,ν) is called an intuitionstic fuzzy norm.

Let $(X,\mu,\nu,*,\Diamond)$ be an IFNS. Then, a sequence $x=(x_n)$ is said to be intuitinstic fuzzy convergent to $L\in X$ if lim $\mu(x_n-L,t)=1$ and lim $\nu(x_n-L,t)=0$ for all $t>0$. In this case we write $x_n\overset{IF}{\to}L$ as $n\to\infty$.

Let $(X,\mu,\nu,*,\Diamond)$ be an IFNS. Then $x=(x_n)$ is said to be intuitinstic fuzzy Cauchy sequence if lim $\mu(x_{n+p}-x_n,t)=1$ and lim $\nu(x_{n+p}-x_n,t)=0$ for all $t>0$ and $p=1,2,\ldots$.

Let $(X,\mu,\nu,*,\Diamond)$ be an IFNS. Then $(X,\mu,\nu,*,\Diamond)$ is said to be complete if every intuitinstic fuzzy Cauchy sequence in $(X,\mu,\nu,*,\Diamond)$ in intuitinstic fuzzy convergent in $(X,\mu,\nu,*,\Diamond)$.

2. Intuitinstic Fuzzy Stability

The functional equation

$$f(3x+y)+f(x+3y)=4f(x)+4f(y) \tag{2.1}$$

is called an additive functional equation, since the function $f(x)=cx$ is its solution. Every solution of the additive functional equation is said to be an additive mapping.

We start with a generalized Hyers-Ulam-Rassias type theorem in IFNS for an additive functional equation.

Theorem 2.1 *Let X be a linear space and let (Z,μ',ν') be an IFNS. Let $\varphi:X\times X\to Z$ be a function such that for some $\alpha>3$*

$$\mu'\left(\varphi\left(\frac{x}{3},0\right),t\right)\geq\mu'(\varphi(x,0),\alpha t)$$

$$\nu'\left(\varphi\left(\frac{x}{3},0\right),t\right)\geq\nu'(\varphi(x,0),\alpha t) \tag{2.2}$$

and

$$\lim_{n\to\infty}\mu'\left(3^n\varphi\left(\frac{x}{3^n},\frac{y}{3^n}\right),t\right)=1$$

$$\lim_{n\to\infty}\nu'\left(3^n\varphi\left(\frac{x}{3^n},\frac{y}{3^n}\right),t\right)=0$$

for all $x,y\in X$ and $t>0$.

Let (Y,μ,ν) be an intuitionstic fuzzy Banach space and let $f:X\to Y$ be a φ-approximately additive mapping and that

$$\mu(3f(3x+y)+f(x+3y)-4f(x)-4f(y)\geq\mu'(\varphi(x,y),t),$$
$$\nu(3f(3x+y)+f(x+3y)-4f(x)-4f(y)\leq\nu'(\varphi(x,y),t) \tag{2.3}$$

for all $t>0$ and all $x,y\in X$. Then there exists a unique additive mapping $A:X\to Y$ such that

$$\mu(A(x)-f(x),t)\geq\mu'\left(\varphi(x,0),\frac{(\alpha-3)t}{2}\right)and$$

$$\nu(A(x)-f(x),t)\leq\nu'\left(\varphi(x,0),\frac{(\alpha-3)t}{2}\right) \tag{2.4}$$

for all $x\in X$ and all $t>o$.

Proof. Put $y=0$ in(2.3). Then for all $x\in X$ and $t>0$

$$\mu(f(3x)-3f(x),t)\geq\mu'(\varphi(x,0),t)$$

this gives that

$$\mu\left(3f\left(\frac{x}{3}\right)-f(x),t\right)\geq\mu'\left(\varphi\left(\frac{x}{3},0\right),t\right)\geq\mu'(\varphi(x,0),\alpha t),$$

$$\nu\left(3f\left(\frac{x}{3}\right)-f(x),t\right)\leq\nu'\left(\varphi\left(\frac{x}{3},0\right),t\right)\leq\nu'(\varphi(x,0),\alpha t). \tag{2.5}$$

Replacing x by $\frac{x}{3^n}$ in(2.5), we get

$$\mu\left(3^{n+1}f\left(\frac{x}{3^{n+1}}\right)-3^nf\left(\frac{x}{3^n}\right),3^nt\right)\geq\mu'\left(\varphi\left(\frac{x}{3^n},0\right),\alpha t\right)\geq\mu'(\varphi(x,0),\alpha^{n+1}t),and$$

$$\nu\left(3^{n+1}f\left(\frac{x}{3^{n+1}}\right)-3^nf\left(\frac{x}{3^n}\right),3^nt\right)\leq\nu'\left(\varphi\left(\frac{x}{3^n},0\right),\alpha t\right)\leq\nu'(\varphi(x,0),\alpha^{n+1}t).$$

Replacing t by $\frac{t}{\alpha^{n+1}}$, we get

$$\mu\left(3^{n+1} f\left(\frac{x}{3^{n+1}}\right)-3^n f\left(\frac{x}{3^n}\right),\frac{3^n t}{\alpha^{n+1}}\right) \geq \mu'(\varphi(x,0),t), and$$

$$\nu\left(3^{n+1} f\left(\frac{x}{3^{n+1}}\right)-3^n f\left(\frac{x}{3^n}\right),\frac{3^n t}{\alpha^{n+1}}\right) \leq \nu'(\varphi(x,0),t). \qquad (2.6)$$

It follows from $3^n f\left(\frac{x}{3^n}\right)-f(x)=\sum_{j=0}^{n-1}\left(3^{j+1} f\left(\frac{x}{3^{j+1}}\right)-3^j f\left(\frac{x}{3^j}\right)\right)$ and (2.6) that

$$\mu\left(3^n f\left(\frac{x}{3^n}\right)-f(x),\sum_{j=0}^{n-1}\frac{3^j t}{\alpha^{j+1}}\right)$$

$$\geq \prod_{j=0}^{n-1}\mu\left(3^{j+1} f\left(\frac{x}{3^{j+1}}\right)-3^j f\left(\frac{x}{3^j}\right),\frac{3^j t}{\alpha^{j+1}}\right) \geq \mu'(\varphi(x,0),t) and$$

$$\nu\left(3^n f\left(\frac{x}{3^n}\right)-f(x),\sum_{j=0}^{n-1}\frac{3^j t}{\alpha^{j+1}}\right)$$

$$\leq \coprod_{j=0}^{n-1}\nu\left(3^{j+1} f\left(\frac{x}{3^{j+1}}\right)-3^j f\left(\frac{x}{3^j}\right),\frac{3^j t}{\alpha^{j+1}}\right) \leq \nu'(\varphi(x,0),t) \qquad (2.7)$$

for all $x \in X, t>0$ and $n>0$ where $\prod_{j=0}^{n-1} a_j = a_1 * a_2 * ... * a_n, \coprod_{j=0}^{n-1} b_j = b_1 \Diamond b_2 \Diamond ... \Diamond b_n$.

By replacing x with $\frac{x}{3^m}$ in(2.7), we obtain

$$\mu\left(3^{n+m} f\left(\frac{x}{3^{n+m}}\right)-3^m f\left(\frac{x}{3^m}\right),\sum_{j=0}^{n-1}\frac{3^{j+m} t}{\alpha^{j+m+1}}\right) \geq \mu'\left(\varphi\left(\frac{x}{3^m},0\right),t\right) \geq \mu'(\varphi(x,0),t), and$$

$$\nu\left(3^{n+m} f\left(\frac{x}{3^{n+m}}\right)-3^m f\left(\frac{x}{3^m}\right),\sum_{j=0}^{n-1}\frac{3^{j+m} t}{\alpha^{j+m+1}}\right) \leq \nu'\left(\varphi\left(\frac{x}{3^m},0\right),t\right) \leq \nu'(\varphi(x,0),t)$$

Thus,

$$\mu\left(3^{n+m} f\left(\frac{x}{3^{n+m}}\right)-3^m f\left(\frac{x}{3^m}\right),\sum_{j=m}^{n+m-1}\frac{3^j t}{\alpha^{j+1}}\right) \geq \mu'(\varphi(x,0),t), and$$

$$\nu\left(3^{n+m} f\left(\frac{x}{3^{n+m}}\right)-3^m f\left(\frac{x}{3^m}\right),\sum_{j=m}^{n+m-1}\frac{3^j t}{\alpha^{j+1}}\right) \leq \nu'(\varphi(x,0),t)$$

for all $x \in X, t>0, m \geq 0$ and $n \geq 0$. Hence

$$\mu\left(3^{n+m} f\left(\frac{x}{3^{n+m}}\right)-3^m f\left(\frac{x}{3^m}\right),t\right) \geq \mu'\left(\varphi(x,0),\frac{t}{\sum_{j=m}^{n+m-1}\frac{3^j t}{\alpha^{j+1}}}\right) and$$

$$\nu\left(3^{n+m} f\left(\frac{x}{3^{n+m}}\right)-3^m f\left(\frac{x}{3^m}\right),t\right) \leq \nu'\left(\varphi(x,0),\frac{t}{\sum_{j=m}^{n+m-1}\frac{3^j t}{\alpha^{j+1}}}\right) \qquad (2.8)$$

for all $x \in X, t>0, m \geq 0$ and $n \geq 0$. Since $\alpha > 3$ and $\sum_{j=0}^{\infty}\left(\frac{3}{\alpha}\right) < \infty$, the Cauchy criterion for convergence in IFNS shows that $3^n f\left(\frac{x}{3^n}\right)$ is a Cauchy sequence in (Y,μ,ν). Since (Y,μ,ν) is complete, this sequence converges to some point A(x) Y. *Fix* x X *and* m=0 *in* (2.8)

$$,we obtain \mu\left(3^n f\left(\frac{x}{3^n}\right)-f(x),t\right)\geq \mu'\left(\varphi(x,0),\frac{t}{\sum_{j=0}^{n-1}\frac{3^j}{\alpha^{j+1}}}\right) and \nu\left(3^n f\left(\frac{x}{3^n}\right)-f(x),t\right)\leq \nu'\left(\varphi(x,0),\frac{t}{\sum_{j=0}^{n-1}\frac{3^j}{\alpha^{j+1}}}\right)$$ for all $t>0$ and

$n>0$. Thus we obtain

$$\mu(A(x)-f(x),t)\geq \mu\left(A(x)-3^n f\left(\frac{x}{3^n}\right),\frac{t}{2}\right)*\mu\left(3^n f\left(\frac{x}{3^n}-f(x)\right),\frac{t}{2}\right)$$

$$\geq \mu'\left(\varphi(x,0),\frac{t}{2\sum_{j=0}^{n-1}\frac{3^j}{\alpha^{j+1}}}\right),$$

$$\nu(A(x)-f(x),t)\leq \nu\left(A(x)-3^n f\left(\frac{x}{3^n}\right),\frac{t}{2}\right)\Diamond\nu\left(3^n f\left(\frac{x}{3^n}-f(x)\right),\frac{t}{2}\right)$$

$$\leq \nu'\left(\varphi(x,0),\frac{t}{2\sum_{j=0}^{n-1}\frac{3^j}{\alpha^{j+1}}}\right),$$

for large n. Taking the limit as $n\to\infty$ and using the definition of IFNS, we get

$$\mu(A(x)-f(x),t)\geq \mu'\left(\varphi(x,0),\frac{(\alpha-3)t}{2}\right)$$

and

$$\nu(A(x)-f(x),t)\leq \nu'\left(\varphi(x,0),\frac{(\alpha-3)t}{2}\right)$$

, for all $x\in X, t>0$. Replace x and y by $\frac{x}{3^n}$ and $\frac{y}{3^n}$, respectively in(2.3), we have

$$\mu\left(3^n f\left(\frac{3x+y}{3^n}\right)+3^n f\left(\frac{x+3y}{3^n}\right)-3^n f\left(\frac{4x}{3^n}\right)-3^n f\left(\frac{4y}{3^n}\right)\right)\geq \mu'\left(\varphi\left(\frac{x}{3^n},\frac{y}{3^n}\right),\frac{t}{3^n}\right)$$

and

$$\nu\left(3^n f\left(\frac{3x+y}{3^n}\right)+3^n f\left(\frac{x+3y}{3^n}\right)-3^n f\left(\frac{4x}{3^n}\right)-3^n f\left(\frac{4y}{3^n}\right)\right)\leq \nu'\left(\varphi\left(\frac{x}{3^n},\frac{y}{3^n}\right),\frac{t}{3^n}\right)$$

for all $x,y\in X, t>0$. Since

$$lim_{n\to\infty}\mu'\left(3^n\varphi\left(\frac{x}{3^n},\frac{y}{3^n}\right),t\right)=1,$$

$$lim_{n\to\infty}\nu'\left(3^n\varphi\left(\frac{x}{3^n},\frac{y}{3^n}\right),t\right)=0,$$

for all $x,y\in X, t>0$. We notice that A satisies (2.1). Therefore A is an additive mapping.

To prove the uniqueness of the additive mapping A, assume that there exists a additve mapping $A':X\to Y$ which satisfies(2.4).

For fix $x \in X$, clearly $3^n A\left(\frac{x}{3^n}\right) = A(x)$ and $3^n A'\left(\frac{x}{3^n}\right) = A'(x)$ for all $n \in N$. It follows from (2.4) that

$$\mu(A(x) - A'(x), t) = \mu\left(3^n A\left(\frac{x}{3^n}\right) - 3^n A'\left(\frac{x}{3^n}\right), t\right)$$
$$\geq \mu\left(3^n A\left(\frac{x}{3^n}\right) - 3^n f\left(\frac{x}{3^n}\right), \frac{t}{2}\right)$$
$$* \mu\left(3^n f\left(\frac{x}{3^n}\right) - 3^n A'\left(\frac{x}{3^n}\right), \frac{t}{2}\right)$$
$$\geq \mu'\left(\varphi\left(\frac{x}{3^n}, 0\right), \frac{(\alpha - 3)t}{2 \cdot 3^n}\right)$$
$$\geq \mu'\left(\varphi(x, 0), \frac{\alpha^n (\alpha - 3)t}{2 \cdot 3^n}\right)$$

and similarly

$$\nu(A(x) - A'(x), t) \leq \nu'\left(\varphi(x, 0), \frac{\alpha^n (\alpha - 3)t}{2 \cdot 3^n}\right).$$

Since $lim_{n \to \infty} \frac{\alpha^n (\alpha - 3)}{2 \cdot 3^n} = \infty$ as $\alpha > 3$, we get $lim_{n \to \infty} \mu'\left(\varphi(x, 0), \frac{\alpha^n (\alpha - 3)t}{2 \cdot 3^n}\right) = 1$, and $lim_{n \to \infty} \nu'\left(\varphi(x, 0), \frac{\alpha^n (\alpha - 3)t}{2 \cdot 3^n}\right) = 0$. Therefore $\mu(A(x) - A'(x), t) = 1$ and $\nu(A(x) - A'(x), t) = 0$, for all $t > 0$. Hence $A(x) = A'(x)$. This completes the proof.

In the following theroem we consider $0 < \alpha < 3$

Theorem 2.2 *Let* X *be a linear space and let* (Z, μ', ν') *be an IFNS. Let* $\varphi : X \times X \to Z$ *be a function such that for some*
$0 < \alpha < 3$

$$\mu'(\varphi(3x, 0), t) \geq \mu'(\alpha\varphi(x, 0), t) and$$
$$\nu'(\varphi(3x, 0), t) \leq \nu'(\alpha\varphi(x, 0), t),$$

$lim_{n \to \infty} \mu'(\varphi(3^n x, 3^n y), 3^n t) = 1$ and $lim_{n \to \infty} \nu'(\varphi(3^n x, 3^n y), 3^n t) = 0$ for all $x, y \in X$ and $t > 0$. Let (Y, μ, ν) be an intuitionistic fuzzy banach space and let $f : X \to Y$ be a φ-approximately additive mapping in the sense that

$$\mu(3f(3x + y) + f(x + 3y) - 4f(x) - 4f(y), t) \geq \mu'(\varphi(x, y), t) and$$
$$\nu(3f(3x + y) + f(x + 3y) - 4f(x) - 4f(y), t) \leq \nu'(\varphi(x, y), t)$$

for all $x, y \in X$ and $t > 0$. Then there exists a unique additive mapping $A : X \to Y$ such that

$$\mu(A(x) - f(x), t) \geq \mu'\left(\varphi(x, 0), \frac{(3 - \alpha)t}{2}\right) and$$
$$\nu(A(x) - f(x), t) \leq \nu'\left(\varphi(x, 0), \frac{(3 - \alpha)t}{2}\right)$$

for all $x \in X$ and $t > 0$.

Proof. The proof of this theoem is similar to Theorem(2.1) . Here we represent the sketch of proof. Put $y = 0$ in(2.3) we get

$$\mu\left(\frac{f(3x)}{3} - f(x), t\right) \geq \mu'(\varphi(x, 0), t) and$$
$$\nu\left(\frac{f(3x)}{3} - f(x), t\right) \leq \nu'(\varphi(x, 0), t),$$

for all $x \in X$ and $t > 0$. So

$$\mu\left(\frac{f(3^{n+1}x)}{3}-f(3^n x),t\right)\geq\mu'\left(\varphi(x,0),\frac{t}{\alpha^n}\right) and$$

$$\nu\left(\frac{f(3^{n+1}x)}{3}-f(3^n x),t\right)\leq\nu'\left(\varphi(x,0),\frac{t}{\alpha^n}\right),$$

for all $x\in X$ and $t>0$. For each $x\in X, n\geq 0, m\geq 0$ and $t>0$, we deduce that

$$\mu\left(\frac{f(3^{n+m}x)}{3^{n+m}}-\frac{f(3^m x)}{3^m},t\right)\geq\mu'\left(\varphi(x,0),\frac{t}{\sum_{j=m}^{n+m-1}\frac{\alpha^j}{3^{j+1}}}\right) and$$

$$\nu\left(\frac{f(3^{n+m}x)}{3^{n+m}}-\frac{f(3^m x)}{3^m},t\right)\leq\nu'\left(\varphi(x,0),\frac{t}{\sum_{j=m}^{n+m-1}\frac{\alpha^j}{3^{j+1}}}\right) \tag{2.9}$$

for all $x\in X$, $t>0$, and $m,n\geq 0.$ Thus, $\left\{\frac{f(3^n x)}{3n}\right\}$ is a Cauchy sequence in intuitionistic fuzzy Banach space. There exist a function $A: X\to Y$ defined by $A(x) = lim_{n\to\infty}\frac{f(3^n x)}{3n}$ and put $m=0$ in (2.9) we obtain

$$\mu\big(A(x)-f(x),t\big)\geq\mu'\left(\varphi(x,0),\frac{(3-\alpha)t}{2}\right) and$$

$$\nu\big(A(x)-f(x),t\big)\leq\nu'\left(\varphi(x,0),\frac{(3-\alpha)t}{2}\right)$$

for all $x\in X$ and $t>0$.

This completes the proof.

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