



FUZZY α -GENERALIZED, FUZZY WEAKLY GENERALIZED BAIRE SPACE AND FUZZY GENERALIZED α -BAIRE SPACE

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A B S T R A C T

RESEARCH ARTICLE

In this paper we introduce fuzzy α -generalized Baire Spaces, fuzzy weakly generalized Baire space, fuzzy generalized α -Baire space and discuss about some of its properties with suitable examples.

Keywords:

Fuzzy α -open sets, fuzzy α -generalized open sets, fuzzy α -generalized Baire space, fuzzy weakly generalized Baire space and fuzzy generalized α -Baire space.

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1. Introduction

The theory of fuzzy sets was initiated by L.A.Zadeh in his classical paper [9] in the year 1965 as an attempt to develop a mathematically precise framework in which to treat systems or phenomena which cannot themselves be characterized precisely. The potential of fuzzy notion was realized by the researchers and has successfully been applied for investigations in all the branches of Science and Technology. The paper of C.L.Chang [2] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

The concepts of α -generalized closed sets have been studied in classical topology in [3]. In this paper we introduce the fuzzy α -generalized, fuzzy weakly generalized and fuzzy generalized α -nowhere dense sets and fuzzy α -generalized, fuzzy weakly generalized and fuzzy generalized α -Baire spaces with suitable examples.

2. Preliminaries:

Now review of some basic notions and results used in the sequel. In this work by (X,T) or simply by X , we will denote a fuzzy topological space due to Chang [2].

Definition 2.1 [1] Let λ and μ be any two fuzzy sets in a fuzzy topological space (X, T) . Then we define:

$$\mu \cup \lambda : X \rightarrow [0,1] \text{ as follows: } \mu \cup \lambda (x) = \max \{ \mu(x), \lambda(x) \};$$

$$\mu \cap \lambda : X \rightarrow [0,1] \text{ as follows: } \mu \cap \lambda (x) = \min \{ \mu(x), \lambda(x) \};$$

$$\mu = \lambda^c \implies \mu(x) = 1 - \lambda(x).$$

For a family $\lambda_i \in I$ of fuzzy sets in (X, T) , the union $\bigcup_i \lambda_i$ and intersection $\bigcap_i \lambda_i$ are defined respectively as $\psi(x) = \sup_i \{ \lambda_i(x), x \in X \}$, and $\delta(x) = \inf_i \{ \lambda_i(x), x \in X \}$.

Definition 2.2 [2]: Let (X,T) be a fuzzy topological space. For a fuzzy set λ of X , the interior and the closure of λ are defined

respectively as $int(\lambda) = \bigvee \{ \mu \mid \mu \leq \lambda, \mu \in T \}$ and $cl(\lambda) = \bigwedge \{ \mu \mid \lambda \leq \mu, \mu \in T \}$.

Definition 2.3 [3]: Let (X,T) be a topological space. For a fuzzy set λ of X is a α -generalized closed set (briefly α -closed) if $cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open in X .

Definition 2.4 [8]: A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy dense if there exists no fuzzy closed set $\mu \in (X,T)$ such that $\lambda < \mu < 1$. That is $cl(\lambda) = 1$.

Definition 2.5 [7]: A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int(cl(\lambda)) = 0$.

Definition 2.6 [7]: Let (X,T) be a fuzzy topological space. A fuzzy set λ in (X,T) is called fuzzy first category set if $\lambda = \bigcup_{i=1}^{\infty} \lambda_i$, where λ_i 's are fuzzy nowhere dense sets in (X,T) . A fuzzy set which is not fuzzy first category set is called a fuzzy second category set in (X,T) .

Definition 2.7 [6] : A fuzzy topological space (X,T) is called fuzzy first category space if $1 = \bigcup_{i=1}^{\infty} \lambda_i$, where λ_i 's are fuzzy nowhere dense sets in (X,T) . A topological space which is not of fuzzy first category is said to be of fuzzy second category space.

Definition 2.8 [7]: Let (X,T) be a fuzzy topological space. Then (X,T) is called a fuzzy Baire space if $int(\bigcup_{i=1}^{\infty} (\lambda_i)) = 0$, where (λ_i) 's are fuzzy nowhere dense sets in (X,T) .

Fuzzy α -generalized nowhere dense sets, Fuzzy weakly generalized nowhere dense sets, Fuzzy generalized α -nowhere dense sets

We introduce fuzzy α -generalized nowhere dense sets, fuzzy weakly generalized nowhere dense sets, fuzzy generalized α -nowhere dense sets with suitable examples.

Definition 3.1: A fuzzy set in a fuzzy topological space (X, T) is called fuzzy α -generalized nowhere dense if there exists no non-zero fuzzy α -generalized open set μ in (X, T) such that $\mu < \alpha\text{-cl}(\mu)$. That is, $\alpha\text{-int}(\alpha\text{-cl}(\mu)) = 0$.

Example 3.1: Let $X = \{a, b, c\}$. The fuzzy sets μ , ν and γ are defined on X as follows:

- $\mu : X \rightarrow [0, 1]$ defined as $\mu(a) = 0.8$; $\mu(b) = 0.7$; $\mu(c) = 0.8$.
- $\nu : X \rightarrow [0, 1]$ defined as $\nu(a) = 0.9$; $\nu(b) = 0.7$; $\nu(c) = 0.7$.
- $\gamma : X \rightarrow [0, 1]$ defined as $\gamma(a) = 0.8$; $\gamma(b) = 0.7$; $\gamma(c) = 0.7$.

Then $T = \{0, \mu, \nu, (\nu \vee \mu), 1\}$ is fuzzy topology on X . The fuzzy sets $\mu, \nu,$ and $\nu \vee \mu$ are fuzzy α -open sets.

Now $1 - \nu \vee \mu$ where $\nu \vee \mu$ is fuzzy α -open then $\alpha\text{-cl}(1 - \lambda) \leq \nu \vee \mu,$

- $1 - \mu \vee \nu$ where $\nu \vee \mu$ is fuzzy α -open then $\alpha\text{-cl}(1 - \mu) \leq \nu \vee \mu,$
- $1 - \nu \vee \mu$ where $\nu \vee \mu$ is fuzzy α -open then $\alpha\text{-cl}(1 - \gamma) \leq \nu \vee \mu,$
- $1 - \nu \vee \mu$ where $\nu \vee \mu$ is fuzzy α -open then $\alpha\text{-cl}(1 - (\lambda \vee \mu)) \leq \nu \vee \mu.$

$1 - \mu, 1 - \nu, 1 - (\nu \vee \mu)$'s are fuzzy generalized closed sets.

- Now $\alpha\text{-int}(\alpha\text{-cl}(1 - \mu)) = \alpha\text{-int}(1 - \mu) = 0,$
- $\alpha\text{-int}(\alpha\text{-cl}(1 - \nu)) = \alpha\text{-int}(1 - \nu) = 0,$
- $\alpha\text{-int}(\alpha\text{-cl}(1 - (\nu \vee \mu))) = \alpha\text{-int}(1 - (\nu \vee \mu)) = 0$

Therefore $1 - \mu, 1 - \nu, 1 - (\nu \vee \mu)$'s are fuzzy α -generalized nowhere dense sets.

Example 3.2 : Let $X = \{a, b, c\}$. The fuzzy sets μ, ν and γ are defined on X as follows:

- $\mu : X \rightarrow [0, 1]$ defined as $\mu(a) = 0.8$; $\mu(b) = 0.6$.
- $\nu : X \rightarrow [0, 1]$ defined as $\nu(a) = 0.9$; $\nu(b) = 0.7$.
- $\gamma : X \rightarrow [0, 1]$ defined as $\gamma(a) = 0.9$; $\gamma(b) = 0.9$.

Then $T = \{0, \mu, \nu, \gamma, 1\}$ is fuzzy topology on X . The fuzzy sets $\mu, \nu,$ and γ are fuzzy α -open sets.

- Now $1 - \gamma$ where $\nu \vee \mu$ is fuzzy α -open then $\alpha\text{-cl}(1 - \lambda) \leq \nu \vee \mu,$
- $1 - \mu$ where $\nu \vee \mu$ is fuzzy α -open then $\alpha\text{-cl}(1 - \mu) \leq \nu \vee \mu,$
- $1 - \nu$ where $\nu \vee \mu$ is fuzzy α -open then $\alpha\text{-cl}(1 - \gamma) \leq \nu \vee \mu,$

Now we define the fuzzy sets μ, ν and γ on X as follows:

- $\mu : X \rightarrow [0, 1]$ defined as $\mu(a) = 0.8$; $\mu(b) = 0.8$.
- $\nu : X \rightarrow [0, 1]$ defined as $\nu(a) = 0.9$; $\nu(b) = 0.8$.
- $\gamma : X \rightarrow [0, 1]$ defined as $\gamma(a) = 0.8$; $\gamma(b) = 0.7$.

The fuzzy subsets $\mu, \nu,$ and γ are not fuzzy α -generalized nowhere dense sets. Since $\alpha\text{-int}(\alpha\text{-cl}(\mu)) = 1 \neq 0,$ $\alpha\text{-int}(\alpha\text{-cl}(\nu)) = 1 \neq 0$ and $\alpha\text{-int}(\alpha\text{-cl}(\gamma)) = 1 \neq 0$. Therefore μ, ν and γ are not of fuzzy α -generalized nowhere dense set.

Definition 3.3: A fuzzy set in a fuzzy topological space (X, T) is called fuzzy weakly generalized nowhere dense if there exists no non-zero fuzzy weakly generalized open set μ in (X, T) such that $\mu < \text{cl}(\mu)$. That is, $\text{int}(\text{cl}(\mu)) = 0$.

Example 3.3: Let $X = \{a, b, c\}$. The fuzzy sets μ, ν and γ are defined on X as follows:

- $\mu : X \rightarrow [0, 1]$ defined as $\mu(a) = 0.9$; $\mu(b) = 0.7$; $\mu(c) = 0.6$.
- $\nu : X \rightarrow [0, 1]$ defined as $\nu(a) = 0.8$; $\nu(b) = 0.9$; $\nu(c) = 0.5$.
- $\gamma : X \rightarrow [0, 1]$ defined as $\gamma(a) = 0.9$; $\gamma(b) = 0.8$; $\gamma(c) = 0.8$.

Then $T = \{0, \mu, \nu, \gamma, (\nu \vee \mu), (\mu \wedge \nu), (\mu \vee \gamma), (\mu \wedge \gamma), 1\}$ is fuzzy topology on X . The fuzzy sets $\mu, \nu, \gamma,$ and $\nu \vee \mu, \mu \wedge \nu, \mu \vee \gamma, \mu \wedge \gamma$ are fuzzy open sets.

Now $(1 - \mu) < \mu \vee \nu \Rightarrow \text{cl}(\text{int}(1 - \mu)) = \text{cl}(0) = \mu \vee \nu,$
 $(1 - \nu) < \mu \vee \nu \Rightarrow \text{cl}(\text{int}(1 - \nu)) = \text{cl}(0) = \mu \vee \nu,$
 $(1 - \gamma) < \mu \vee \nu \Rightarrow \text{cl}(\text{int}(1 - \gamma)) = \text{cl}(0) = \mu \vee \nu,$
 $(1 - \lambda \vee \mu) < \mu \vee \nu \Rightarrow \text{cl}(\text{int}(1 - \nu \vee \mu)) = \text{cl}(0) = \mu \vee \nu,$
 $(1 - \mu \wedge \nu) < \mu \vee \nu \Rightarrow \text{cl}(\text{int}(1 - \mu \wedge \nu)) = \text{cl}(0) = \mu \vee \nu,$
 $(1 - \mu \vee \gamma) < \mu \vee \nu \Rightarrow \text{cl}(\text{int}(1 - \mu \vee \gamma)) = \text{cl}(0) = \mu \vee \nu,$
 $(1 - \mu \wedge \gamma) < \mu \vee \nu \Rightarrow \text{cl}(\text{int}(1 - \mu \wedge \gamma)) = \text{cl}(0) = \mu \vee \nu.$
 Where $1 - \mu, 1 - \nu, 1 - (\nu \vee \mu), 1 - (\mu \wedge \nu), 1 - \mu \vee \gamma, 1 - \mu \wedge \gamma$'s are fuzzy weakly generalized closed set.

- Now $\text{Int}(\text{cl}(1 - \mu)) = \text{int}(1 - \mu) = 0,$
- $\text{Int}(\text{cl}(1 - \nu)) = \text{int}(1 - \nu) = 0,$
- $\text{Int}(\text{cl}(1 - \gamma)) = \text{int}(1 - \gamma) = 0,$
- $\text{Int}(\text{cl}(1 - \nu \vee \mu)) = \text{int}(1 - \nu \vee \mu) = 0,$
- $\text{Int}(\text{cl}(1 - \mu \wedge \nu)) = \text{int}(1 - \mu \wedge \nu) = 0,$
- $\text{Int}(\text{cl}(1 - \mu \vee \gamma)) = \text{int}(1 - \mu \vee \gamma) = 0,$
- $\text{Int}(\text{cl}(1 - \mu \wedge \gamma)) = \text{int}(1 - \mu \wedge \gamma) = 0$

Therefore $1 - \mu, 1 - \nu, 1 - (\nu \vee \mu), 1 - (\mu \wedge \nu), 1 - (\mu \vee \gamma), 1 - (\mu \wedge \gamma)$'s are fuzzy weakly generalized nowhere dense set.

Example 3.4: Let $X = \{a, b, c\}$. The fuzzy sets μ, ν and γ are defined on X as follows:

- $\mu : X \rightarrow [0, 1]$ defined as $\mu(a) = 0.8$; $\mu(b) = 0.5$.
- $\nu : X \rightarrow [0, 1]$ defined as $\nu(a) = 0.9$; $\nu(b) = 0.6$.
- $\gamma : X \rightarrow [0, 1]$ defined as $\gamma(a) = 0.9$; $\gamma(b) = 0.8$.

Then $T = \{0, \mu, \nu, \gamma, 1\}$ is fuzzy topology on X . The fuzzy sets $\mu, \nu,$ and γ are fuzzy open sets.

- Now $1 - \gamma < \gamma \Rightarrow \text{cl}(\text{int}(1 - \gamma)) = 0 = \gamma,$
- $1 - \mu < \gamma \Rightarrow \text{cl}(\text{int}(1 - \mu)) = 0 = \gamma,$
- $1 - \nu < \gamma \Rightarrow \text{cl}(\text{int}(1 - \nu)) = 0 = \gamma,$

Now we define the fuzzy sets μ, ν and γ on X as follows:

- $\mu : X \rightarrow [0, 1]$ defined as $\mu(a) = 0.8$; $\mu(b) = 0.6$.
- $\nu : X \rightarrow [0, 1]$ defined as $\nu(a) = 0.8$; $\nu(b) = 0.7$.
- $\gamma : X \rightarrow [0, 1]$ defined as $\gamma(a) = 0.9$; $\gamma(b) = 0.7$.

The fuzzy subsets $\mu, \nu,$ and γ are not fuzzy weakly generalized nowhere dense sets. Since $\text{int}(\text{cl}(\mu)) = 1 \neq 0,$ $\text{int}(\text{cl}(\nu)) = 1 \neq 0$ and $\text{int}(\text{cl}(\gamma)) = 1 \neq 0$. Therefore $\mu, \nu,$ and γ are not of fuzzy weakly generalized nowhere dense set.

Definition 3.5: A fuzzy set in a fuzzy topological space (X, T) is called fuzzy generalized α -nowhere dense if there exists no non-zero fuzzy generalized α -open set μ in (X, T) such that $\mu < \text{cl}(\mu)$. That is, $\text{int}(\text{cl}(\mu)) = 0$.

Example 3.5: Let $X = \{a, b, c\}$. The fuzzy sets μ, ν and γ are defined on X as follows:

- $\mu : X \rightarrow [0, 1]$ defined as $\mu(a) = 0.9$; $\mu(b) = 0.9$; $\mu(c) = 0.8$.
- $\nu : X \rightarrow [0, 1]$ defined as $\nu(a) = 0.8$; $\nu(b) = 0.7$; $\nu(c) = 0.6$.
- $\gamma : X \rightarrow [0, 1]$ defined as $\gamma(a) = 0.9$; $\gamma(b) = 0.8$; $\gamma(c) = 0.6$.

Then $T = \{0, \mu, \nu, \gamma, (\nu \vee \mu), 1\}$ is fuzzy topology on X . The fuzzy sets $\mu, \nu, \gamma,$'s are fuzzy open sets.

- Now $1 - \gamma < \mu \Rightarrow \mu$ is open $\Rightarrow \text{cl}(1 - \gamma) = \mu,$
- $1 - \mu < \mu \Rightarrow \mu$ is open $\Rightarrow \text{cl}(1 - \mu) = \mu,$
- $1 - \nu < \mu \Rightarrow \mu$ is open $\Rightarrow \text{cl}(1 - \nu) = \mu.$
- $1 - \mu, 1 - \nu, 1 - \gamma$'s are fuzzy generalized α -closed set.

- Now $\text{Int}(\text{cl}(1 - \gamma)) = \text{int}(1 - \gamma) = 0,$
- $\text{Int}(\text{cl}(1 - \mu)) = \text{int}(1 - \mu) = 0,$
- $\text{Int}(\text{cl}(1 - \nu)) = \text{int}(1 - \nu) = 0.$

Now $1-\lambda$, $1-\mu$, $1-\nu$'s are fuzzy generalized α -nowhere dense set.

Example 3.6: Let $X = \{a,b,c\}$. The fuzzy sets λ , μ and ν are defined on X as follows:

- $\lambda : X \rightarrow [0,1]$ defined as $\lambda(a) = 0.7$; $\lambda(b) = 0.6$.
- $\mu : X \rightarrow [0,1]$ defined as $\mu(a) = 0.8$; $\mu(b) = 0.6$.
- $\nu : X \rightarrow [0,1]$ defined as $\nu(a) = 0.9$; $\nu(b) = 0.7$.

Then $T = \{0, \lambda, \mu, \nu, 1\}$ is fuzzy topology on X . The fuzzy sets λ , μ , and ν are fuzzy open sets.

- Now $1-\lambda < \Rightarrow \text{cl}(1-\lambda) = \lambda$,
- $1-\mu < \Rightarrow \text{cl}(1-\mu) = \mu$,
- $1-\nu < \Rightarrow \text{cl}(1-\nu) = \nu$,

Now we define the fuzzy sets α and β on X as follows:

- $\alpha : X \rightarrow [0,1]$ defined as $\alpha(a) = 0.7$; $\alpha(b) = 0.6$.
- $\beta : X \rightarrow [0,1]$ defined as $\beta(a) = 0.8$; $\beta(b) = 0.6$.
- $\gamma : X \rightarrow [0,1]$ defined as $\gamma(a) = 0.9$; $\gamma(b) = 0.7$.

The fuzzy subsets α and β are not fuzzy generalized α -nowhere dense sets. Since $\text{int}(\text{cl}(\alpha)) = 1 \neq 0$, $\text{int}(\text{cl}(\beta)) = 1 \neq 0$ and $\text{int}(\text{cl}(\gamma)) = 1 \neq 0$. Therefore α and β are not of fuzzy generalized α -nowhere dense set.

4. Fuzzy α -generalized Baire spaces, Fuzzy weakly generalized Baire space, Fuzzy generalized α -Baire space

We introduce fuzzy α -generalized Baire space, fuzzy weakly generalized Baire space, fuzzy generalized α -Baire space with suitable examples.

Definition 4.1: A fuzzy topological space (X,T) is called fuzzy α -generalized Baire space if $\text{int}(\cup_{i=1}^{\infty} (\lambda_i)) = 0$, where (λ_i) 's are fuzzy α -generalized nowhere dense sets in (X,T) .

In example 3.1, The fuzzy sets $1-\lambda$, $1-\mu$, $1-\nu$, $1-(\nu\mu)$'s are fuzzy α -generalized nowhere dense sets. Now $\text{int}[(1-\lambda) \vee (1-\mu) \vee (1-\nu) \vee (1-(\nu\mu))] = \text{int}(1-\lambda) = 0$. Therefore the fuzzy topological space (X,T) is fuzzy α -generalized Baire space.

Definition 4.2: A fuzzy topological space (X,T) is called fuzzy weakly generalized Baire space if $\text{int}(\cup_{i=1}^{\infty} (\lambda_i)) = 0$, where (λ_i) 's are fuzzy weakly generalized nowhere dense sets in (X,T) .

In example 3.3, The fuzzy sets $1-\lambda$, $1-\mu$, $1-\nu$, $1-(\nu\mu)$, $(1-\lambda\mu)$, $1-(\mu\nu)$, $1-(\mu\lambda)$'s are fuzzy weakly generalized nowhere dense sets. Now $\text{int}[(1-\lambda) \vee (1-\mu) \vee (1-\nu) \vee (1-(\nu\mu)) \vee (1-(\lambda\mu)) \vee (1-(\mu\nu)) \vee (1-(\mu\lambda))] = \text{int}(1-(\lambda\mu)) = 0$. Therefore the fuzzy topological space (X,T) is fuzzy weakly generalized Baire space.

Definition 4.3: A fuzzy topological space (X,T) is called fuzzy generalized α -Baire space if $\text{int}(\cup_{i=1}^{\infty} (\lambda_i)) = 0$, where (λ_i) 's are fuzzy generalized α -nowhere dense sets in (X,T) .

In example 3.5, The fuzzy sets $1-\lambda$, $1-\mu$, $1-\nu$'s are fuzzy generalized α -nowhere dense sets. Now $\text{int}[(1-\lambda) \vee (1-\mu) \vee (1-\nu)] = \text{int}(1-\mu) = 0$. Therefore the fuzzy topological space (X,T) is fuzzy generalized α -Baire space.

5. Some relations of fuzzy α -generalized, fuzzy weakly generalized Baire space and fuzzy generalized α -Baire space:

Proposition 5.1: A fuzzy generalized α -Baire space is also a fuzzy α -generalized Baire space.

Consider the following example.

Let $X = \{a,b,c\}$. The fuzzy sets λ , μ and ν are defined on X as follows:

- $\lambda : X \rightarrow [0,1]$ defined as $\lambda(a) = 0.9$; $\lambda(b) = 0.7$; $\lambda(c) = 0.6$.
- $\mu : X \rightarrow [0,1]$ defined as $\mu(a) = 0.8$; $\mu(b) = 0.5$; $\mu(c) = 0.4$.
- $\nu : X \rightarrow [0,1]$ defined as $\nu(a) = 0.7$; $\nu(b) = 0.6$; $\nu(c) = 0.6$.

Then $T = \{0, \lambda, \mu, \nu, (\mu\nu), (\mu\lambda), 1\}$ is fuzzy topology on X .

- $1-\lambda < \Rightarrow$ is open $\Rightarrow \text{cl}(1-\lambda) = \lambda$,
- $1-\mu < \Rightarrow$ is open $\Rightarrow \text{cl}(1-\mu) = \mu$,
- $1-\nu < \Rightarrow$ is open $\Rightarrow \text{cl}(1-\nu) = \nu$,
- $1-\mu\nu < \Rightarrow$ is open $\Rightarrow \text{cl}(1-\mu\nu) = \mu\nu$,
- $1-\mu\lambda < \Rightarrow$ is open $\Rightarrow \text{cl}(1-\mu\lambda) = \mu\lambda$.

$(1-\lambda), (1-\mu), (1-\nu), (1-\mu\nu), (1-\mu\lambda)$'s are fuzzy generalized α -closed set.

- $\text{Int}(\text{cl}(1-\lambda)) = \text{int}(1-\lambda) = 0$,
- $\text{Int}(\text{cl}(1-\mu)) = \text{int}(1-\mu) = 0$,
- $\text{Int}(\text{cl}(1-\nu)) = \text{int}(1-\nu) = 0$,
- $\text{Int}(\text{cl}(1-\mu\nu)) = \text{int}(1-\mu\nu) = 0$,
- $\text{Int}(\text{cl}(1-\mu\lambda)) = \text{int}(1-\mu\lambda) = 0$.

$(1-\lambda), (1-\mu), (1-\nu), (1-\mu\nu), (1-\mu\lambda)$'s are fuzzy generalized α -nowhere dense set. Now

- $\text{Int}[(1-\lambda) \vee (1-\mu) \vee (1-\nu) \vee (1-\mu\nu) \vee (1-\mu\lambda)] = 0$
- $\text{Int}(1-\mu\lambda) = 0$.

Therefore (X,T) is a fuzzy generalized α -Baire space. Now to say that it is fuzzy α -generalized Baire space we have to show that μ is α -open $\text{Int}(\text{cl}(\mu)) = \mu$.

- $\text{Int}(\text{cl}(\mu)) = \text{int}(\text{cl}(\mu)) = \text{int}(1) = 1$,
- $\text{Int}(\text{cl}(\mu)) = \text{int}(\text{cl}(\mu)) = \text{int}(1) = 1$,
- $\text{Int}(\text{cl}(\mu)) = \text{int}(\text{cl}(\mu)) = \text{int}(1) = 1$,
- $\text{Int}(\text{cl}(\mu\nu)) = \text{int}(\text{cl}(\mu\nu)) = \text{int}(1) = 1$,
- $\text{Int}(\text{cl}(\mu\lambda)) = \text{int}(\text{cl}(\mu\lambda)) = \text{int}(1) = 1$.

Thus generalized α -Baire space is also a α -generalized Baire space.

Proposition 5.2: A weakly generalized Baire space is also a generalized α -Baire space.

Consider an example.

Let $X = \{a,b,c\}$. The fuzzy sets λ , μ and ν are defined on X as follows:

- $\lambda : X \rightarrow [0,1]$ defined as $\lambda(a) = 0.9$; $\lambda(b) = 0.9$; $\lambda(c) = 0.8$.
- $\mu : X \rightarrow [0,1]$ defined as $\mu(a) = 0.8$; $\mu(b) = 0.7$; $\mu(c) = 0.6$.
- $\nu : X \rightarrow [0,1]$ defined as $\nu(a) = 0.9$; $\nu(b) = 0.8$; $\nu(c) = 0.6$.

Then $T = \{0, \lambda, \mu, \nu, 1\}$ is fuzzy topology on X .

- Now $1-\lambda < \Rightarrow \text{cl}(\text{int}(1-\lambda)) = \text{cl}(0) = 0$,
- $1-\mu < \Rightarrow \text{cl}(\text{int}(1-\mu)) = \text{cl}(0) = 0$,
- $1-\nu < \Rightarrow \text{cl}(\text{int}(1-\nu)) = \text{cl}(0) = 0$.

$(1-\lambda), (1-\mu), (1-\nu)$'s are fuzzy weakly generalized closed set.

- $\text{Int}(\text{cl}(\text{int}(1-\lambda))) = \text{int}(1-\lambda) = 0$,
- $\text{Int}(\text{cl}(\text{int}(1-\mu))) = \text{int}(1-\mu) = 0$,
- $\text{Int}(\text{cl}(\text{int}(1-\nu))) = \text{int}(1-\nu) = 0$.

$(1-\lambda), (1-\mu), (1-\nu)$'s are fuzzy weakly generalized nowhere dense set.

- $\text{Int}([(1-\lambda) \vee (1-\mu) \vee (1-\nu)]) = 0$

$\text{Int}(1-\mu) = 0$. Therefore (X,T) is a fuzzy weakly generalized Baire spaces. Now to say that it is generalized α -Baire space we have to show that $\text{cl}(\mu) \supseteq \mu \forall \mu$ is open in X .

- $(1-\lambda) < \Rightarrow \mu$ is open $\Rightarrow \text{cl}(1-\lambda) = \lambda$,
- $(1-\mu) < \Rightarrow \mu$ is open $\Rightarrow \text{cl}(1-\mu) = \mu$,

$(1-\gamma) < \forall \mu \Rightarrow \mu$ is open $\Rightarrow \text{cl}(1-\gamma) = \mu$.
 $(1-\mu), (1-\mu), (1-\gamma)$'s are fuzzy generalized γ -closed set.
 $\text{Int cl int}(1-\gamma) = \text{int}(1-\gamma) = 0$,
 $\text{Int cl int}(1-\mu) = \text{int}(1-\mu) = 0$,
 $\text{Int cl int}(1-\mu) = \text{int}(1-\mu) = 0$.
 $(1-\mu), (1-\mu), (1-\gamma)$'s are fuzzy generalized γ -nowhere dense set. Now

$\text{Int} [((1-\mu) \vee (1-\mu) \vee (1-\gamma))] = 0$
 $\text{Int}(1-\mu) = 0$. Therefore (X, T) is a fuzzy generalized γ -Baire spaces. Thus weakly generalized Baire space is also a generalized γ -Baire space.

Proposition 5.3: A weakly generalized Baire space is also a γ -generalized Baire space.

Consider an example.
 Let $X = \{a, b, c\}$. The fuzzy sets μ, ν and γ are defined on X as follows:

- $\mu : X \rightarrow [0, 1]$ defined as $\mu(a) = 0.9; \mu(b) = 0.7; \mu(c) = 0.6$.
- $\nu : X \rightarrow [0, 1]$ defined as $\nu(a) = 0.8; \nu(b) = 0.8; \nu(c) = 0.5$.
- $\gamma : X \rightarrow [0, 1]$ defined as $\gamma(a) = 0.8; \gamma(b) = 0.8; \gamma(c) = 0.8$.

Then $T = \{0, \mu, \nu, \lambda\nu\mu, \lambda\mu, \lambda\lambda\gamma, 1\}$ is fuzzy topology on X .

- $1-\mu < \forall \mu \Rightarrow \text{cl int}(1-\mu) = \text{cl}(0) = 0 \leq \forall \mu$,
- $1-\mu < \forall \mu \Rightarrow \text{cl int}(1-\mu) = \text{cl}(0) = 0 \leq \forall \mu$,
- $1-\gamma < \forall \mu \Rightarrow \text{cl int}(1-\gamma) = \text{cl}(0) = 0 \leq \forall \mu$,
- $1-\nu\mu < \forall \mu \Rightarrow \text{cl int}(1-\nu\mu) = \text{cl}(0) = 0 \leq \forall \mu$,
- $1-\lambda\mu < \forall \mu \Rightarrow \text{cl int}(1-\lambda\mu) = \text{cl}(0) = 0 \leq \forall \mu$,
- $1-\lambda\gamma < \forall \mu \Rightarrow \text{cl int}(1-\lambda\lambda\gamma) = \text{cl}(0) = 0 \leq \forall \mu$.

$(1-\mu), (1-\mu), (1-\gamma), (1-\lambda\gamma), (1-\nu\mu), (1-\lambda\mu)$'s are fuzzy weakly generalized closed set.

- $\text{Int cl}(1-\mu) = \text{int}(1-\mu) = 0$,
- $\text{Int cl}(1-\mu) = \text{int}(1-\mu) = 0$,
- $\text{Int cl}(1-\gamma) = \text{int}(1-\gamma) = 0$,
- $\text{Int cl}(1-\nu\mu) = \text{int}(1-\nu\mu) = 0$,
- $\text{Int cl}(1-\lambda\mu) = \text{int}(1-\lambda\mu) = 0$,
- $\text{Int cl}(1-\lambda\gamma) = \text{int}(1-\lambda\gamma) = 0$.

$(1-\mu), (1-\mu), (1-\gamma), (1-\lambda\gamma), (1-\nu\mu), (1-\lambda\mu)$'s are fuzzy weakly generalized nowhere dense set.

$\text{Int} [(1-\mu) \vee (1-\mu) \vee (1-\gamma) \vee (1-\lambda\mu) \vee (1-\nu\mu) \vee (1-\lambda\lambda\gamma)] = 0$

$\text{Int}(1-\lambda\mu) = 0$. (X, T) is a fuzzy weakly generalized Baire space. Now to say that it is γ -generalized Baire space we have to show that it μ is open and $\text{cl}(\mu) = \mu$.

- $1-\mu < \forall \mu \Rightarrow \forall \mu$ is γ -open $\Rightarrow \text{cl}(1-\mu) = \mu$,
- $1-\mu < \forall \mu \Rightarrow \forall \mu$ is γ -open $\Rightarrow \text{cl}(1-\mu) = \mu$,
- $1-\gamma < \forall \mu \Rightarrow \forall \mu$ is γ -open $\Rightarrow \text{cl}(1-\gamma) = \mu$,
- $1-\nu\mu < \forall \mu \Rightarrow \forall \mu$ is γ -open $\Rightarrow \text{cl}(1-\nu\mu) = \mu$,
- $1-\lambda\mu < \forall \mu \Rightarrow \forall \mu$ is γ -open $\Rightarrow \text{cl}(1-\lambda\mu) = \mu$,

$1-\lambda\gamma < \forall \mu \Rightarrow \forall \mu$ is γ -open $\Rightarrow \text{cl}(1-\lambda\gamma) = \mu$.
 $(1-\mu), (1-\mu), (1-\gamma), (1-\lambda\gamma), (1-\nu\mu), (1-\lambda\mu)$'s are fuzzy generalized closed set.

- $\text{Int cl}(1-\mu) = \text{int}(1-\mu) = 0$,
- $\text{Int cl}(1-\mu) = \text{int}(1-\mu) = 0$,
- $\text{Int cl}(1-\gamma) = \text{int}(1-\gamma) = 0$,
- $\text{Int cl}(1-\nu\mu) = \text{int}(1-\nu\mu) = 0$,
- $\text{Int cl}(1-\lambda\mu) = \text{int}(1-\lambda\mu) = 0$,
- $\text{Int cl}(1-\lambda\gamma) = \text{int}(1-\lambda\gamma) = 0$.

$(1-\mu), (1-\mu), (1-\gamma), (1-\lambda\gamma), (1-\nu\mu), (1-\lambda\mu)$'s are fuzzy generalized nowhere dense set.

$\text{Int} [(1-\mu) \vee (1-\mu) \vee (1-\gamma) \vee (1-\lambda\mu) \vee (1-\nu\mu) \vee (1-\lambda\lambda\gamma)] = 0$

$\text{Int}(1-\lambda\mu) = 0$. (X, T) is a fuzzy γ -generalized Baire space. Thus weakly generalized Baire space is also γ -generalized Baire space.

Conclusion

In this paper, we introduce a concept fuzzy γ -generalized, fuzzy weakly generalized Baire space and fuzzy generalized γ -Baire space, Further some relation between them and illustrated with some examples.

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