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A STUDY ON SOME OPERATIONS OVER INTUITIONISTIC FUZZY GRAPHS OF SECOND TYPE

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A B S T R A C T **RESEARCH ARTICLE**

In this paper, we define some operations on intuitionistic fuzzy graphs ofsecond type and establish some of their properties.

Keywords:

Intuitionistic fuzzy sets, Intuitionistic fuzzy sets of second type,Intuitionistic fuzzy graphs, Intuitionistic fuzzy graphs of second type, union, join, constant.

2010 AMS Subject Classification: 05C72, 03E72

1. Introduction

Fuzzy sets were introduced by Lotfi. A. Zadeh[11] in 1965 as a generalization of classical (crisp) sets. Further the fuzzy sets are generalized by Krassimir.T.Atanassov [1] in which he has taken non-membership values also into consideration and introduced Intuitionistic fuzzy sets [IFS] and their extensions like Intuitionisticfuzzy sets of second type [IFSST], Intuitionistic L-fuzzy sets [ILFS] and TemporalIntuitionistic fuzzy sets [TIFS]. A. Shannon and K. T. Atanassov [7] discussedthe theory of Intuitionistic fuzzy graphs. R. Parvathi and M. G. Karunambigai[3,5,6] introduced Intuitionistic Fuzzy Graphs [IFG] elaborately and analyzed itscomponents also introduced and studied the concept of union, join and constant onIFG. Further S. Ismail Mohideen, A. NagoorGani, B. Fathima Kani and C.Yasmin [2] discussed the properties of operations on intuitionistic fuzzygraphs. The present authors [8,9,10] introduced the extension of IFG namelyIntuitionistic Fuzzy Graphs of Second Type [IFGST] and defined some basic concepts.

In section 2, we give some basicdefinitions and in section 3, we define some new operations namely union and join onIFGST. Also establish some of their properties.The paper is concluded in section 4.

2. Preliminaries:

In this section, we give some basic definitions.

Definition 2.1 [5] An Intuitionistic Fuzzy Graph [IFG]is of the form $G = [V, E]$where

(i) $V = \{v_1, v_2, \dots v_n\}$such that $\mu_1: V \to [0,1]$ and $\nu_1: V \to [0,1]$denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and
$0 \le \mu_1(v_i) + \nu_1(v_i) \le 1$for every
$v_i \in V, \ (i = 1,2, \dots n)$
(ii) $E \subseteq V \times V$where$\mu_2: V \times V \to [0,1]$ and $\nu_2: V \times V \to [0,1]$ are such that
$\mu_2(v_i, v_j) \le min[\mu_1(v_i), \mu_1(v_j)]$,
$\nu_2(v_i, v_j) \le max[\nu_1(v_i), \nu_1(v_j)]$
and$0 \le \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \le 1$ for every $(v_i, v_j) \in E, \ (i, j = 1,2, \dots n)$

Definition 2.2 [6] Let $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ be two IFGs with $V_1 \cap V_2 \neq \phi$ and $G = G_1 \cup G_2 = [V_1 \cup V_2, E_1 \cup E_2]$ be the union of $G_1$and G_2. Then the union of IFGs $G_1$and G_2 is an IFG defined by,

$$(\mu_1 \cup \mu_1')(u) = \begin{cases} \mu_1(u), & if \ u \in V_1 - V_2 \\ \mu_1'(u), & if \ u \in V_2 - V_1 \\ max(\mu_1(u), \mu_1'(u)), & if \ u \in V_1 \cap V_2 \end{cases}$$
$$(\nu_1 \cup \nu_1')(u) = \begin{cases} \nu_1(u), & if \ u \in V_1 - V_2 \\ \nu_1'(u), & if \ u \in V_2 - V_1 \\ min(\nu_1(u), \nu_1'(u)), & if \ u \in V_1 \cap V_2 \end{cases}$$

$$(\mu_2 \cup \mu_2')(u_iu_j) = \begin{cases} \mu_2(u_iu_j), & if \ u_iu_j \in E_1 - E_2 \\ \mu_2'(u_iu_j), & if \ u_iu_j \in E_2 - E_1 \\ max(\mu_2(u_iu_j), \mu_2'(u_iu_j)), & if \ u_iu_j \in E_1 \cap E_2 \end{cases}$$
$$(\nu_2 \cup \nu_2')(u_iu_j) = \begin{cases} \nu_2(u_iu_j), & if \ u_iu_j \in E_1 - E_2 \\ \nu_2'(u_iu_j), & if \ u_iu_j \in E_2 - E_1 \\ min(\nu_2(u_iu_j), \nu_2'(u_iu_j)), & if \ u_iu_j \in E_1 \cap E_2 \end{cases}$$

where (μ_1, ν_1) and (μ_1', ν_1') are the vertex degree of membership and non membership of the elements of V_1in $G_1$and V_2in G_2 respectively. Also (μ_2, ν_2) and (μ_2', ν_2') are the edge degree of membership and non-membership of the elements of E_1in $G_1$and E_2in G_2 respectively.

Definition 2.3 [6] The join of two intuitionistic fuzzy graphs $G_1 = [V_1, E_1]$ and $G_2 = [V_2, E_2]$ is an IFG, $G = G_1 + G_2 = [V_1 \cup V_2, E_1 \cup E_2 \cup E']$ where $E'$is the set of all edges joining the vertices of V_1 with the vertices of $V_2$is defined by

$(\mu_1 + \mu_1')(u) = (\mu_1 \cup \mu_1')(u)$
$if \ u \in V_1 \cup V_2$
$(\nu_1 + \nu_1')(u) = (\nu_1 \cup \nu_1')(u)$
$if \ u \in V_1 \cup V_2$

$$(\mu_2 + \mu_2')(u_iu_j) = \begin{cases} (\mu_2 \cup \mu_2')(u_iu_j), & if\ u_iu_j \in E_1 \cup E_2 \\ min\,(\mu_1(u_i), \mu_1'(u_j)), & if\ u_iu_j \in E' \end{cases}$$

$$(\nu_2 + \nu_2')(u_iu_j) = \begin{cases} (\nu_2 \cup \nu_2')(u_iu_j), & if\ u_iu_j \in E_1 \cup E_2 \\ max\,(\nu_1(u_i), \nu_1'(u_j)), & if\ u_iu_j \in E' \end{cases}$$

Definition 2.4 [3]

Let $G = [(\mu_{1i},\ \nu_{1i}),\ (\mu_{2ij},\ \nu_{2ij})]$ be an IFG. If $d_\mu(v_i) = k_i$ and $d_\nu(v_j) = k_j$ for all $v_i,\ v_j \in V$ then G is called as $(k_i,\ k_j)$– Constant IFG (or) Constant IFG of degree $(k_i,\ k_j)$.

Definition 2.5 [8] An Intuitionistic Fuzzy Graphs of Second Type [IFGST]is of the form $G = [V, E]$where

(i) $V = \{v_1, v_2, \dots v_n\}$such that $\mu_1: V \to [0,1]$ and $\nu_1: V \to [0,1]$denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and
$0 \le \mu_1(v_i)^2 + \nu_1(v_i)^2 \le 1$ for every $v_i \in V$, $(i = 1,2, \dots n)$
(ii) $E \subseteq V \times V$where$\mu_2: V \times V \to [0,1]$ and $\nu_2: V \times V \to [0,1]$ are such that

$$\mu_2(v_i, v_j) \le min\left[\mu_1(v_i)^2, \mu_1(v_j)^2\right],$$
$$\nu_2(v_i, v_j) \le max\left[\nu_1(v_i)^2, \nu_1(v_j)^2\right]$$

and$0 \le \mu_2(v_i, v_j)^2 + \nu_2(v_i, v_j)^2 \le 1$ for every $(v_i, v_j) \in E$, $(i, j = 1,2, \dots n)$

Definition 2.6 [10] Let $G = [V,\ E]$ is an IFGST. Suppose $d_\mu(v_i) = k_i$and $d_\nu(v_j) = k_j$ for all $v_i,\ v_j \in V$ then the graph G is called as constant IFGST of degree $(k_i,\ k_j)$ or $(k_i,\ k_j)$ - constant IFGST.

3. Some Operations on Intuitionistic Fuzzy Graphs of Second Type

In this section, we define the new union and join operations over intuitionistic fuzzy graphs of second type. Also establish some of their properties.

Definition 3.1 Let $G_1 = [V_1,\ E_1]$ and $G_2 = [V_2,\ E_2]$ be two IFGST with $V_1 \cap V_2 \ne \phi$ and $G = G_1 \cup G_2$ where $G_1 \cup G_2 = [V_1 \cup V_2,\ E_1 \cup E_2]$ then the union of G_1 and $G_2$is also an IFGST defined by,

$$(\mu_1 \cup \mu_1')(u) = \begin{cases} \mu_1(u), & if\ u \in V_1 - V_2 \\ \mu_1'(u), & if\ u \in V_2 - V_1 \\ max(\mu_1(u), \mu_1'(u)), & if\ u \in V_1 \cap V_2 \end{cases}$$

$$(\nu_1 \cup \nu_1')(u) = \begin{cases} \nu_1(u), & if\ u \in V_1 - V_2 \\ \nu_1'(u), & if\ u \in V_2 - V_1 \\ min(\nu_1(u), \nu_1'(u)), & if\ u \in V_1 \cap V_2 \end{cases}$$

$$(\mu_2 \cup \mu_2')(u_iu_j) = \begin{cases} \mu_2(u_iu_j), & if\ u_iu_j \in E_1 - E_2 \\ \mu_2'(u_iu_j), & if\ u_iu_j \in E_2 - E_1 \\ max(\mu_2(u_iu_j), \mu_2'(u_iu_j)), & if\ u_iu_j \in E_1 \cap E_2 \end{cases}$$

$$(\nu_2 \cup \nu_2')(u_iu_j) = \begin{cases} \nu_2(u_iu_j), & if\ u_iu_j \in E_1 - E_2 \\ \nu_2'(u_iu_j), & if\ u_iu_j \in E_2 - E_1 \\ min(\nu_2(u_iu_j), \nu_2'(u_iu_j)), & if\ u_iu_j \in E_1 \cap E_2 \end{cases}$$

where (μ_1, ν_1) and (μ_1', ν_1') are the vertex degree of membership and non membership of the elements of V_1in $G_1$and V_2in G_2 respectively. Also (μ_2, ν_2) and (μ_2', ν_2') are the edge degree of membership and non-membership of the elements of E_1in $G_1$and E_2in G_2 respectively.

Example 3.1

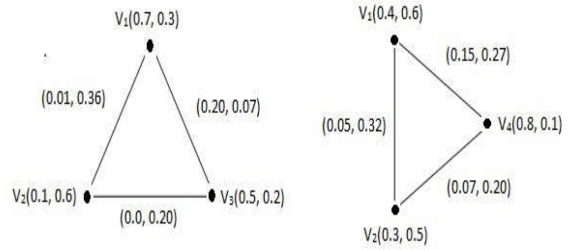

Fig 1 G_1andG_2

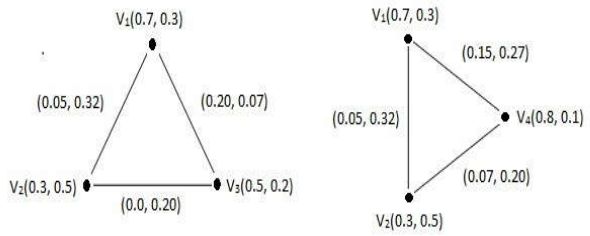

Fig 2 $G_1 \cup G_2$

Definition 3.2 Let $G_1 = [V_1,\ E_1]$ and $G_2 = [V_2,\ E_2]$ be two IFGST and $G = G_1 + G_2$ where $G_1 + G_2 = [V_1 \cup V_2,\ E_1 \cup E_2 \cup E']$ here $E'$is the set of all edges joining the vertices of V_1 with the vertices of V_2 then the union of G_1 and $G_2$is also an IFGST defined by,

$(\mu_1 + \mu_1')(u) = (\mu_1 \cup \mu_1')(u)$
$if\ u \in V_1 \cup V_2$
$(\nu_1 + \nu_1')(u) = (\nu_1 \cup \nu_1')(u)$
$if\ u \in V_1 \cup V_2$

$$(\mu_2 + \mu_2')(u_iu_j) = \begin{cases} (\mu_2 \cup \mu_2')(u_iu_j), & if\ u_iu_j \in E_1 \cup E_2 \\ min\,(\mu_1^2(u_i), (\mu_1')^2(u_j)), & if\ u_iu_j \in E' \end{cases}$$

$$(\nu_2 + \nu_2')(u_iu_j) = \begin{cases} (\nu_2 \cup \nu_2')(u_iu_j), & if\ u_iu_j \in E_1 \cup E_2 \\ max\,(\nu_1^2(u_i), (\nu_1')^2(u_j)), & if\ u_iu_j \in E' \end{cases}$$

Example 3.2

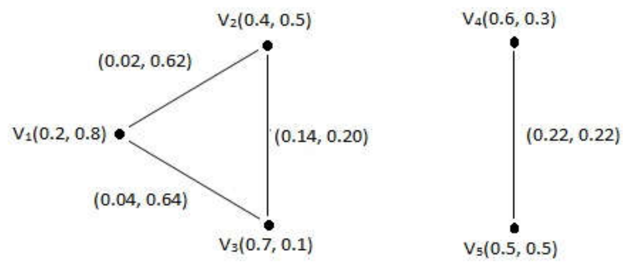

Fig 3 G_1andG_2

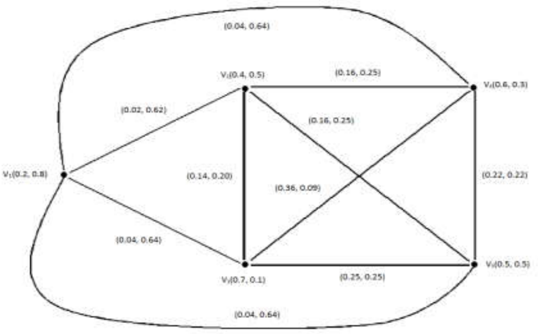

Fig 4 $G_1 + G_2$

Theorem 3.1: Let G_1 and G_2 be any two IFGST with$V_1 \cap V_2 = \phi$. Then $G_1 \cup G_2$ is $(k_1,\ k_2)$ constant IFGST if and only if G_1 and G_2 are $(k_1,\ k_2)$constant IFGST.

Proof: Given that $G_1 \cup G_2$ is $(k_1,\ k_2)$constant IFGST then we have,

$d_{G_1 \cup G_2}(v) = d_{G_1}(v)ifv \in V_1$ and $d_{G_1 \cup G_2}(v) = d_{G_2}(v)$ if $v \in V_2$

Since$V_1 \cap V_2 = \phi$, therefore we have,

$$d_{G_1 \cup G_2}(v) = (k_1,\ k_2) \Leftrightarrow d_{G_1}(v) = (k_1,\ k_2) \text{and}$$
$$d_{G_1 \cup G_2}(v) = (k_1,\ k_2) \Leftrightarrow d_{G_1}(v) = (k_1,\ k_2)$$

This completes the proof.

Theorem 3.2: Let G_2 be an IFGST and G_1 be a subgraph of G_2 and $G_1 \cup G_2$ is a$(k_1,\ k_2)$ constant IFGST then G_2 is a $(k_1,\ k_2)$ constant IFGST.

Proof: Let G_1 be a subgraph of an IFGST G_2 and $G_1 \cup G_2$ is a $(k_1,\ k_2)$ constant IFGST.

We have, $\mu_1(v_i) \leq \mu_1'(v_i), \nu_1(v_i) \geq \nu_1'(v_i)$
and$\mu_2(v_i, v_j) \leq \mu_2'(v_i, v_j)$,
$\nu_2(v_i, v_j) \geq \nu_2'(v_i, v_j)$

where$((\mu_1, \nu_1), (\mu_2, \nu_2))$ be the degree of membership and non-membership of the elements $v_i \in V_1$ in G_1 and $((\mu_1', \nu_1'), (\mu_2', \nu_2'))$ be the degree of membership and non-membership of the elements $v_i \in V_2$ in G_2.

Clearly, $(\mu_1 \cup \mu_1')(v_i) = \mu_1'(v_i)$,
$(\nu_1 \cup \nu_1')(v_i) = \nu_1'(v_i)$
and$(\mu_2 \cup \mu_2')(v_i) = \mu_2'(v_i)$,
$(\nu_2 \cup \nu_2')(v_i) = \nu_2'(v_i)$
Therefore $G_1 \cup G_2 = ((\mu_1', \nu_1'), (\mu_2', \nu_2'))$
$G_1 \cup G_2 = G_2$
So G_2 is a $(k_1,\ k_2)$ constant IFGST.
This completes the proof.

Theorem 3.3: Let G_1 and G_2 be two IFGST with $V_1 \cap V_2 \neq \phi$ and none of V_1andV_2 is a subset of others. If $G_1 \cup G_2$ is a $(k_1,\ k_2)$constant IFGST then G_1 and $G_2$are not a constant IFGST.

Proof: Let G_1 and G_2 be two IFGST and $V_1 \cap V_2 \neq \phi$ then
We may have either $E_1 \cap E_2 = \phi$ (or) $E_1 \cap E_2 \neq \phi$
We prove this theorem in two cases.

Case 1

If $E_1 \cap E_2 = \phi$
Given that $G_1 \cup G_2$ is a constant IFGST
We have $d_{G_1 \cup G_2}(v) = (k_1, k_2)$

$$d_{G_1 \cup G_2}(v) = \begin{cases} d_{G_1}(v), & if\ \ v \in V_1 - V_2 \\ d_{G_2}(v), & if\ \ v \in V_2 - V_1 \\ d_{G_1}(v) + d_{G_2}(v), & if\ \ v \in V_1 \cap V_2 \end{cases}$$

Since none of $V_1$and V_2 are subset of others and $V_1 \cap V_2 = \phi$
So both $V_1 - V_2$ and $V_2 - V_1$ are non-empty.
Let $v \in V_1 - V_2$then $d_{G_1}(v) = (k_1, k_2)$ and if $u \in V_2 - V_1$then $d_{G_2}(u) = (k_1, k_2)$
Now, let $w \in V_1 \cap V_2$
Then $d_{G_1}(w) + d_{G_2}(w) = (k_1, k_2)$
$\Rightarrow d_{G_1}(w) < (k_1, k_2)$
and $d_{G_2}(w) < (k_1, k_2)$
Therefore$d_{G_1}(v) \neq d_{G_1}(w)$
and $d_{G_2}(u) \neq d_{G_2}(w)$
This gives the contradiction to the definition of constant IFGST.
Hence G_1 and G_2 are not constant IFGST.

Case 2

If $E_1 \cap E_2 \neq \phi$
If some edge incident to v is lies in $E_1 \cap E_2$ then

$$d_{G_1 \cup G_2} = d_{G_1}(v) + d_{G_2}(v) - \sum_{vu \in E_1 \cap E_2} min\ \{\mu_2(vu), \mu_2'(vu)\}$$
$$(k_1, k_2) = d_{G_1}(v) + d_{G_2}(v) - \sum_{vu \in E_1 \cap E_2} min\ \{\mu_2(vu), \mu_2'(vu)\}$$

Therefore we have $(k_1, k_2) > d_{G_1}(v)$ and $(k_1, k_2) > d_{G_2}(v)$
Now,$u \in V_1 - V_2$ and $w \in V_2 - V_1$ then
$d_{G_1}(u) = (k_1, k_2)$and$d_{G_1}(w) = (k_1, k_2)$
So, $d_{G_1}(v) \neq d_{G_1}(u)$; $d_{G_2}(v) \neq d_{G_2}(w)$
Hence G_1 and G_2 are not constant IFGST.
This completes the proof.

Theorem 3.4: Let G_1 and G_2 be two IFGST with $V_1 \cap V_2 \neq \phi$. If G_1 and G_2 areconstant IFGST then $G_1 \cup G_2$ is not constant IFGST.

Proof: Given that G_1 and G_2 be two IFGST and $V_1 \cap V_2 \neq \phi$ also G_1 and G_2 areconstant IFGST thenwe have, $d_{G_1}(v_1) = (k_1, k_2)$ for all $v_1 \in V_1$ and $d_{G_2}(v_2) = (k_3, k_4)$ for all $v_2 \in V_2$
Now,

$d_{G_1 \cup G_2}(v) = d_{G_1}(v) = (k_1, k_2)$if for all
$v \in V_1 - V_2$ and
$d_{G_1 \cup G_2}(v) = d_{G_2}(v) = (k_3, k_4)$if for all
$v \in V_2 - V_1$
Suppose there is a vertex $w \in V_1 \cap V_2$ such that

$$d_{G_1 \cup G_2}(w) = d_{G_1}(w) + d_{G_2}(w)$$
$$d_{G_1 \cup G_2}(w) = (k_1, k_2) + (k_3, k_4)$$
$$d_{G_1 \cup G_2}(w) = ((k_1 + k_3), (k_2 + k_4))$$

Therefore $d_{G_1 \cup G_2}(v) \neq d_{G_1 \cup G_2}(w)$
Hence $G_1 \cup G_2$ is not constant IFGST.
This completes the proof.

Theorem 3.5: Let G_1 and G_2 be two IFGST with $V_1 \cap V_2 = \phi$. If $G_1 + G_2$ is (k_1, k_2) constant IFGST then G_1 and G_2 are constant IFGST.

Proof: Let G_1 and G_2 be two IFGST such that $V_1 \cap V_2 = \phi$. Also given that $G_1 + G_2$ is (k_1, k_2) constant IFGST.
For any $v \in V_1$ then we have,

$$d_{G_1+G_2}(v) = d_{G_1}(v) + \sum_{vw \in E'} \begin{bmatrix} min(\mu_1^2(v), (\mu_1')^2(w)), \\ max(\nu_1^2(v), (\nu_1')^2(w)) \end{bmatrix}$$

$$(k_1 + k_2) = d_{\mu_1,\nu_1}(v) + \sum_{vw \in E'} \begin{bmatrix} min(\mu_1^2(v), (\mu_1')^2(w)), \\ max(\nu_1^2(v), (\nu_1')^2(w)) \end{bmatrix}$$

$$k_1 = d_{\mu_1}(v) + \sum_{vw \in E'} min(\mu_1^2(v), (\mu_1')^2(w))$$

Since $G_1 + G_2$ is a constant IFGST.
We have

$$\sum_{vw \in E'} min(\mu_1^2(v), (\mu_1')^2(w)) = k$$

Then
$k_1 = d_{\mu_1}(v) + k \Rightarrow d_{\mu_1}(v) = k_1 - k = k_3$ forall $v \in V_1$
Similarly we have,
$k_2 = d_{\nu_1}(v) + l \Rightarrow d_{\nu_1}(v) = k_2 - l = k_4$ forall $v \in V_1$
Hence G_1 is (k_3, k_4) constant IFGST.
In a similar way we can prove G_2 is (k_5, k_6) constant IFGST.
This completes the proof.

Conclusion

In this paper, we have defined the union and join operations on IFGST and established some of their properties. In future we will study some more properties and applications of IFGST in real life situations.

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