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BIVARIATE OPTIMAL REPLACEMENT POLICY FOR A MULTI STATE DEGENERATIVE SYSTEM WITH VARYING COST STRUCTURES UNDER QUASI RENEWAL PROCESS

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A B S T R A C T **RESEARCH ARTICLE**

In this paper, we introduce and study the bivariate optimal replacement policy for a multistate degenerative system with varring cost structure under quasi renewal process. Explicit expression for the cost structure under the bivariate replacement policy (T,N) is derived. Minimization of the cost expression is also deduced.

Keywords:

Policy (T, N), Multistate system, Mean sequence, Description of the model.

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1. Introduction

The maintenance Problem of a multistate degenerative system with k -working states having k -distinct rewards and l -failure states with l -different repair costs is considered in this paper. The long-run average cost for a multistate degenerative system under the bivariate replacement policy (T, N) where T is the working age of the system and N is the number of failures of the system in derived. Existence of Optimality under the bivariate replacement policy is deduced.

The rest of the paper is structured as follows: In the section 2, we give a general description of the model. We also present the monotone process model of a one-component multistate system and the relevant results regarding their Probability Structure. In the section 3, we derive an experssion for the long-run average cost per unit time for this model under a bivariate replacement policy (T, N). Existence of optimality under the bivariate replacement policy is also deduced in this section. Finally, conclusion is given in section 4.

2. Description of the Model

In this section, we describe the model of a one-component multistate system. We also evaluate the conditional probabilities of the operating time and failure time given the state of the system.

Consider a one-component multistate system with $(k+l)$ states k -workers states and l -failure states.

The system state at time t is given by

$$S(t) = \begin{cases} i, & \text{if the system is in the } i^{th} \text{ working state at time } t \ (i = 1,2,\ldots,k) \\ k+j, & \text{if the system is in the } j^{th} \text{ failure state at time } t \ (j = 1,2,\ldots,l) \end{cases}$$

The set of working state is $\Omega_1 = \{1,2,\ldots k\}$ the set of failure states is $\Omega_2 = \{k+1, k+2, \ldots k+l\}$ and the state space is $\Omega = \Omega_1 \cup \Omega_2$. Initially, assume that a new system in working state 1 is installed. Whenever the system fails, it will be repaired. Let t_n be the completion time of the n^{th} repair, $n = 0,1,2\ldots$ with $t_0 = 0$ and let S_n be the time of occurrence of the n^{th} failure, $n = 1,2,\ldots$ then

$t_0 < S_1 < t_1 < \ldots < s_n < t_n < \ldots$

Consider a monotone process model for a multistate one-component system described in this section and make the following package of assumptions 2.1-2.8

2.1 At the beginning, a new system is installed. The system has $(k+l)$ possible states, where the states $1,2,\ldots k$ denotes, respectively, the first-type working state, the second-type working state,$\ldots k^{th}$ type working state and the states $(k+1),(k+2),\ldots,(k+l)$ denote, respectively, the first-type failure state, the second-type failure state,$\ldots$ and the l^{th} type failure state of the system. The occurrances of these types of failures are stochastic and mutually exclusive.

2.2 Whenever the system fails in any of the failure states, it will be repaired. The system will be repalaced by an identical one some times later.

2.3 Let X_n be the survival time of the system after $(n-1)^{st}$ repair. then $\{X_n, n = 1,2,\ldots\}$ forms a non-increasing geometric process with parameter $a > 1$ and

$E(X_1) = \lambda > 0$

2.4 Let Y_n be the repair time of the system after n^{th} failure. Then $\{Y_n, n = 1,2,\ldots\}$ forms a non- decreasing geometric process with parameter $b, 0 < b < 1$ and $E(Y_1) = \mu \geq 0$. Here $\mu = 0$ mean that repair time is negligible.

2.5 If the system in working state i is operating, then let the reword rate be r_i if the system in failure state $(k = i)$ is under repair, the repair cost is c_i. The replacement cost comprises two parts. One part is the replacement cost R and the other proportional to the replacement time Z at rate C_p. In other words, the replacement cost is given by $R + C_p Z.$

2.6 Assume that $1 \leq a_1 \leq a_2 \leq \ldots \leq a_k$ and $1 \geq b_1 \geq b_2 \geq \ldots \geq b_k > 0$.

2.7 Assume that $F_n(t)$ is the cumulative distribution of $L_n = \sum_{i=1}^{n} X_i$ and $G_n(t)$ be the cumulative distribution of $M_n = \sum_{i=1}^{n} Y_i$.

2.8 The survival time X_n, the repair time Y_n and the replacement time $Z, (n = 1,2,\ldots k)$ are independent random variables. We now describe the probability structure of the model.

Assume that the transition probability from working state $i, i = 1,2,\ldots k$ to failure state $(k + j), j = 1,2,\ldots,l$ is

$$P(S(s_{n+1}) = k + j/S(t_n) = i) = q_j,$$

with $\sum_{j=1}^{l} q_j = l$. Moreover, the transition probability from failure state $k + j, j = 1,2,\ldots,l$ to working state $i, i = 1,2,\ldots k$ is given by

$$P(S(t_n) = i/S(s_n) = k + j) = P_i$$

with $\sum_{i=1}^{k} p_i = 1$.

Assume that there exists a life-time distribution $U(t)$ and $a_i > 0, i = 1,2,\ldots k$ such that

$$P(X_1 \leq t) = U(t)$$

(2.1)

and

$$P(X_2 \leq t/S(t_1) = i) = U(a_i, t), i = 1,2,\ldots,k$$

(2.2)

where $1 \leq a_1 \leq a_2 \leq \ldots \leq a_k$.

In general for $i_j \in \{1,2,\ldots,k\}$

$$P(X_n \leq t/S(t_1) = i_1,\ldots,S(t_{n-1}) = i_{n-1}) = U(a_{i_1},\ldots,a_{i_{n-1}} t)$$

(2.3)

Similarly, assume that there exists a life-time distribution $V(t)$ and $b_i > 0,$ $i = 1,2,\ldots,l$ such that

$$P(Y_1 \leq t/S(s_1) = k + i) = V(b_i t)$$

(2.4)

where $1 \geq b_1 \geq b_2 \geq \ldots \geq b_l > 0$ and in general, for $i_j \in \{1,2,\ldots l\}$

$$P(Y_n \leq t/S(s_1) = k + i,\ldots,S(s_n) = k + i_n) = V(b_{i_1},\ldots,b_{i_n} t)$$

(2.5)

3. The Policy (T, N)

In this section, we introduce and study a bivariate replacement policy (T, N) for the multistate degenerative system, under which system is replaced at working age T or at the time of N-th failure, whichever occurs first. The problem is to choose an optimal replacement policy $(T, N)^*$ such that the long-run average cost per unit time is minimized. The working age T of the system at time is the commutation life-time given by

$$T(t) = \begin{cases} t - M_n, & if\ L_n + M_n \leq t < L_{n+1} + M_n \\ L_{n+1}, & if\ L_{n+1} + M_n \leq t < L_{n+1} + M_{n+1} \end{cases}$$

where $L_n = \sum_{i=1}^{n} X_i$ and $M_n = \sum_{i=1}^{n} Y_i$ and $L_0 = M_0 = 0$

Following Lam [2005], the distribution of the survival time X_n in assumption 2.3 and the distribution of the repair time Y_n in assumption 2.4 are given by

$$P(X_n \leq t) = \sum_{\sum_{i=1}^{k} j_1 = n-1} \frac{(n-1)!}{j_1!\ldots j_k!}\ p_1^{j_1}\ldots p_k^{j_k}\ U(a_1^{j_1}\ldots a_k^{j_k} t)$$

(3.1)

where $j_1, j_2, \ldots j_k \in \mathbb{Z}^+$ and

$$P(Y_n \leq t) = \sum_{\sum_{i=1}^{l} j_1 = n} \frac{n!}{j_1!\ldots j_l!}\ q_1^{j_1}\ldots q_l^{j_l}\ V(b_1^{j_1}\ldots b_l^{j_l} t)$$

(3.2)

where $j_1, j_2, \ldots j_l \in \mathbb{Z}^+$, If $E(X_1) = \lambda$, then the mean survival time is $E(X_n) = \lambda\, \alpha^{n-1}$, for $n > 1$,

where $\alpha = \left[\sum_{i=1}^{k} \frac{p_i}{a_i}\right]$ and If $E(Y_1) = \mu$ then the mean repair time is $E(Y_n) = \beta^n \mu$

for $n > 1$, where

$$\beta = \left[\sum_{j=1}^{n} \frac{q_j}{b_j} \right]$$

Further if $R_n = r_i$ denotes the reward earned after the n^{th} repair, where $S(s_{n-1}) = i, i = 1,2,\ldots k$ then mean reward earned after $(n-1)$ -st repair is $E(R_1 X_1) = r\lambda$ and for $n \geq 2$ then expected reward after installation is given by

$$E(R_n X_n) = r\lambda\alpha^{n-1}$$
(3.3)

where $r = \sum_{i=1}^{k} \frac{r_i p_i}{a_i}$ and if $C_n = c_i$ denote the repair cost after the n^{th} failure, where $S(s_n) = k + i, i = 1,2,\ldots l$ then mean repair cost after n^{th} failure is

$$E(C_n Y_n) = C\mu\beta^n$$
(3.4)

where $C = \sum_{i=1}^{l} \frac{c_i q_i}{b_i}$

The length of a cycle under the bivariate replacement policy (T, N) is

$$W = \left(\sum_{i=1}^{N} X_i + \sum_{i=1}^{N-1} Y_i \right) \chi(L_N \leq T) + \left(T + \sum_{i=1}^{\eta} Y_i \right) \chi(L_N > T) + Z$$

where $\eta = 0,1,2,\ldots, N-1$ is the number of failures before the working age of the system exceeds T and $\chi(.)$ denote the indicate functions.From Leung [2006], we have

$$E[\chi_{(L_i \leq T \leq L_N)}] = P(L_i \leq T \leq L_N)$$
$$= F_i(T) - F_N(T)$$

Lemma 3.1 The mean length of a cycle under the policy (T, N) is

$$E(w) = \int_0^T \overline{F}_N(u)du + \sum_{i=1}^{N-1} \mu\, \beta^{i-1} F_i(T) + \tau$$
(3.5)

Proof. Consider

$$E(w) = E\left[\left(\sum_{i=1}^{N} X_i + \sum_{i=1}^{N-1} Y_i \right) \chi_{(L_N \leq T)} \right] + E\left[\left(T + \sum_{i=1}^{\eta} Y_i \right) \chi_{(L_N > T)} \right] + E(Z)$$
$$= E\left\{ E\left[\left(\sum_{i=1}^{N} X_i + \sum_{i=1}^{N-1} Y_i \right) \chi_{(L_N \leq T)\backslash} L_N \right] \right\} + E[T\chi_{(L_N > T)}]$$
$$+ E\left[\left(\sum_{i=1}^{\eta} Y_i \right) \chi_{(L_N > T)} \right] + E(Z)$$
$$= \int_0^T u dF_N(u) + \int_0^T \sum_{i=1}^{N-1} E(Y_i) dF_N(u) + T\overline{F}_N(T)$$
$$+ \sum_{i=1}^{N-1} \mu\beta^{i-1} E[\chi_{(L_1 \leq T < L_N)}] + \tau$$
$$= \int_0^T u dF_N(u) + \sum_{i=1}^{N-1} \mu\beta^{i-1} F_N(T) + T\overline{F}_N(T) + \sum_{i=1}^{N-1} \mu\beta^{i-1} P[\chi_{(L_1 \leq T \leq L_N)}] + \tau$$
$$= T\overline{F}_N(T) + \int_0^T u dF_N(u) + \sum_{i=1}^{N-1} \mu\beta^{i-1} [F_i(T) - F_N(T)] + \sum_{i=1}^{N-1} \mu\beta^{i-1} F_N(T) + \tau$$
$$= \int_0^T \overline{F}_N(u)du + \sum_{i=1}^{N-1} \mu\beta^{i-1} F_i(T) + \tau$$

which is (3.5)

Lemma 3.2 If $L_N \leq T$ and $n \geq 2$, then the expected reward earned is

$$E\left[\left(\sum_{n=2}^{N} R_N X_N \right) \chi_{(L_N \leq T)} \right] = \sum_{n=2}^{N} r\lambda\alpha^{n-2} \int_0^T u dF_N(u)$$
(3.6)

Proof. Consider

$$E\left[\left(\sum_{n=2}^{N} R_N X_N \right) \chi_{(L_N \leq T)} \right] = E\left\{ E\left[\left(\sum_{n=2}^{N} R_N X_N \right) \chi_{(L_N \leq T)} / L_N \right] \right\}$$

$$= \int_0^T E\left(\sum_{n=2}^{N} R_N X_N / L_N = u \right) dF_N(u)$$

$$= \sum_{n=2}^{N} r\lambda\alpha^{n-2} \int_0^T u dF_N(u)$$

$$= \left(\frac{1-\alpha^{N-1}}{1-\alpha} \right) r\lambda \int_0^T u dF_N(u)$$

which is (3.6)

Lemma 3.3 If $L_N > T$ and $n \geq 2$, then the expected reward earned is

$$E\left[\left(\sum_{n=2}^{N} R_N X_N \right) \chi_{(L_N > T)}\right] = \sum_{n=2}^{N} r\lambda\alpha^{n-2}[F_n(T) - F_N(T)]$$

(3.7)

Proof. Consider

$$E\left[\left(\sum_{n=2}^{N} R_N X_N \right) \chi_{(L_N > T)}\right] = E\left[\left(\sum_{n=2}^{N} R_N X_N \right) \chi_{(L_n < T < L_N)}\right]$$

$$= \sum_{n=2}^{N} E(R_n X_n) E[\chi_{(L_n < T < L_N)}]$$

$$= \sum_{n=2}^{N} r\lambda\alpha^{n-2} p[L_n < T < L_N]$$

$$= \sum_{n=2}^{N} r\lambda\alpha^{n-2}[F_n(T) - F_N(T)]$$

which is (3.7)

Lemma 3.4 If $L_N \leq T$, then the expected repair cost is

$$E\left[\left(\sum_{n=1}^{N-1} C_n Y_n \right) \chi_{(L_N \leq T)}\right] = \sum_{n=1}^{N-1} C\mu\, \beta^{n-1} F_N(T)$$

(3.8)

Proof. Consider

$$E\left[\left(\sum_{n=1}^{N-1} C_n Y_n \right) \chi_{(L_N \leq T)}\right] = E\left[E\left(\sum_{n=1}^{N-1} C_n Y_n / L_N \right) \chi_{(L_N \leq T)}\right]$$

$$= \int_0^T E\left(\sum_{n=1}^{N-1} C_n Y_n / L_N \right) dF_n(u)$$

$$= \int_0^T \sum_{n=1}^{N-1} E(C_n Y_n) dF_N(u)$$

$$= \sum_{n=1}^{N-1} E(C_n Y_n) \int_0^T dF_N(u)$$

$$= \sum_{n=1}^{N-1} c\mu\beta^{n-1} F_N(T)$$

$$= c\mu\left(1 + \beta + \beta^2 + \ldots + \beta^{N-2}\right) F_N(T)$$

$$= c\mu\left(\frac{\beta^{N-1} - 1}{\beta - 1} \right) F_N(T)$$

which on simplification yields (3.8)

Lemma 3.5 If $L_N > T$, then the expected repair cost is

$$E\left[\left(\sum_{n=1}^{\eta-1} C_n Y_n\right)\chi_{(L_N>T)}\right] = \sum_{n=1}^{N-1} C\mu\,\beta^{n-1}[F_n(T)-F_N(T)]$$

(3.9)

Proof. Consider

$$E\left[\left(\sum_{n=1}^{\eta-1} C_n Y_n\right)\chi_{(L_N>T)}\right] = E\left[\left(\sum_{n=1}^{N-1} C_n Y_n\right)\chi_{(L_N<T<L_N)}\right]$$

$$= \sum_{n=1}^{N-1} E(C_n Y_n) E[\chi_{(L_N<T<L_N)}]$$

$$= \sum_{n=1}^{N-1} C\mu\beta^{n-1} P[L_n < T < L_N]$$

$$= \sum_{n=1}^{N-1} C\mu\beta^{n-1}[F_n(T)-F_N(T)]$$

which is (3.9)

Let T_1 be the first replacement time and let $T_n(n \geq 2)$ be the time between $(n-1)^{st}$ replacement and n^{th} replacement. Then the sequence $T_n, n=1,2,\ldots$ forms a renewal process. The interarrival time between two consecutive replacements is renewal cycle. By the renewal reward theorem, the long-run average cost per unit time under the multistate bivariate replacement policy (T,N) is given by

$$\mathrm{C}(T,N) = \frac{\text{The expected cost incurred in a cycle}}{\text{The expected length of a cycle}}$$

$$= \frac{\left[\begin{array}{l} E\left\{\left(\sum_{n=1}^{N-1} C_n Y_n - \sum_{n=1}^{N} R_n X_n\right)\chi_{(L_N \leq T)}\right\} + R \\ + E\left\{\left(\sum_{n=1}^{\eta} C_n Y_n - T\sum_{n=1}^{\eta} R_n\right)\chi_{(L_N>T)}\right\} + C_p E(z)\end{array}\right]}{[E(w)]}$$

using Lemma 3.1 to 3.5, we obtain

$$\mathrm{C}(T,N) = \frac{\left[\begin{array}{l}\sum_{n=1}^{N-1} C\mu\,\beta^{n-1}F_N(T) - \sum_{n=2}^{N} r\lambda\alpha^{n-2}\int_0^T u\,dF_N(u) + C_p\tau + R - r_1\lambda \\ + \sum_{n=1}^{N-1} C\mu\,\beta^{n-1}[F_n(T)-F_N(T)] - \sum_{n=2}^{N} r\lambda\,\alpha^{n-2}[F_n(T)-F_N(T)] - r_1 T\end{array}\right]}{\left[\int_0^T \overline{F}_N(u)du + \sum_{n=1}^{N-1}\mu\,\beta^{n-1}F_n(T) + \tau\right]}$$

On Summarizing the above facts, we have the following results.

For the model described in section 2, under the assumptions 1.1 to 1.8 the long-run average cost per unit time under the bivariate replacement policy (T,N) for a multistate degenerative system is given by

$$\mathrm{C}(T,N) = \frac{\left[\begin{array}{l}\sum_{n=1}^{N-1} C\mu\,\beta^{n-1}F_n(T) - \sum_{n=2}^{N} r\lambda\,\alpha^{n-2}\int_0^T u\,dF_N(u) - r_1(\lambda+T) \\ + \sum_{n=2}^{N} r\lambda\,\alpha^{n-2}[F_n(T)-F_N(T)] + C_p\,\tau + R\end{array}\right]}{\left[\int_0^T \overline{F}_N(u)du + \sum_{n=1}^{N-1}\mu\,\beta^{n-1}F_n(T) + \tau\right]}$$

4. Deductions

Here $\mathrm{C}(T,N)$ is a bivariate function. Obviously, when N is fixed, $\mathrm{C}(T,N)$ is a function of T for fixed $N = m$, it can be written as

$$\mathrm{C}(T,N) = C_m(T), m = 1,2,\ldots$$

Thus for a fixed m, we can fixed T_m^* by analytical or numerical methods such that $C_m(T_m^*)$ is minimized. That is when $N = 1,2,\ldots m,\ldots$ we can fixed

$T_1^*, T_2^*, T_3^*, \ldots, T_m^*, \ldots$ respectively, such that corresponding $C_1(T_1^*), C_2(T_2^*), \ldots, C_m(T_m^*) \ldots$ are minimized.

Because the total life time of a multistate degenerative system is limited, the minimum of the long-run average cost per unit time exists. So we can determine the minimum of the long run average cost per unit time based on $C_1(T_1^*), C_2(T_2^*), \ldots, C_m(T_m^*) \ldots$ for example, if the minimum is denoted by $C_n(T_n^*)$ we obtain the bivariate optimal replacement policy $(T, N)^*$ such that

$$C((T,N)^*) = \min_n C_n(T_n^*)$$

CONCLUSION

By concluding a repairable system for a monotone process model of a one component multistate degenerate system explicit expression for the long-run average cost per unit time under the bivariate replacement policy (T, N) is derived. Existence of optimality under the bivariate replacement policy is also deduced.

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