International Journal of Current Advanced Research
ISSN: O: 2319-6475, ISSN: P: 2319-6505, Impact Factor: SJIF: 6.614
Available Online at www.journalijcar.org
Volume: 7| Issue: 1| Special Issue January: 2018 | Page No. 86-90
DOI: http://dx.doi.org/10.24327/IJCAR

DEFUZZIFICATION FOR PENTAGONAL FUZZY NUMBERS

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A B S T R A C T

RESEARCH ARTICLE

Fuzzy numbers are widely used tool to represent impreciseness in the natural phenomenon. Pentagonal Fuzzy Number (PFN) is a type of fuzzy number, where the left and right continuous functions have the variations in the α-level. In this paper, we made a study on traditional defuzzification methods where the inputs are of pentagonal fuzzy numbers. Through this paper, we introduce a new defuzzification formula to calculate the crisp output from two vague pentagonal fuzzy number inputs.

Keywords:

Pentagonal fuzzy number, defuzzification, centroid, fuzzy center of PFN, area of PFN, center of sums.

1. Introduction

Impreciseness in the natural phenomenon is inflexible to quantify. To handle this inflexible nature, Lotfi A. Zadeh developed a theory called fuzzy set theory in the year 1965 [9], which has its roots in eastern logic. Fuzzy number is a tool to represent the vague inflexible notion numerically. The real life circumstance with impreciseness is controlled by fuzzy control system. Fuzzy control system [1] [8] is made up of three major components such as, fuzzification (structuring crisp inputs into linguistic fuzzy inputs), fuzzy inference system (processing linguistic fuzzy input variables by programming with suitable inference rules, typically fuzzy IF-THEN rules) and defuzzification [3] (commonly referred as a reverse process of fuzzification). In any imprecise situation, human and their subjective judgments (opinions) are more complex to process and compute. Fuzzy number evolution reduced the complex intricacy that persists while processing the human subjective judgments (opinions). Most of the fuzzy control system has adopted fuzzy number as a tool to define the linguistic input variables. Triangular and trapezoidal fuzzy membership functions are the commonly used fuzzy membership functions to define the linguistic fuzzy inputs [2-4]. In this paper, we made an extension on defuzzifying the linguistic fuzzy inputs defined by pentagonal fuzzy membership function. Throughout this paper, we adopt $\widetilde{A}$ notation for representing the fuzzy set, whereas in our previous paper [6] we have used '~' below the letter *A*.

T. Pathinathan and K. Ponnivalavan [5] introduced pentagonal fuzzy number in the year 2014 along with certain operational properties. Also they proposed the generalized form [6] of the pentagonal fuzzy number in the year 2015. The concept of fuzzy center of pentagonal fuzzy number was developed by T. Pathinathan and Rajkumar in the year 2015 [7]. In this paper, we made a study on traditional defuzzification methods where the input variables are pentagonal fuzzy numbers. Also, we have analyzed the results obtained from the various traditional defuzzification methods with suitable illustrations. The paper is organized as follows. Section two provides the basic preliminaries of pentagonal fuzzy numbers. Section three discusses the three defuzzification methods which has pentagonal fuzzy number as the input variables. Section four gives the conclusion.

2. Basic Definitions

In this section, we provide basic definitions and graphical illustrations of pentagonal fuzzy number. Also we present some of the concepts like centroid and fuzzy center for pentagonal fuzzy number.

2.1 Fuzzy Set

A Fuzzy set $\widetilde{A}$ of X is defined by the following pair as,

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) / x \in X\}$$

where, x is a element in the universe of discourse X, $\mu_{\widetilde{A}} : X \to [0,1]$ is a mapping called the degree of membership function and $\mu_{\widetilde{A}}(x)$ is the membership value of $x \in X$.

2.2 Fuzzy Number

A Fuzzy number $\widetilde{A}$ is a fuzzy set on the real line R, must satisfy the following conditions.

(i) $\mu_{\widetilde{A}}(x)$ is piecewise continuous.

(ii) There exists atleast one $x \in R$ with $\mu_{\widetilde{A}}(x) = 1$.

(iii) $\widetilde{A}$ must be normal and convex.

2.3 Pentagonal Fuzzy Number [6]

A Pentagonal Fuzzy Number $\widetilde{A}_P = (a_1, b_1, c_1, d_1, e_1; k, w)$ is a fuzzy subset of real line R, where a_1, b_1, c_1, d_1 and e_1 are real numbers with $a_1 \le b_1 \le c_1 \le d_1 \le e_1$, k be the $\alpha - cut$ value of the pentagonal fuzzy number with $0 < k < 1$ and w be the core (height) of the pentagonal fuzzy number with $0 < w \le 1$. Then the pentagonal fuzzy membership function is defined as;

$$\mu_{\tilde{A}_P}(x)=\begin{cases} k\left(\dfrac{x-a_1}{b_1-a_1}\right) & ; a_1 \le x \le b_1 \\ k+(w-k)\left(\dfrac{x-b_1}{c_1-b_1}\right) & ; b_1 \le x \le c_1 \\ w & ; x=c_1 \\ k+(w-k)\left(\dfrac{d_1-x}{d_1-c_1}\right) & ; c_1 \le x \le d_1 \\ k\left(\dfrac{e_1-x}{e_1-d_1}\right) & ; d_1 \le x \le e_1 \\ 0 & ; x \le a_1 \text{ and } x \ge e_1 \end{cases} \quad (1)$$

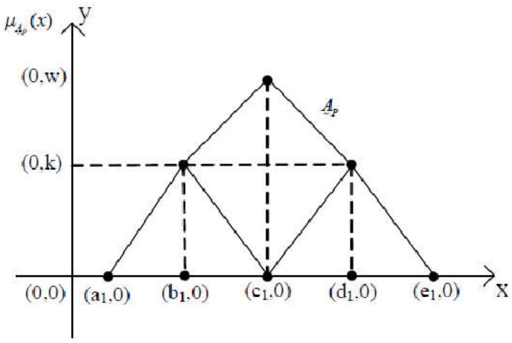

Figure 1 Pentagonal Fuzzy Number

2.4 Centroid of a pentagonal fuzzy number [6]

The centroid of a pentagonal fuzzy number $\tilde{A}_P=(a_1,b_1,c_1,d_1,e_1;0.5,w_1)$ is defined by the formula,

$$G(\bar{x}_0,\bar{y}_0)=\left(\frac{a_1+b_1+5c_1+d_1+e_1}{9},\frac{5}{18}\right)$$

(2)

For generalized pentagonal fuzzy number, the centroid is given by the formula,

$$G(\bar{x}_0,\bar{y}_0)=\left(\frac{a_1+b_1+5c_1+d_1+e_1}{9},\frac{5w_1}{18}\right)$$

(3)

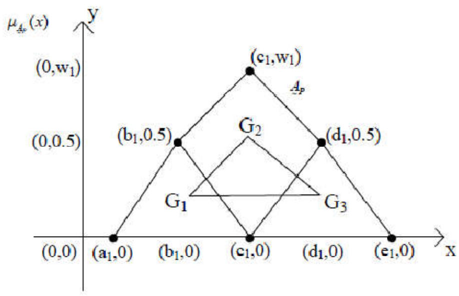

Figure 2 Centroid of a pentagonal fuzzy number

2.5 Fuzzy center value of a pentagonal fuzzy number [7]

Let $\tilde{A}_P=(a_1,b_1,c_1,d_1,e_1)$ be a pentagonal fuzzy number and $\mu_{\tilde{A}_P}(x)$ be its membership function with c_1 is the core (maximum membership value) of a pentagonal fuzzy number. Then the fuzzy center value for pentagonal fuzzy number $\tilde{A}_P$ is given by the formula,

$$F_{\tilde{C}}=\frac{c_1}{2}+\frac{1}{4}(a_1+e_1)$$

(4)

3. Defuzzification Methods for Pentagonal Fuzzy Numbers

In this section, we present some of the traditional defuzzification methods to process the pentagonal fuzzy number inputs and to produce a crisp output. This section discusses three common types of traditional defuzzification methods, such as; center of gravity method, weighted average method and center of sums method. Also, this section introduces new defuzzification formulas which incorporate the concept of centroid, fuzzy centre value and area of the pentagonal fuzzy number to process the linguistic fuzzy inputs.

3.1 Center of Gravity Method

Center of gravity method is one of the widely used traditional defuzzification method to defuzzify the fuzzy output into a crisp value. The center of gravity method is defined by the following formula,

$$x^*=\frac{\int_a^b x\mu_{\tilde{C}}(x)dx}{\int_a^b \mu_{\tilde{C}}(x)dx}$$

(5)

This method does not have any limiting condition on non-symmetric pentagonal fuzzy number. The following example illustrates the defuzzification of two generalized symmetric pentagonal fuzzy number inputs by using center of gravity method.

3.1.1 Illustration

If $\tilde{A}_P=(1,2,3,4,5;0.8)$ and $\tilde{B}_P=(3,4,5,6,7;0.7)$ are defined to be two generalized symmetric pentagonal fuzzy numbers with core value 0.8 and 0.7 respectively. Figure 3 represents the geometrical illustration of two generalized symmetric pentagonal fuzzy number inputs. Figure 4 represents the combined form of two fuzzy pentagonal fuzzy number inputs.

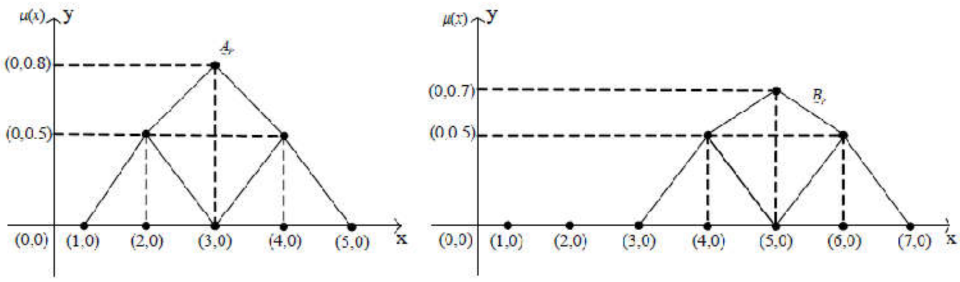

Figure 3 Pentagonal fuzzy number inputs $\tilde{A}_p$ and $\tilde{B}_p$.

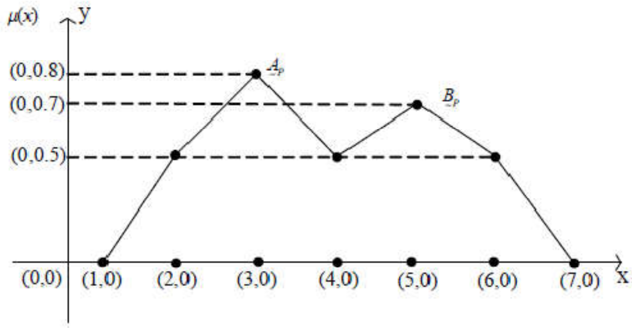

Figure 4 Combined pentagonal fuzzy number

Then by equation (5), the defuzzification of two pentagonal fuzzy number inputs is calculated as follows,

$$\int_a^b x\mu_{\tilde{C}}(x)dx = \int_1^2 0.5\,x(x-1)\,dx + \int_2^3 0.8\,x(x-2)\,dx + \int_3^4 0.8\,x(4-x)\,dx + \int_4^5 0.7\,x(x-4)\,dx + \int_5^6 0.7\,x(6-x)\,dx + \int_6^7 0.5\,x(7-x)dx$$

$$\int_a^b x\mu_{\tilde{C}}(x)dx = \int_1^2 0.5(x-1)\,dx + \int_2^3 0.8\,(x-2)\,dx + \int_3^4 0.8\,(4-x)\,dx + \int_4^5 0.7\,(x-4)\,dx + \int_5^6 0.7(6-x)\,dx + \int_6^7 0.5\,(7-x)\,dx$$

$$x^* = \frac{\dfrac{(0.5\times 5)+(0.5\times 5)+(0.5\times 5)+(0.5\times 5)+(0.5\times 5)+(0.5\times 5)}{6}}{\dfrac{(0.5+0.8+0.8+0.7+0.7+0.5)}{2}} = 3.95$$

The below diagram (Figure 5) shows the defuzzified crisp output value representation for the two pentagonal fuzzy number inputs.

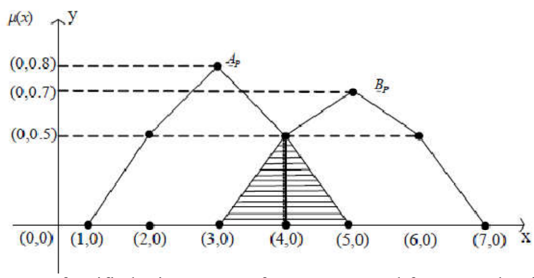

Figure 5 Defuzzified crisp output of two pentagonal fuzzy number inputs

3.2 Weighted Average Method

Weighted average method is the most suitable method to defuzzify two or more symmetric fuzzy inputs. The weighted average method is formed by taking average of each membership function in the output by its respective degree of importance (also known as maximum membership value / core value of the respective membership function). The weighted average defuzzifying formula is given by,

$$x^* = \frac{\sum \bar{x}_0\,\mu_{\tilde{G}}(\bar{x}_0)}{\sum \mu_{\tilde{G}}(\bar{x}_0)} \quad (6)$$

where $\bar{x}_0$ is the centroid of the symmetric fuzzy input. Here we have symmetric fuzzy inputs as pentagonal fuzzy numbers. Then defuzzification of two pentagonal fuzzy number inputs is defined by the formula,

$$x^* = \frac{\sum \bar{x}_0\,\mu_{\tilde{G}}(\bar{x}_0)}{\sum \mu_{\tilde{G}}(\bar{x}_0)} \quad (7)$$

where, $\bar{x}_0 = \dfrac{a_1 + b_1 + 5c_1 + d_1 + e_1}{9}$ represents the centroid of the pentagonal fuzzy number. This method is more suitable for symmetric pentagonal fuzzy number input.

3.2.1 Illustration 1

If $\widetilde{A}_P = (1,2,3,4,5;1)$ and $\widetilde{B}_P = (3,4,5,6,7;1)$ are defined to be two normalized symmetric pentagonal fuzzy numbers. Then centroid of the two defined normalized symmetric pentagonal fuzzy numbers is calculated by using the equation (2) as follows;

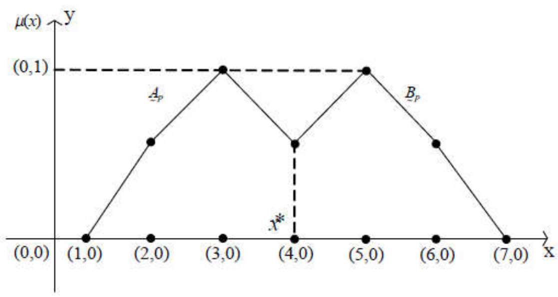

Figure 6 Normalized symmetric pentagonal fuzzy number inputs

Centroid of the pentagonal fuzzy number $\widetilde{A}_P$, derived using equation (2) is 3. Centroid of the pentagonal fuzzy number $\widetilde{B}_P$, derived using equation (2) is 5.

And the defuzzified value of the crisp output is calculated by equation (7) as follows,

$$x^* = \frac{(3\times 1)+(5\times 1)}{1+1} = 4 \, .$$

3.2.2 Illustration 2

If $\widetilde{A}_P = (1,2,3,4,5;0.8)$ and $\widetilde{B}_P = (3,4,5,6,7;0.7)$ are defined to be two generalized symmetric pentagonal fuzzy numbers with core value 0.8 and 0.7 respectively.

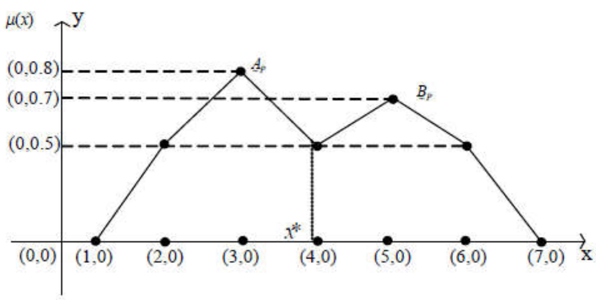

Figure 7 Normalized symmetric pentagonal fuzzy number inputs

Then the centroid of the two generalized pentagonal fuzzy numbers is calculated as follows: Centroid of the pentagonal fuzzy number $\widetilde{A}_P$, derived using equation (2) is 3. Centroid of the pentagonal fuzzy number $\widetilde{B}_P$, derived using equation (2) is 5.

Then by using equation (7), the defuzzified value is calculated as follows,

$$x^* = \frac{(3\times 0.8)+(5\times 0.7)}{0.8+0.7} = 3.9333 \, .$$

The above figure (7) shows the defuzzified value of the two generalized pentagonal fuzzy number inputs and it is represent by dotted line.

3.2.3 Illustration 3

If $\widetilde{A}_P = (1,2,3,4,5;0.6)$, $\widetilde{B}_P = (3,4,5,6,7;0.47)$ and $\widetilde{C}_P = (2,2.25,3,3.75,4;0.45)$ are defined to be the three generalized symmetric pentagonal fuzzy numbers with the core value 0.6, 0.47 and 0.45 respectively.

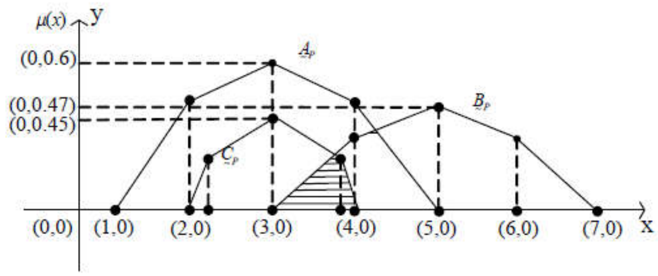

Figure 8 Generalized symmetric pentagonal fuzzy number inputs

Then by equation (7), the defuzzified value of the above pentagonal fuzzy number output is calculated as follows,

$$x^* = \frac{(3\times 0.6)+(5\times 0.47)+(3\times 0.45)}{0.6+0.47+0.45} = 3.6184$$

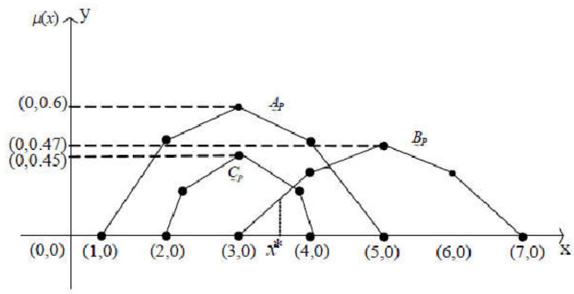

Figure 9 Defuzzified output crisp value for $\tilde{A}_p$, $\tilde{B}_p$ and $\tilde{C}_p$.

The result obtained by the weighted average defuzzification method which uses centroid (Definition 2.4) of a two pentagonal fuzzy number inputs is equal to the concept of fuzzy center of a pentagonal fuzzy number (Definition 2.5).

3.3 Center of Sums Method

Center of sums method is one of the powerful defuzzifying techniques, which uses area to calculate the defuzzification value from two non-symmetric pentagonal fuzzy number inputs. If $\widetilde{A}_P = (a_1, b_1, c_1, d_1, e_1; w)$ be the generalized pentagonal fuzzy number and the area of the pentagonal fuzzy number is calculated by the formula;

$$Area(\widetilde{A}_P) = \frac{(a_5 - a_1) + 2(a_4 - a_2)}{4}$$

(8)

Then the defuzzified value of the two pentagonal fuzzy number inputs is calculated by the formula is stated as follows,

$$x^* = \frac{\sum_{i=1}^{n} F_{c_i} A_i}{\sum_{i=1}^{n} A_i}$$

(9)

where F_{ci} denote the fuzzy center value of the pentagonal fuzzy numbers and A_i represents the area of a pentagonal fuzzy numbers (Equation 8).

3.3.1 Illustration

If $\widetilde{A}_P = (1,2,3,4,6;0.7)$ and $\widetilde{B}_P = (3,4,5,6,6.5;0.47)$ are defined to be two generalized non-symmetric pentagonal fuzzy numbers with the core value 0.8 and 0.7 respectively.

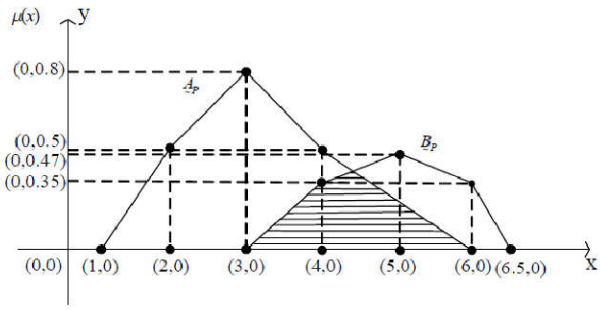

Figure 10 Non-symmetric pentagonal fuzzy numbers $\tilde{A}_p$ and $\tilde{B}_p$

Now, Area$(\widetilde{A}_P)$=2.25, Area$(\widetilde{B}_P)$=1.875. Then the fuzzy centre value of the two pentagonal fuzzy number is calculated by using the equation 4 as follows, $F_{\widetilde{A}_P} = 3.25$ and $F_{\widetilde{B}_P} = 3.25$.

Then the defuzzified value of the two pentagonal fuzzy number inputs is calculated by using the formula defined in the equation 9 as follows,

$$x^* = \frac{(2.25 \times 3.25) + (1.875 \times 4.875)}{2.25 + 1.875} = 3.9886 .$$

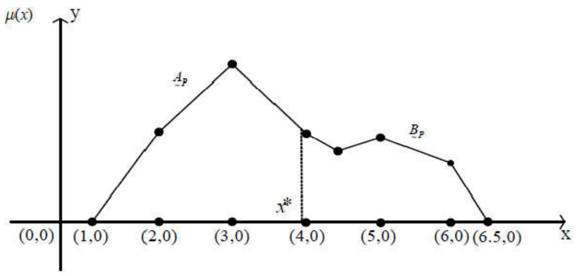

Figure 9 Defuzzified output crisp value for non-symmetric pentagonal fuzzy number $\tilde{A}_p$ and $\tilde{B}_p$.

Conclusion

In this study, we made an analysis of three traditional defuzzification methods where the input variables are observed to be pentagonal fuzzy numbers. Also we have introduced various defuzzification methods for calculating crisp output from two vague pentagonal fuzzy number inputs. Also, we provided the geometrical illustration for the proposed defuzzification methods with pentagonal fuzzy number inputs.

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