International Journal of Current Advanced Research
ISSN: O: 2319-6475, ISSN: P: 2319-6505, Impact Factor: SJIF: 6.614
Available Online at www.journalijcar.org
Volume: 7| Issue: 1| Special Issue January: 2018 | Page No. 62-65
DOI: http://dx.doi.org/10.24327/IJCAR

ON (1,2)- TWO OUTDEGREE EQUITABLE DOMINATION NUMBER OF SOME MIDDLE GRAPH FAMILIES

Ameenal Bibi K and Rajakumari P

P.G. and Research Department of Mathematics, D.K.M.College for Women (Autonomous), Vellore-632001, Tamilnadu, India

A B S T R A C T **RESEARCH ARTICLE**

A (1,2)- two outdegree equitable dominating set in a graph G=(V,E) is a set S having the property that for every vertex v in S at distance 1 from v and a second vertex in S at distance almost 2 from v and if for any two vertices $u, v \in D$ such that $|od_D(u) - od_D(v)| \leq 2$. The order of the smallest (1,2)-two outdegree equitable dominating set of G is called (1,2)- two outdegree equitable domination number of G and denoted by $\gamma_{(1,2)2oe}(G)$. This paper is aimed to attain the (1,2)- two outdegree equitable domination number in the middle graphs of G_n, W_n, $F_{1,n}$ and $K_{1,n,n}$.

2010AMS Subject Classification: 03E72

Keywords:

(1,2) domination number, two-out degree equitable domination number, Middle graph, Gear graph, Wheel graph, Fan graph and Double star graph.

AMS Subject Classification: 05C76, 05C7C 05C69.

1. Introduction

By a graph G=(V,E) we mean a finite connected, undirected graph with neither loops nor multiple edges. The order and size of G are denoted by n and m respectively. For any graph theoretic terminology, we refer chartrand and Lesniak [4]. Let G=(V,E) be a graph and let $v \in V$. The open neighborhood of $v \in V$ denoted by $N(v) = \{u \in V: uv \in E\}$. If $S \subseteq V$ then $N(S) = \cup_{v \in S} N(v)$ and $N[S] = N(S) \cup S$.

Domination in graphs has become an important area of research in graph theory as evidenced by the results contained in the two books by Haynes, Hedetniemi and Slater (1998) [6].Dominating queens is the origin of the study of dominating set in graphs. Berge [2] and Ore [9] were the pioneers to define dominating sets. A non-empty subset S of V is called a dominating set if N[S]=V. The minimum (maximum) cardinality of a minimal dominating set of G is called the domination number (upper domination nu0 of G and is denoted by $\gamma(G)(\Gamma(G))$.

A new type of dominating set namely (1,2) dominating set is introduced by Steve Hedetniemi and Sandee Hedetniemi [10]. N.Murugesan, Deepa and S.Deepa and Nair introduced (1,2)-domination in the line graphs of C_n, P_n *and* $K_{1,n}$, (1,2)-domination in the middle and central graphs of $K_{1,n}, C_n$ *and* P_n and also (1,2)-domination in some Harmonious graphs.

Vernold Vivin.J (2010) studied the harmonious coloring of line graph, middle graph and central graph of certain special graphs [12]. Venkatkrishanan and Swaminathan (2010)[11] have studied color class domination number of middle graph and central graph of $K_{1,n}, C_n$ *and* P_n. Ali Sahal and V.Mathad (Sahal 2013) introduced the concept of two outdegree equitable domination number in graphs.

2.Preliminaries

2.1 Wheel graph [12]

The Wheel graph W_n on $n+1$ vertices is defined as $W_n = C_n + K_1$ where C_n is n- cycle. Let $V(W_n) = \{v_i: 1 \leq i \leq n-1\} \cup \{v\}$ and $E(W_n) = \{u_i = v_i v_{i+1}: 1 \leq i \leq n-1, subscripts\ modulo\ n\} \cup \{e_i = vv_i: 1 \leq i \leq n-1\}$.

2.2 Gear graph[8]

The Gear graph G_n is a wheel graph $W_{1,n}$ with a vertex added between each pair of adjacent vertices of the outer cycle.

2.3 Fan graph[8]

A Fan graph $W_{m,n}$ is defined as the join of two graphs, $\overline{Km} + Pn$ where $\overline{Km}$ is the empty graph on m vertices and P_n is the path graph on n vertices.

2.4 Double star graph

A Double star graph is a graph formed by starting with 2 vertices and joining them together $V(K_{1,n,n}) = \{v\} \cup \{v_i: 1 \leq i \leq n\} \cup \{u_i: 1 \leq i \leq n\}$.

2.5 Middle graph[12]

Let G be a graph with vertex set V(G) and edge set E(G). The middle graph of G, denoted by M(G) is defined as follows. The vertex set of M(G) is $V(G) \cup E(G)$. Two vertices x, y of M(G) are adjacent in M(G) in case one of the following holds

(i)x, y are in E(G) and x, y are adjacent in G.
$(ii) x$ is in V(G), y is in E(G) and x, y are incident in G.

3. Main Results

In this section, we attained the (1,2)-two outdegree equitable domination numbers of the middle graphs of Gear, Wheel, Fan and Double star graphs.

Definition 3.1

A (1,2) – two outdegree equitable dominating set having property that for every vertex v in $V - S$ there is atleast one vertex in S at distance 1 from v and a second vertex in S at distance almost 2 from v and if for any two vertices $u, v \in D$ such that $|od_D(u) - od_D(v)| \leq 2$, where $od_D(u) = |N(v) \cap (V - D)|$. The order of the smallest (1,2)- two outderee equitable dominating set of G is called (1,2)- two outdegree equitable domination number of G and denoted by $\gamma_{(1,2)2oe}(G)$.

Example 3.2

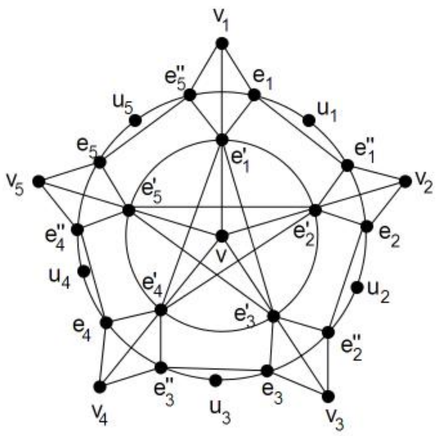

Fig 1 ($M(G_5)$)

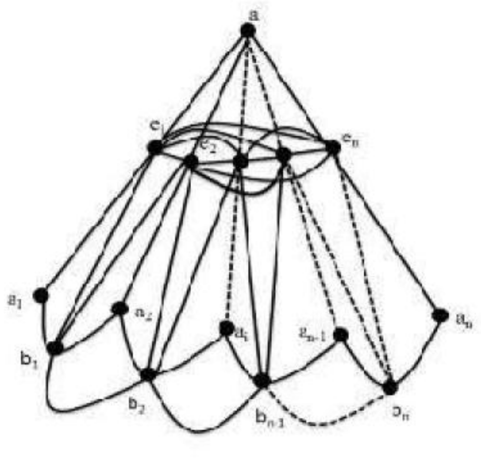

Fig 2 ($M(F_{1,n})$)

Theorem 3.3

For the Middle graph of Gear graph G_n, the (1,2)- two out degree equitable domination number is : $\gamma_{(1,2)2oe}(M(G_n)) = 2n$.

Proof

By the definition of middle graph of middle graph $V(MG_n) = V(G_n) \cup E(G_n)$, $|V(M(G_n))| = 5n + 1$ in which the set $\{e_i': 1 \leq i \leq n\} \cup \{v\}$ induces a clique K_{n+1} of order $n + 1$ and for each $i\,(1 \leq i \leq n)$ the set of vertices $\{e_i'', e_{i+1}', e_{i+1}, v_{i+1}: subscript\ modulo\ n\}$ induces a clique.

In $M(G_n)$,the vertex e_i' is adjacent to $v \cup \{v_i\}$ $(1 \leq i \leq n)$ and each u_i is adjacent to $\{e_i \cup e_i'': 1 \leq i \leq n\}$. So $\{e_i' \cup u_i: 1 \leq i \leq n\}$ will form a dominating set.

Let $D = \{e_i' \cup u_i: 1 \leq i \leq n\}$ be an dominating set of $M(G_n)$ and $V - D = \{v \cup \{e_i \cup e_i'' \cup v_i\}\ \ 1 \leq i \leq n\}$. For every vertex v in $V - D$ there is atlest one vertex in D at distance1 from v and a second vertex in D at distance atmost 2 from v. So, $\{e_i' \cup u_i: 1 \leq i \leq n\}$ will form a (1,2) dominating set.

Now, $e_i' \in D$ then $od_D(e_i') = |N(e_i') \cap V - D|\ for\ i = 1$
$= |\{(e_{i+1}', e_{i+2}', \ldots e_n') \cup (v \cup \{e_i, v_i, e_n''\})\}\cap\{v \cup (e_i \cup e_i'' \cup v_i) : 1 \leq i \leq n\}|$
$= |(v \cup (e_i, v_i, e_n'') = 4$
$for\ i = 2, e_2' \in D$ then $od_D(e_2') = |N(e_2') \cap V - D|$
$= |\{(e_{i-1}', e_{i+1}', \ldots e_n') \cup (v \cup \{e_i, v_i, e_{i-1}''\})\}\cap\{v \cup (e_i \cup e_i'' \cup v_i) : 1 \leq i \leq n\}|$
$= |(v \cup (e_i, v_i, e_{i-1}'')$
$= 4$
$for\ i = n, e_n' \in D$ then $od_D(e_n') = |N(e_n') \cap V - D|$
$= |\{(e_1', e_2', \ldots e_{i-1}') \cup (v \cup \{e_i, v_i, e_{i-1}''\})\}\cap\{v \cup (e_i \cup e_i'' \cup v_i) : 1 \leq i \leq n\}|$
$= |(v \cup (e_i, v_i, e_{i-1}'')$
$= 4$
Similarly, if $u_i \in D$ then
$od_D(u_i) = |N(u_i) \cap V - D|$
$= |(e_i, e_i'') \cap (v \cup (e_i \cup e_i'' \cup v_i))|$
$= |(e_i, e_i'')|$
$= 2$
Then $|od_D(e_i) - od_D(u_i)| \leq 2$, for any $e_i, u_i \in D$. Therefore D is the minimum (1,2)- two outdegree equitable dominating set. Hence, $\gamma_{(1,2)2oe}(M(G_n)) = 2n$.

Observation 3.4

(i) For any $i(M(G_n)) = n$
(ii) For any $\Gamma(M(G_n)) = 2n + 1$

Theorem 3.5

For the Middle graph of Wheel graph W_n, the (1,2)- two out degree equitable domination number is : $\gamma_{(1,2)2oe}(M(W_n)) = n$.

Proof

Let $V(W_n) = \{v, v_1, v_2, \ldots v_{n-1}\}$ and $V(M(W_n)) = \{v, v_1, v_2, \ldots v_{n-1}\} \cup \{e_1, e_2, \ldots e_{n-1}\} \cup \{u_1, u_2, \ldots u_{n-1}\}$, where u_i is the vertex of $M(W_n)$ corresponding to the edge $v_i v_{i+1}$ of $W_n\ (1 \leq i \leq n - 1)$.

By the definition of middle graph, the vertices v and $e_i: 1 \leq i \leq n - 1$ induce a clique of order n in $M(W_n)$. In $M(W_n)$ the vertex e_i is adjacent to $v \cup \{u_i \cup v_i: 1 \leq i \leq n\}$

Let $D = \{e_i: 1 \leq i \leq n\}$ be an dominating set of $M(W_n)$ and $V - D = \{v \cup \{u_i \cup v_i: 1 \leq i \leq n\}\}$.

For every vertex v in $V - D$ there is atlest one vertex in D at distance 1 from v and a second vertex in D at distance atmost 2 from v. So, $\{e_i: 1 \le i \le n\}$ will form a (1,2) dominating set.

Now, $e_i \in D$ then $od_D(e_i) = |N(e_i) \cap V - D|$ $for\ i = 1$
$= |\{v \cup e_{i+1}, e_{i+2}, \dots e_n \cup u_n, v_i, u_i\} \cap \{v \cup u_i \cup v_i: 1 \le i \le n\}|$
$= |(u_n, v_i, u_i, v|$
$= 4$
$od_D(e_i) = |N(e_i) \cap V - D|$ $for\ i = 2$
$= |\{v \cup e_{i-1}, e_{i+1}, \dots e_n \cup u_{i-1}, v_i, u_i\} \cap \{v \cup u_i \cup v_i: 1 \le i \le n\}|$
$= |(u_{i-1}, v_i, u_i, v|$
$= 4$
$od_D(e_n) = |N(e_n) \cap V - D|$ $for\ i = n$
$= |\{v \cup e_1, e_2, \dots e_n \cup u_{i-1}, v_i, u_i\} \cap \{v \cup u_i \cup v_i: 1 \le i \le n\}|$
$= |(u_{i-1}, v_i, u_i, v|$
$= 4$

Then $|od_D\ e_i - od_D\ e_i| \le 2$, for any $e_i \in D$. Therefore, D is the minimum (1,2)- two outdegree equitable dominating set. Hence, $\gamma_{(1,2)2oe}\ M\ W_n = n$

Observation 3.6

(i) For any Middle graph of W_n, $i\ M\ W_n = n + 1$
(ii) For any Middle graph of W_n,Γ $M\ W_n = n + 1$

Theorem 3.7

For the Middle graph of Fan graph $F_{1,n}$, the (1,2)- two out degree equitable domination number is : $\gamma_{1,2\ 2oe}\ M\ F_{1,n} = n \quad if\ n = 3, n = 4. n = 5, n = 6$
$does\ not\ exist \quad if\ n > 6$

Proof

Consider the Fan graph $F_{1,n}$ denote the vertices $V\ F_{1,n} = a \cup a_1, a_2, a_3, .. a_{n-1}, a_n \cup b_1, b_2, b_3, \dots b_{n-1}, b_n$ and $E\ F_{1,n} = e_i: 1 \le i \le n$, where e_i is the edge aa_i $1 \le i \le n$. By the definition of middle graph $V(M\ F_{1,n} = V(F_{1,n}) \cup E(F_{1,n}) = a_i: 1 \le i \le n \cup b_i: 1 \le i \le n \cup e_i: 1 \le i \le n$
In $M\ F_{1,n}$, each vertex a is adjacent in $\{e_i: 1 \le i \le n\}$ and each b_{i-1} is adjacent to $a_i: 1 \le i \le n$.

Let $D = \{a \cup b_{i-1}: 2 \le i \le n\}$ be the dominating set of $M\ F_{1,n}$ and $V - D = \{a_i \cup e_i: 1 \le i \le n\}$. For every vertex v in $V - D$ there is atlest one vertex in D at distance 1 from v and a second vertex in D at distance atmost 2 from v. So $\{a \cup b_{i-1}\}$ will form a (1,2) dominating set.

Now, $a \in D$ then
$od_D\ a = |N\ a \cap V - D|$
$= |e_i: 1 \le i \le n \cap \{a_i \cup e_i: 1 \le i \le n\}|$
$= |e_i|$
$= n$
If $b_{i-1} \in D$ then
$od_D(b_{i-1}) = |N(b_{i-1}) \cap V - D|$
$for\ i = 1, b_1 \in D$ then
$od_D(b_1) = |N(b_1) \cap V - D| = |(a_{i-1}, a_i, e_{i-1}, e_i, b_i) \cap (a_i \cup e_i: 1 \le i \le n)|$
$= |(a_{i-1}, a_i, e_{i-1}, e_i)|$
$= 4$
$for\ i = 2, b_2 \in D$ then $od_D(b_2) = |N(b_2) \cap V - D| = |(a_{i-1}, a_i, e_{i-1}, e_i, b_{i-2}, b_i) \cap (a_i \cup e_i: 1 \le i \le n)| = |(a_{i-1}, a_i, e_{i-1}, e_i)|$
$= 4$
$for\ i = n, b_{n-1} \in D$ then $od_D(b_{n-1}) = |N(b_{n-1}) \cap V - D|$
$= |(a_{i-1}, a_i, e_{i-1}, b_{i-2}) \cap (a_i \cup e_i: 1 \le i \le n)| = |(a_{i-1}, a_i, e_{i-1}, e_i)|$
$= 4$

Then $|od_D(a) - od_D\ b_{i-1}| \le 2$, for any $b_i \in D$. Therefore, D is the minimum (1,2)- two outdegree equitable dominating set. Hence, $\gamma_{(1,2)2oe}\ M\ F_{1,n} = n\ if\ n = 3,4,5,6$
Suppose $if\ n > 6$, if for any two vertices $u, v \in D$ such that the $|od_D(u) - od_D(v)| \ne 2$ where
$od_D\ u = |N\ v \cap V - D$. This shows that $M\ F_{1,n}$ is a graph that does not satisfy the 1,2)- two outdegree equitable domination condition.

Observation 3.8

(i) For any Middle graph of $F_{1,n}$, $i\ M\ F_{1,n} = n + 1$

(ii) For any Middle graph$F_{1,n}$,Γ $M(F_{1,n} = n$

Theorem 3.9

For the Middle graph of Double star graph $K_{1,n,,n}$, the (1,2)- two out degree equitable domination number is :
$\gamma_{1,2\ 2oe}\ M(K_{1,n,,n}) = 2n$

Proof

Let $V(K_{1,n,n} = v \cup v_i: 1 \le i \le n \cup u_i: 1 \le i \le n$. By the definition of middle graph, each edge vv_i and vv_i $(1 \le i \le n)$ in $K_{1,n,,n}$ are subdivided by the vertices u_i and s_i in $M(F_{1,n})$.(i.e) $V\ M\ K_{1,n,,n} = v \cup v_i: 1 \le i \le n \cup u_i: 1 \le i \le n \cup e_i: 1 \le i \le n \cup s_i: 1 \le i \le n$ the vertices $v, e_1, e_2, \dots e_n$ induce a clique of order $n + 1$ (say K_{n+11}) in $M\ K_{1,n,,n}$.
In $M(K_{1,n,,n})$ the vertices e_i is adjacent to $v \cup v_i: 1 \le i \le n$ and u_i is adjacent to $s_i: 1 \le i \le n$.
Let $D = \{e_i \cup u_i: 1 \le i \le n\}$ be the dominating set of $M\ K_{1,n,,n}$ and $V - D = v \cup v_i \cup s_i : 1 \le i \le n$.
Now, $e_i \in D$ then $od_D(e_i) = |N(e_i) \cap V - D|$ $for\ i = 1$ $= |\{v \cup e_{i+1}, e_{i+2}, \dots e_n \cup v_i, s_i\} \cap \{v \cup v_i \cup s_i : 1 \le i \le n\}|$
$= |\{v \cup v_i \cup s_i\}|$
$= 3$
$for\ i = 2$, $e_2 \in D$ then $od_D(e_2) = |N(e_2) \cap V - D|$ $= |\{v \cup e_{i-1}, e_{i+1}, \dots e_n \cup v_i, s_i\} \cap \{v \cup v_i \cup s_i : 1 \le i \le n\}| = |\{v \cup v_i \cup s_i\}| = 3$
$for\ i = n$, $e_n \in D$ then $od_D(e_n) = |N(e_n) \cap V - D|$
$= v$ □ $e_1, e_2, \dots e_{i-1}$ □ $v_i, s_i \cap v$ □ v_i □ $s_i : 1 \le i \le n$
$= |\{v$ □ v_i □ $s_i\}|$
$= 3$
Similarly, if u_i □ D then $od_D(u_i) = |N(u_i) \cap V - D|$
$= |\{s_i: 1 \le i \le n\} \cap \{v$ □ v_i □ $s_i : 1 \le i \le n\}| = |s_i|$
$= 4$
Then $|od_D(e_i) - od_D\ u_i| \le 2$, for any e_i, u_i □ D. Therefore, D is the minimum (1,2)- two outdegree equitable dominating set. Hence, $\gamma_{1,2\ 2oe}\ M(K_{1,n,,n}) = 2n$

Observation 3.10

(i) For any Middle graph of $K_{1,n,n}$, $i\ M(K_{1,n,n}) = n + 1$
(ii) For any Middle graph $K_{1,n,n}$,ɼ $M(K_{1,n,n}) = n + 1$

Conclusion

In this paper, we attained the exact values of the (1,2)- two outdegree equitable domination number for the middle graphs of $G_n, W_n, F_{1,n}$ and $K_{1,n,n}$. We further extended this study on middle graphs of some more special classes of graphs.

References

[1] A.Sahal and V.Mathad “Two-out degree equitable domination in graphs” Transaction on combinatorics vol 2.No. 3 (2013)

[2] Berge C. Theory of graph and its Applications, Methuen London 1962.

[3] Cockayne, E.J. and Hedetniemi S.T. (1977). Towardsa theory of domination in graphs. Networks, 7. pp.241-267

[4] G. Chartrand and L. Lesniak, Graphs and Digraphs, Chapman and Hall. CRC, 4th edition, 2005.

[5] Frank Harary, Graph Theory, Narosa Publishing home,(1969).

[6] Haynes T.W., Hedetniemi S.T. and Slater P.J., Fundamentals of domination in Graphs, Marcel Dekker, New York, 1998.

[7] Murugesan N and Deepa S. Nair,(1,2)-domination in Middle and Central graphs of Cn,Pn,K1,n. Applied Mathematics, 2013,3(4) 144-151.

[8] Manjunath.G, Murali R, B. Shanmukha, “Hamiltonian laceability in middle graphs”. International *Journal of Engineering Sciences Research Technology* (2015).

[9] Ore O, Theory of graphs, Amerarica, Math. Soc, Collog, Publications 38, Providence, 1962.

[10] Steve Hedetniemi, Sandee Hedetniemi, (1,2) - Domination in Graphs.

[11] Venketakrishnan Y.B and Swaminathan .V,”Color class domination number of middle graph and central graph of $K_{1,n}$. C_n and P_n", AMO Vol.12, No.2, 2010

[12] Vernold Vivin.J, Venkatachalam. M, ”On b-chromatic number of sunletgraph and wheel graph families”.

******