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INTUITIONISTIC DOUBLE LAYERED FUZZY PLANAR GRAPH

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A B S T R A C T **RESEARCH ARTICLE**

Infuzzy graph,intuitionistic double layered fuzzy graph and intuitionistic fuzzy planar graph have been defined already by different authors. In this paper intuitionistic double layered fuzzy planar graph is defined with examples. We introduce the notions of intuitionistic double layered fuzzy planar graph and theoretical concepts of their interesting properties.

Keywords:

Fuzzy graphs, intuitionistic fuzzy planar graph, intuitionisticdouble layered fuzzy planar graph, intuitionisticweak double layered fuzzy planar graph, intuitionisticfuzzy faces.

1. Introduction

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975[2]. During the same time Yeh and Bang have also introduced various concepts in connectedness with fuzzy graph[1].Abdul-jabbar and Naoom[8] introduced the concept of fuzzy planar graph. Also, Nirmala and Dhanabal [7] defined special fuzzy planar graph. A.Pal, S.Samanta and M.Pal[4] have defined fuzzy planar graph in a different concept where crossing of edge are allowed. Intuitionistic fuzzy planar graph was introduces by Noura Alshehri and Muhammad Akram[3]. The first definition of intuitionistic fuzzy relations and intuitionistic fuzzy graphs were introduced by Atanassov in 1986[8]. The double layered fuzzy graph was introduced Pathinathan and Jesintha Rosline[6]. The Intuitionistic double layered fuzzy graph is given by Jesintha Roseline and Pathinathan[9]. In this paper we defineIntuitionistic Double Layered Fuzzy Planar Graph $(IDLFPG)$and we discuss some properties.

2. Preliminaries

Definition 2.1 [2]: A fuzzy graph $G=(V,\pm,\ll)$ is a non-empty set V together with a pair of functions $\pm : V \to [0,1]$ and $\ll : V \times V \to [0,1]$ such that for all $a,b \in V$, $\ll(a,b) \le \pm(a) \wedge \pm(b)$, where $\pm(a)$ a$\pm(b)$ and $\ll(a,b)$ represent the membership values of the vertex a and of the edge (a,b) in G respectively.

Definition 2.2 [4]: Let Ψ be a fuzzy multigraph and for a certain geometrical representation $P_1, P_2, ...P_N$ be the point of intersections between the edges Ψ is said to be fuzzy planar graph with fuzzy planarity value f , where

$$f = \frac{1}{1+\{P_1+P_2+.....+P_N\}}$$

It is obvious that f is bounded and the range of f is $0< f \le 1$.

Definition 2.3[13]: Strength of the intuitionistics double layered fuzzy edge xy can be measured by the value.

$$DLI_{xy} = \left(DLM_{xy}, DLN_{xy}\right)$$

$$= \left(\frac{E_B(xy)_i}{E_A(x) \wedge E_A(y)}, \frac{V_B(xy)_i}{V_A(x) \vee V_A(y)} \right)$$

3. Intuitionistic Double layered fuzzy planar graph (IDLFPG)

Definition 3.1: Let Ψ be an intuitionistic double layered fuzzy planar graph with the underlying crisp graph Ψ^***.** The vertex set of $IDL(\Psi)$be $\left\langle E_{DL_1}, V_{DL_1} \right\rangle$.the geometrical representation $IDLP_1, IDLP_2, ...IDLP_N$ be the points of intersections between the edges $IDL(\Psi)$ is said to be intuitionistic double layered fuzzy planar graph with intuitionistic double layered fuzzy planarity value $f_{IDL} = \left(M_{f_{IDL}}, N_{f_{IDL}}\right)$

$$= \left(\frac{1}{1+\left(M_{IDL_1P}+M_{IDL_2P}+...+M_{IDL_NP}\right)}, \frac{1}{1+\left(N_{IDL_1P}+N_{IDL_2P}+...+N_{IDL_NP}\right)} \right)$$

Clearly, $f_{IDL} = \left(M_{f_{IDL}}, N_{f_{IDL}}\right)$ is bounded and $0 < M_{f_{IDL}} \leq 1$.

Remark: 3.1.1. We only consider minimal intersecting points of the intuitionistic double layered fuzzy planar graph.

Example: 3.1.2. Consider the intuitionistic fuzzy planar graph Ψ, whose crisp graph Ψ^* is a cycle with n vertices.

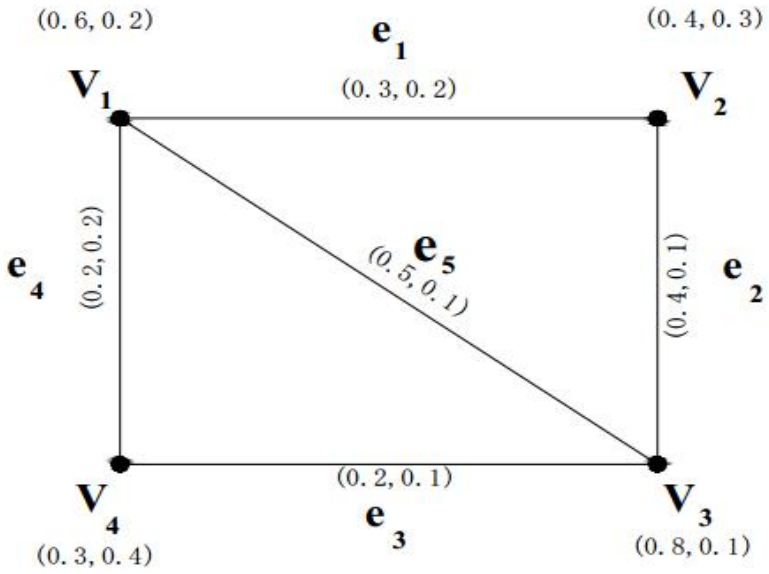

Figure1 A Intuitionistic Fuzzy Planar graph

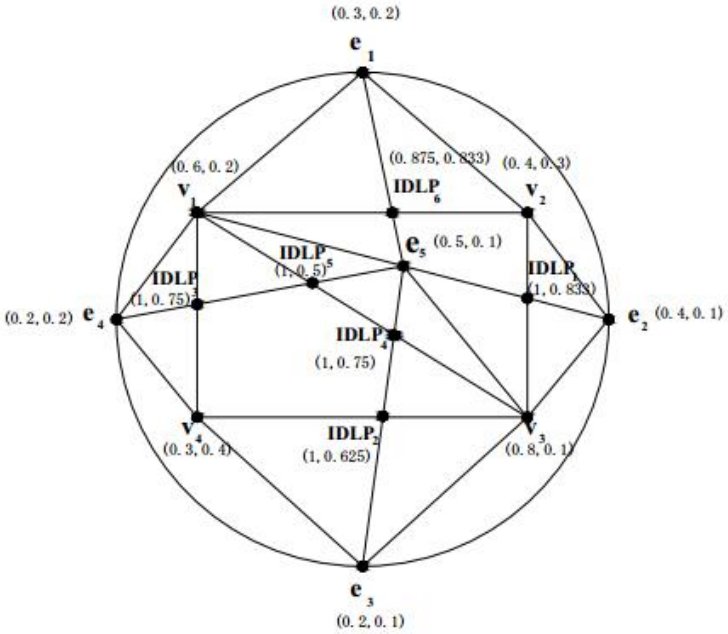

Figure 2 Intuitionistic Double layered fuzzy planar graph

Consider theintuitionistic fuzzy planar graph with n=5 vertices.

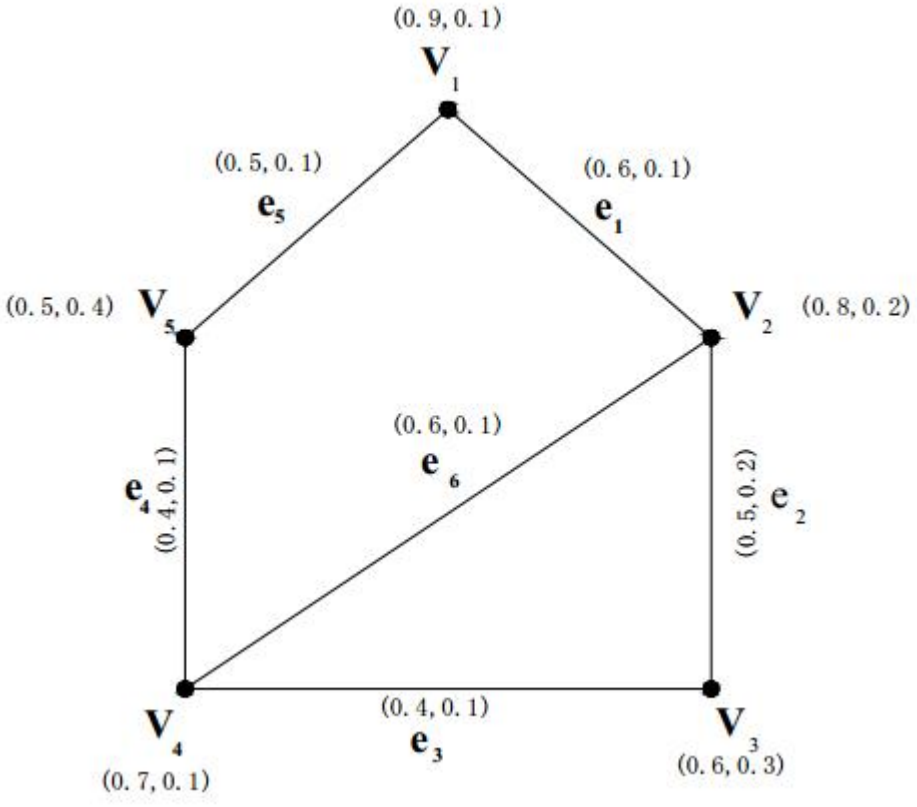

Figure 3 AIntuitionistic Fuzzy Planar graph

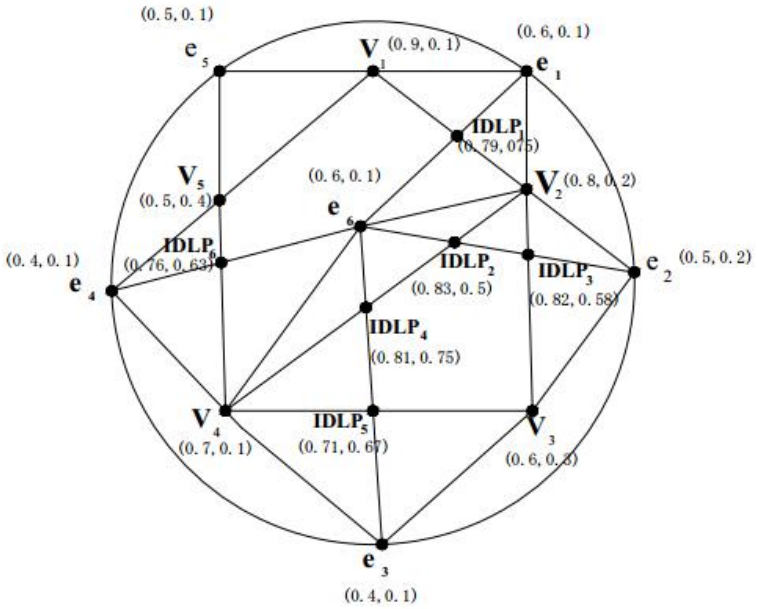

Figure 4 Intuitionistic Double layered fuzzy planar graph

Consider the intuitionistic fuzzy planar graph with n=6 vertices.

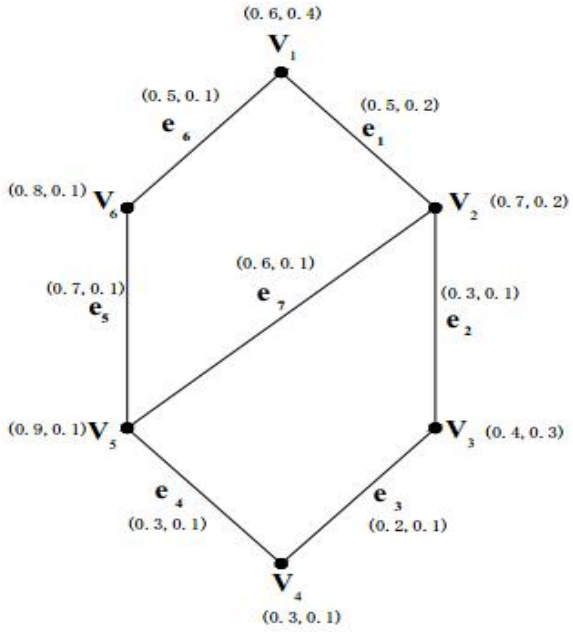

Figure 5 AIntuitionistic Fuzzy Planar graph

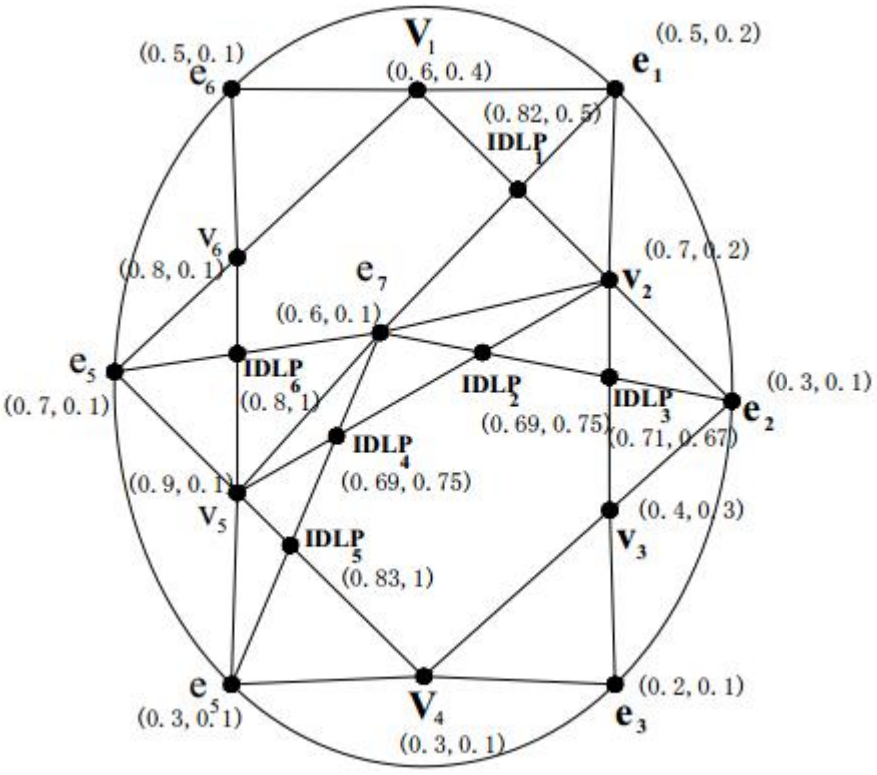

Figure 6 Intuitionistic Double layered fuzzy planar graph

Intuitionstic double layered fuzzy planarity value for the intuitionistic double layered fuzzy complete multigraph is given below.

Theorem: Let Ψ be an intuitionistic double layered fuzzy complete multigraph. The intuitionistic double layered fuzzy planarity value $f_{IDL} = \left(M_{f_{IDL}}, N_{f_{IDL}}\right)$ of Ψ is given by $M_{f_{IDL}} = \frac{1}{\left(1+DLN_P\right)}$ and $N_{f_{IDL}} = \frac{1}{\left(1+DLN_P\right)}$ Such that $M_{f_{IDL}} + N_{f_{IDL}} \leq 1$, where DLN_P is the number of points of intersections between the edges in Ψ.

Proof: Let be an intuitionistic double layered fuzzy complete multigraph, For the double layered fuzzy complete multigraph $\left\{\ll_A(x) \wedge \ll_A(y)\right\} = \ll_B(xy)_i, \left\{V_A(x) \wedge V_A(y)\right\} = V_B(xy)_i$ for all $i = 1,2,....m$ and for all $x, y \in V$ for each intersecting edge (xy) and $i = 1,2,.....P_{xy}$

Let $IDLP_1, IDLP_2, ...IDLP_k$ be the double layered intersections points between the edge in Ψ, k being an integer. For any double layered intersecting edge (ab) in Ψ. $DLI_{ab} = (DLM_{ab}, DLN_{ab}) = 1.IDLP_1$ Therefore, for the point of intersection between the edges (ab) and , $\mathcal{J}_{IDLP_1}$ is equal to $\left(\frac{1+1}{2}, \frac{1+1}{2}\right) = (1,1)$

Hence $\mathcal{J}_{IDLP_i} = 1$ for $i = 1,2,.....k$

$$f_{IDL} = \left(M_{f_{IDL}}, N_{f_{IDL}}\right)$$

$$= \left(\frac{1}{1+\left(M_{IDL_1P}+M_{IDL_2P}+....+M_{IDL_kP}\right)}, \frac{1}{1+\left(N_{IDL_1P}+N_{IDL_2P}+....+N_{IDL_kP}\right)}\right)$$

$$= \left(\frac{1}{1+(1+1+.....+1)}, \frac{1}{1+(1+1+.....+1)}\right)$$

$$= \left(\frac{1}{1+n_p}, \frac{1}{1+n_p}\right)$$

Where n_p is the number of point of intersections between the edges in Ψ.

Definition:3.2

An intuitionistic double layered fuzzy planar graph $LDL(\Psi)$ is called weak intuitionistic double layered fuzzy planar graph if the intuitionistic double layered fuzzy planarity value $f_{IDL} = \left(M_{f_{IDL}}, N_{f_{IDL}}\right)$ of the graph is $M_{f_{IDL}}$ less than 0.5 and $N_{f_{IDL}}$ also less than 0.5.

Theorem 2: Let $LDL(\Psi)$ be a weak intuitionistic double layered fuzzy planar graph. The number of intuitionistic double layered intersecting points between weak edges in $LDL(\Psi)$ is $IDLP_1, IDLP_2, ...IDLP_N$

Proof: Let $LDL(\Psi)$ be a weak intuitionistic double layered fuzzy planar graph.

Let, if possible, $LDL(\Psi)$ has one point of intersecting between tow strong edges $LDL(\Psi)$.

For any strong edges $(ab, \ll_B(ab)_i, \neg_B(ab)_i)$

$$\ll_B(ab)_i \geq \frac{1}{2}\min\{\ll_A(a), \ll_B(b)\}$$

$$\neg_B(ab)_i \leq \frac{1}{2}\max\{\neg_A(a), \neg_A(b)\}.$$

This shows that $M_{ab} \geq 0.5$ or $N_{ab} \leq 0.5$. Thus for two intuitionistic double layered intersecting strong edges $(ab, \ll_B(ab)_i, \neg_B(ab)_i)$ and $(cd, \ll_B(cd)_j, \neg_B(cd)_j)$

$$\frac{M_{ab}+M_{cd}}{2} < 0.5, \frac{N_{ab}+N_{cd}}{2} < 0.5$$

That is , $M_{IDLP_1} < 0.5, N_{IDLP_1} < 0.5$

This implies that, $1 + M_{IDLP_1} < 1.5, 1 + N_{IDLP_2} < 1.5$

There fore

$$M_{f_{IDL}} = \frac{1}{1+M_{IDLP_1}} > 0.5, N_{f_{IDL}} = \frac{1}{1+N_{IDLP_1}} > 0.5$$

It contradicts the fact that the intuitionistic fuzzy graph is a week intuitionistic double layered fuzzy planar graph. So number of points of intersections between strong edges cannot be one.

Obviously, if the number of points of intersections of strong intuitionisticdouble layered fuzzy edges increases, the intuitionisticdouble layered fuzzy planarity value decreases.

Similarly, if the number of points of intersection of strong edges is one, then the intuitionisticdouble layered fuzzy planarity value $M_{f_{IDL}} > 0.5, N_{f_{IDL}} < 0.5.$

Any intuitionisticdouble layered fuzzy planar graph without any crossing between edges is a strong intuitionisticdouble layered fuzzy planar graph.

But intuitionistic double layered fuzzy planar graph contains large number of intersecting points. So, the intuitionistic double layered fuzzy planar graph is a week double layered fuzzy planar graph.

Definition: 3.3

Let $IDL(\Psi)$ be a intuitionistic double layered fuzzy planar graph and $E_{IDL} = \{(ab, \ll_B(ab)_i, \neg_B(ab)_i; i = 1,2,.....,m \mid ab \in V \times V\}.$

An intuitionistic double layered fuzzy edges $E'_{IDL} \subset E_{IDL}$ of a geometric representation of $IDL(\Psi)$. The membership and non membership value of the intuitionistic double layered fuzzy face are,

$$\wedge\left\{\frac{\ll_B(ab)_i}{\{\ll_A(a) \wedge \ll_A(b)\}}, i = 1,2,....., m \mid ab \in E'_{IDL}\right\}$$

$$\vee\left\{\frac{\neg_B(ab)_i}{\{\neg_A(a) \vee \neg_A(b)\}}, i = 1,2,....., m \mid ab \in E'_{IDL}\right\}$$

An intuitionistic double layered fuzzy planar graph face is called weak intuitionistic double layered fuzzy planar fuzzy face if its membership value is < 0.5 and non membership value is < 0.5 otherwise strong intuitionistic double layered fuzzy planar fuzzy face.

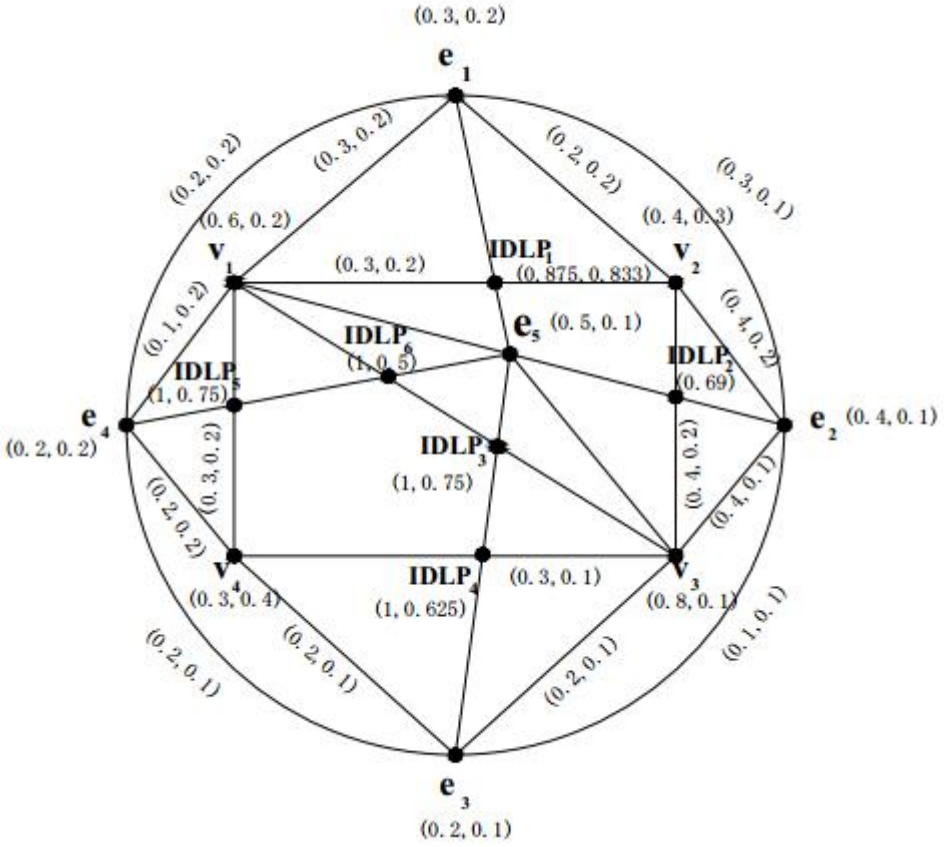

Figure 7 Intuitionistic Double layered fuzzy planar graph

Example: Consider an intuitionistic double layered fuzzy planar graph as shown in figure 7. the intuitionistic double layered fuzzy planar graph has the following faces:

(i) Intuitionistic double layered fuzzy planar graph face F_1 is bounded by the edges $(e_1v_2,0.2,0.2),(v_2e_2,0.4,0.2)and(e_1e_2,0.3,0.1)$

(ii)Outer intuitionistic double layered fuzzy planar graph face F_{22} is surrounded by edges $(e_1e_2,0.3,0.1),(e_2e_3,0.1,0.1),(e_3e_4,0.2,0.1)and(e_4e_1,0.2,0.2)$

(iii)Similarly we can fined $F_2, F_3, F_4, ...F_{21}$.

Conclusion

Fuzzy graph theory has numerous applications to problems in systems analysis, operations research, economics, and transportation. However, in many cases, some aspects of a graphtheoretic problem may be vague or uncertain. It is natural to deal with the vagueness and uncertainty using the methods of fuzzy sets. Since intuitionistic fuzzy set has shown advantages in handling vagueness and uncertainty compared to fuzzy set, we have applied the concept of intuitionistic fuzzy sets to intuitionistic double layered fuzzy planar graph in this paper. The natural extension of this research work is application of intuitionistic double layered fuzzy planar graphs in the area of applied soft computing including neural networks, decision making, and geographical information systems.

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