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A STUDY ON INTERVAL VALUED INTUITIONISTIC FUZZY SETS OF SECOND TYPE AND THEIR OPERATORS

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A B S T R A C T **RESEARCH ARTICLE**

In this paper, we introduce some operators over Interval Valued Intuitionistic Fuzzy Sets of Second Type and establish some of their properties.

Keywords:

Intuitionistic Fuzzy Sets, Intuitionistic Fuzzy Sets of Second Type, Interval Valued Fuzzy Sets, Interval Valued Intuitionistic Fuzzy Sets, Interval Valued Intuitionistic Fuzzy Sets of Second Type.

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1. Introduction

An Intuitionistic Fuzzy Set for a given underlying set X were introduced by K. T. Atanassov [2] which is the generalization of ordinary Fuzzy Sets. K. T. Atanassov and G. Gargov [3] further introduced the concepts of Interval Valued Intuitionistic Fuzzy Set (IVIFS).

The present authors further introduced the new extension of IVIFS namely Interval Valued Intuitionistic Fuzzy Sets of Second Type (IVIFSST) and established some of their properties [4]. The rest of the paper is designed as follows: In Section 2, we give some basic definitions. In Section 3, we introduce some new operators over Interval Valued Intuitionistic Fuzzy Sets of Second Type and establish some of their properties. This paper is concluded in section 4.

2. Preliminaries

In this section, we give some basic definitions.

Definition 2.1[2] Let X be a nonempty set. An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the following form.

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$$

Where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$.

$$0 \le \mu_A(x) + \nu_A(x) \le 1$$

Definition 2.2 [2] Let a set X be fixed. An Intuitionistic Fuzzy Set of Second Type (IFSST) A in X is defined as an object of the following form.

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$.

$$0 \le \mu_A{}^2(x) + \nu_A{}^2(x) \le 1$$

Definition 2.3 [3] An Interval Valued Intuitionistic Fuzzy Sets (IVIFS) A in X is given by

$$A = \{\langle x, M_A(x), N_A(x)\rangle \mid x \in X\}$$

where $M_A : X \to [0,1]$, $N_A : X \to [0,1]$. The intervals $M_A(x)$ and $N_A(x)$ denote the degree of membership and the degree of non-membership of the element x in X, where $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)]$ with the condition that

$$M_{AU}(x) + N_{AU}(x) \le 1 \ \forall \ x \in X$$

Definition 2.4 [4] An Interval Valued Intuitionistic Fuzzy Sets of Second Type (IVIFSST) A in X is given by

$$A = \{\langle x, M_A(x), N_A(x)\rangle \mid x \in X\}$$

where $M_A: X \to [0,1]$, $N_A: X \to [0,1]$. The intervals $M_A(x)$ and $N_A(x)$ denote the degree of membership and the degree of non-membership of the element x in X, where $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)]$ with the condition that $M^2{}_{AU}(x) + N^2{}_{AU}(x) \le 1 \ \forall x \in X$.

Definition 2.5 [4] For every two IVIFSST A and B, we have the following relations and operations

1. $\bar{A} = \{\langle x, [N_{AL}(x), N_{AU}(x)], [M_{AL}(x), M_{AU}(x)]\rangle \mid x \in X\}$
2. $A \cup B = \{\langle x, [\max(M_{AL}(x), M_{BL}(x)), \max(M_{AU}(x), M_{BU}(x))], [\min(N_{AL}(x), N_{BL}(x)), \min(N_{AU}(x), N_{BU}(x))]\rangle \mid x \in X\}$
3. $A \cap B = \{\langle x, [\min(M_{AL}(x), M_{BL}(x)), \min(M_{AU}(x), M_{BU}(x))], [\max(N_{AL}(x), N_{BL}(x)), \max(N_{AU}(x), N_{BU}(x))]\rangle \mid x \in X\}$
4. $A + B$
$= \{\langle x, [M^2{}_{AL}(x) + M^2{}_{BL}(x) - M^2{}_{AL}(x)M^2{}_{BL}(x), M^2{}_{AU}(x) + M^2{}_{BU}(x) - M^2{}_{AU}(x)M^2{}_{BU}(x)],$

$[N^2_{AL}(x)N^2_{BL}(x), N^2_{AU}(x)N^2_{BU}(x)]\rangle \mid x \in X\}$

5. $A.B = \{\langle x, [M^2_{AL}(x)M^2_{BL}(x), M^2_{AU}(x)M^2_{BU}(x)],$
$N^2_{AL}(x) + N^2_{BL}(x) - N^2_{AL}(x)N^2_{BL}(x),$
$N^2_{AU}(x) + N^2_{BU}(x) - N^2_{AU}(x)N^2_{BU}(x)]\rangle \mid x \in X\}$

6. $A @ B =$

$$\left\{\langle x, \left[\frac{M^2_{AL}(x) + M^2_{BL}(x)}{2}, \frac{M^2_{AU}(x) + M^2_{BU}(x)}{2}\right] \left[\frac{N^2_{AL}(x) + N^2_{BL}(x)}{2}, \frac{N^2_{AU}(x) + N^2_{BU}(x)}{2}\right]\rangle \mid x \in X\right\}$$

Definition 2.6 [5] For every IVIFSST, we have the following

Necessity operator

$$\Box A = \{\langle x, [M_{AL}(x), M_{AU}(x)], \left[N_{AL}(x), \sqrt{1 - M^2_{AU}(x)}\right]\rangle \mid x \in X\}$$

Possibility operator

$$\Diamond A = \{\langle x, \left[M_{AL}(x), \sqrt{1 - N^2_{AU}(x)}\right], [N_{AL}(x), N_{AU}(x)]\rangle \mid x \in X\}$$

3. Some Operators Over IVIFSST

In this section, we define some new operators over IVIFSSTs and establish some of their properties.

Definition 3.1 Let X be a non empty set and for every IVIFSST A, we define four operators which map an IFS to an IVIFSST as follows

$*_1 A = \{\langle x, M_{AL}(x), N_{AL}(x)\rangle \mid x \in X\}$,
$*_2 A = \{\langle x, M_{AL}(x), N_{AU}(x)\rangle \mid x \in X\}$,
$*_3 A = \{\langle x, M_{AU}(x), N_{AL}(x)\rangle \mid x \in X\}$,
$*_4 A = \{\langle x, M_{AU}(x), N_{AU}(x)\rangle \mid x \in X\}$.

Theorem 3.1 *Let X be a non empty set and for every IVIFSST A, we have the following*

i. $*_1 \Box A = *_1 A,$
ii. $*_3 \Box A = *_3 A,$
iii. $*_1 \Diamond A = *_1 A,$
iv. $*_2 \Diamond A = *_2 A.$

Proof:

Let

$A = \{\langle x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\rangle \mid x \in X\}$

Then,

$\Box A = \{\langle x, [M_{AL}(x), M_{AU}(x)],$

$\left[N_{AL}(x), \sqrt{1 - M^2_{AU}(x)}\right]\rangle \mid x \in X\},$

$\Diamond A = \{\langle x, \left[M_{AL}(x), \sqrt{1 - N^2_{AU}(x)}\right],$

$[N_{AL}(x), N_{AU}(x)]\rangle \mid x \in X\}$

(i) Apply the operator $*_1$ in $\Box A$ we have

$*_1 \Box A = *_1 \{\langle x, [M_{AL}(x), M_{AU}(x)],$

$\left[N_{AL}(x), \sqrt{1 - M^2_{AU}(x)}\right]\rangle \mid x \in X\}$

$= \{\langle x, M_{AL}(x), N_{AL}(x)\rangle \mid x \in X\}$

$= *_1 A$

(ii) Apply the operator $*_3$ in $\Box A$ we have

$*_3 \Box A = *_3 \{\langle x, [M_{AL}(x), M_{AU}(x)],$

$\left[N_{AL}(x), \sqrt{1 - M^2_{AU}(x)}\right]\rangle \mid x \in X\}$

$= \{\langle x, M_{AU}(x), N_{AL}(x)\rangle \mid x \in X\}$

$= *_3 A$

Other proofs are similar.

Theorem 3.2 *Let X be a non empty set and for every IVIFSST A, we have the following*

i. $\overline{*_1 \bar{A}} = *_1 A,$
ii. $\overline{*_2 \bar{A}} = *_3 A,$
iii. $\overline{*_3 \bar{A}} = *_2 A,$
iv. $\overline{*_4 \bar{A}} = *_4 A.$

Proof:

Let

$A = \{\langle x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\rangle \mid x \in X\}$

Then,

$\bar{A} = \{\langle x, [N_{AL}(x), N_{AU}(x)], [M_{AL}(x), M_{AU}(x)]\rangle \mid x \in X\}$

(i) Apply the operator $*_1$ in $\bar{A}$ we have

$*_1 \bar{A} = *_1 \{\langle x, [N_{AL}(x), N_{AU}(x)], [M_{AL}(x), M_{AU}(x)]\rangle \mid x \in X\}$

$= \{\langle x, N_{AL}(x), M_{AL}(x)\rangle \mid x \in X\}$

$\overline{*_1 \bar{A}} = \overline{\{\langle x, N_{AL}(x), M_{AL}(x)\rangle \mid x \in X\}}$

$= \{\langle x, M_{AL}(x), N_{AL}(x)\rangle \mid x \in X\}$

$= *_1 A$

(ii) Apply the operator $*_2$ in $\bar{A}$ we have

$*_2 \bar{A} = *_2 \{\langle x, [N_{AL}(x), N_{AU}(x)], [M_{AL}(x), M_{AU}(x)]\rangle \mid x \in X\}$

$= \{\langle x, N_{AL}(x), M_{AU}(x)\rangle \mid x \in X\}$

$\overline{*_2 \bar{A}} = \overline{\{\langle x, N_{AL}(x), M_{AU}(x)\rangle \mid x \in X\}}$

$= \{\langle x, M_{AU}(x), N_{AL}(x)\rangle \mid x \in X\}$

$= *_3 A$

The other proofs are similar.

Theorem 3.3 *Let X be a non empty set and for every IVIFSST A and B and for* $1 \le i \le 4$, *we have the following*

(i) $*_1 (A + B) = *_1 A + *_1 B,$
(ii) $*_1 (A.B) = *_1 A . *_1 B,$
(iii) $*_1 (A @ B) = *_1 A @ *_1 B.$

Proof:

(i) We prove the result for $i = 1$

Let

$A = \{\langle x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\rangle \mid x \in X\}$

and

$B = \{\langle x, [M_{BL}(x), M_{BU}(x)], [N_{BL}(x), N_{BU}(x)]\rangle \mid x \in X\}$

Then,

$A + B = \{\langle x, [M^2_{AL}(x) + M^2_{BL}(x) M^2_{AL}(x)M^2_{BL}(x),$
$M^2_{AU}(x) + M^2_{BU}(x) - M^2_{AU}(x)M^2_{BU}(x)],$
$[N^2_{AL}(x)N^2_{BL}(x), N^2_{AU}(x)N^2_{BU}(x)]\rangle \mid x \in X\}$

Apply the operator $*_1$ we have

$*_1(A + B)$

$= *_1 \{\langle x, [M^2_{AL}(x) + M^2_{BL}(x) - M^2_{AL}(x)M^2_{BL}(x),$
$M^2_{AU}(x) + M^2_{BU}(x) - M^2_{AU}(x)M^2_{BU}(x)],$
$[N^2_{AL}(x)N^2_{BL}(x), N^2_{AU}(x)N^2_{BU}(x)]\rangle \mid x \in X\}$

$= \{\langle x, M^2_{AL}(x) + M^2_{BL}(x) - M^2_{AL}(x)M^2_{BL}(x), N^2_{AL}(x)N^2_{BL}(x)\rangle \mid x \in X\} \ldots.(1)$

By the definition

$*_1A = \{\langle x, M_{AL}(x), N_{AL}(x)\rangle \mid x\in X\}$

$*_1B = \{\langle x, M_{BL}(x), N_{BL}(x)\rangle \mid x\in X\}$

$*_1 A +*_1 B$

$= \{\langle x, M^2_{AL}(x) + M^2_{BL}(x) - M^2_{AL}(x).M^2_{BL}(x), N^2_{AL}(x)N^2_{BL}(x)\rangle \mid x \in X\} \ldots.(2)$

From equation (1) and (2) we have

$*_1 (A + B) = *_1 A +*_1 B$

Similarly for $i = 2,3$ & 4 we have

$*_i (A + B) = *_i A +*_i B$

(ii) We prove the result for $i = 2$

Let

$A = \{\langle x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\rangle \mid x \in X\}$

$B = \{\langle x, [M_{BL}(x), M_{BU}(x)], [N_{BL}(x), N_{BU}(x)]\rangle \mid x \in X\}$

Then,

$A.B = \{\langle x, [M^2_{AL}(x)M^2_{BL}(x), M^2_{AU}(x)M^2_{BU}(x)] [N^2_{AL}(x) + N^2_{BL}(x) - N^2_{AL}(x)N^2_{BL}(x), N^2_{AU}(x) + N^2_{BU}(x) - N^2_{AU}(x)N^2_{BU}(x)],\rangle \mid x \in X\}$

Apply the operator $*_2$ we have

$*_2(A.B) = *_2 \{\langle x, [M^2_{AL}(x)M^2_{BL}(x), M^2_{AU}(x)M^2_{BU}(x)] [N^2_{AL}(x) + N^2_{BL}(x) - N^2_{AL}(x)N^2_{BL}(x), N^2_{AU}(x) + N^2_{BU}(x) - N^2_{AU}(x)N^2_{BU}(x)],\rangle \mid x \in X\}$

$= \{\langle x, M^2_{AL}(x)M^2_{BL}(x), N^2_{AU}(x) + N^2_{BU}(x) - N^2_{AU}(x)N^2_{BU}(x)],\rangle \mid x \in X\} \ldots(3)$

By the definition

$*_2A = \{\langle x, M_{AL}(x), N_{AU}(x)\rangle \mid x\in X\}$

$*_2B = \{\langle x, M_{BL}(x), N_{BU}(x)\rangle \mid x\in X\}$

$*_1 A \; . *_2 B = \{\langle x, M^2_{AL}(x)M^2_{BL}(x), N^2_{AU}(x) + N^2_{BU}(x) - N^2_{AU}(x)N^2_{BU}(x)],\rangle \mid x \in X\} \ldots(4)$

From equation (3) and (4) we have

$*_2 (A \,.\, B) = *_2 A \,.\, *_2 B$

Similarly for $i = 1, 3$ & 4 we have

$*_i (A \,.\, B) = *_i A \,.\, *_i B$

(iii) We prove the result for $i = 3$

$A = \{\langle x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)]\rangle \mid x \in X\}$

$B = \{\langle x, [M_{BL}(x), M_{BU}(x)], [N_{BL}(x), N_{BU}(x)]\rangle \mid x \in X\}$

Then,

$$A @ B = \left\{\langle x, \left[\frac{M^2_{AL}(x) + M^2_{BL}(x)}{2}, \frac{M^2_{AU}(x) + M^2_{BU}(x)}{2}\right] \left[\frac{N^2_{AL}(x) + N^2_{BL}(x)}{2}, \frac{N^2_{AU}(x) + N^2_{BU}(x)}{2}\right]\rangle \mid x \in X\right\}$$

Apply the operator $*_3$ we have

$$*_3 (A @ B) = *_3 \left\{\langle x, \left[\frac{M^2_{AL}(x) + M^2_{BL}(x)}{2}, \frac{M^2_{AU}(x) + M^2_{BU}(x)}{2}\right] \left[\frac{N^2_{AL}(x) + N^2_{BL}(x)}{2}, \frac{N^2_{AU}(x) + N^2_{BU}(x)}{2}\right]\rangle \mid x \in X\right\}$$

$$= \left\{\langle x, \frac{M^2_{AU}(x) + M^2_{BU}(x)}{2}, \frac{N^2_{AL}(x) + N^2_{BL}(x)}{2}\rangle \mid x \in X\right\} \ldots\ldots(5)$$

By the definition

$*_3A = \{\langle x, M_{AU}(x), N_{AL}(x)\rangle \mid x\in X\}$

$*_3B = \{\langle x, M_{BU}(x), N_{BL}(x)\rangle \mid x\in X\}$

$*_3 A @ *_3 B$

$$= \left\{\langle x, \frac{M^2_{AU}(x) + M^2_{BU}(x)}{2}, \frac{N^2_{AL}(x) + N^2_{BL}(x)}{2}\rangle \mid x \in X\right\} \ldots\ldots(6)$$

From equation (5) and (6) we have

$*_3 (A @ B) = *_3 A @ *_3 B$

Similarly for $i = 1, 2$ & 4 we have

$*_i (A @ B) = *_i A @ *_i B$

Conclusion

We have introduced some operators over Interval Valued Intuitionistic Fuzzy Sets of Second Type and established some of their properties. It is still open to define some more operators on IVIFSST.

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