International Journal of Current Advanced Research
ISSN: O: 2319-6475, ISSN: P: 2319-6505, Impact Factor: SJIF: 6.614
Available Online at www.journalijcar.org
Volume: 7| Issue: 1| Special Issue January: 2018 | Page No.39-47
DOI: http://dx.doi.org/10.24327/IJCAR

GENERALISED ULAM-HYERS STABILITY OF A $n-$ DIMENSIONAL ADDITIVE-QUADRATIC FUNCTIONAL EQUATION IN ANTI-INTUITIONISTIC FUZZY NORMED SPACES

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A B S T R A C T **RESEARCH ARTICLE**

In this paper, the authors introduced and investigate the generalized Ulam $-$ Hyers stability of a $n-$ dimensional additive-quadratic functional equation

$$\sum_{k=0}^{n}\left[\sum_{l=1}^{2}h\left(\frac{d_{2k}x_{2k}+(-1)^{l}d_{2k+1}x_{2k+1}}{2}\right)\right]$$

$$=\sum_{k=0}^{n}\left[\sum_{l=1}^{2}\left[\left(\frac{d_{2k}}{2}\right)^{l}\left[h(x_{2k})+(-1)^{l}h(-x_{2k})\right]\right]+\left(\frac{d_{2k+1}}{2}\right)^{2}\left[h(x_{2k+1})+h(-x_{2k+1})\right]\right]$$

where d_i is positive integer with $d_i \neq 0$ in anti-intuitionistic fuzzy normed spaces using Hyers method.

1. Introduction

Stability problem of a functional equation was first posed by S.M. Ulam [45] which was answered by D.H. Hyers [23] and then generalized by T. Aoki [2], Th.M. Rassias [37], J.M. Rassias [34] for additive mappings and linear mappings, respectively. Further generalizations on the above stability results was given in [15, 20, 21, 39]. Since then several stability problems for various functional equations have been investigated in [1, 3, 4, 6, 7, 8, 9, 10, 11, 16, 24, 32, 35, 38, 46]; various fuzzy stability results concerning Cauchy, Jensen, quadratic and cubic functional equations were discussed in [18, 19, 27, 28, 29, 30, 42, 43, 44].

The solution and stability of following Mixed type additive quadratic functional equations

$$f(x+2y+3z)+f(x-2y+3z)+f(x+2y-3z)+f(x-2y-3z)$$
$$=4f(x)+8\left[f(y)+f(-y)\right]+8\left[f(z)+f(-z)\right] \quad (1)$$

$$f(3x+2y+z)+f(3x-2y+z)+f(3x+2y-z)+f(3x-2y-z)$$
$$=12f(x)+12\left[f(x)+f(-x)\right]+8\left[f(y)+f(-y)\right]+2\left[f(z)+f(-z)\right] \quad (2)$$

$$f\left(l^{-1}u+m^{-1}v+n^{-1}w\right)+f\left(l^{-1}u-m^{-1}v+n^{-1}w\right)+f\left(l^{-1}u+m^{-1}v-n^{-1}w\right)+f\left(l^{-1}u-m^{-1}v-n^{-1}w\right)$$
$$=4f\left(l^{-1}u\right)+2\left[f\left(m^{-1}v\right)+f\left(-m^{-1}v\right)\right]+2\left[f\left(n^{-1}w\right)+f\left(-n^{-1}w\right)\right] \quad (3)$$

$$f\left(\sum_{i=1}^{n}r_ix_i\right)=\sum_{i=1}^{n}\left(\sum_{j=1}^{2}\frac{r_i^j}{2}\left[f(x_i)+(-1)^j f(x_i)\right]\right)+\sum_{1\le i<j\le n}\frac{r_ir_j}{4}\left(\sum_{p=0}^{1}\left(\sum_{q=0}^{1}(-1)^{p+q}f\left[(-1)^p x_i+(-1)^q x_j\right]\right)\right) \quad (4)$$

were investigated by M. Arunkumar, P. Agilan [6, 7, 8, 9, 10, 11, 12, 13, 36].

In this paper, the authors introduced and investigate the generalized Ulam $-$ Hyers stability of a $n-$ dimensional additive-quadratic functional equation

$$\sum_{k=0}^{n}\left[\sum_{l=1}^{2}h\left(\frac{d_{2k}x_{2k}+(-1)^{l}d_{2k+1}x_{2k+1}}{2}\right)\right]$$
$$=\sum_{k=0}^{n}\left[\sum_{l=1}^{2}\left[\left(\frac{d_{2k}}{2}\right)^{l}\left[h(x_{2k})+(-1)^{l}h(-x_{2k})\right]\right.\right.$$
$$\left.\left.+\left(\frac{d_{2k+1}}{2}\right)^{2}\left[h(x_{2k+1})+h(-x_{2k+1})\right]\right]\right] \quad (5)$$

where d_i is positive integer with $d_i \neq 0$ in Anti-intuitionistic fuzzy normed spaces using Hyers method.

In Section 2, basic definition and preliminaries of Anti-intuitionistic fuzzy normed space is present, In Section 3, the generalized Ulam - Hyers stability of the functional equation (5) is proved via Hyers method.

2. Preliminaries of Anti-Intuitionistic Fuzzy Normed Spaces

In this section, some preliminaries about Anti-intuitionistic fuzzy normed space. For definations and notations about intuitionistic fuzzy normed space one can refer [17, 14, 42].

Definition 2.1 [42] Let « and ¬ be membership and nonmembership degree of an anti-intuitionistic fuzzy set from $X\times(0,+\infty)$ to $[0,1]$ such that $«_x(t)+¬_x(t)\leq 1$ for all $x\in X$ and all $t>0$. The triple $(X,P_{«,¬},T)$ is said to be an Anti-intuitionistic fuzzy normed space (briefly AIFN-space) if X is a vector space, T is a continuous $t-$representable and $P_{«,¬}$ is a mapping $X\times(0,+\infty)\rightarrow L^{*}$ satisfying the following conditions: for all $x,y\in X$ and $t,s>0$,

$(IFN1)$ $P_{«,¬}(x,0)=0_{L^*}$;

$(IFN2)$ $P_{«,¬}(x,t)=1_{L^*}$ if and only if $x=0$;

$(IFN3)$ $P_{«,¬}(\sim x,t)=P_{«,¬}\left(x,\frac{t}{|\sim|}\right)$ for all $\sim\neq 0$;

$(IFN4)$ $P_{«,¬}(x+y,t+s) \leq_{L^*} T\left(P_{«,¬}(x,t),P_{«,¬}(y,s)\right).$

In this case, $P_{«,¬}$ is called an Anti-intuitionistic fuzzy norm. Here, $P_{«,¬}(x,t)=\left(«_x(t),¬_x(t)\right)$.

3. Stability Results: Direct Method

In this section, the authors present the generalized Ulam-Hyers stability of the Additive-quadratic functional equation (5) in Anti-intuitionistic fuzzy normed spaces. Now we use the following notation for a given mapping $Df: X\rightarrow Y$ such that

$$Df(x_0,x_1,\cdots x_{2n},x_{2n+1})$$
$$\sum_{k=0}^{n}\left[\sum_{l=1}^{2}h\left(\frac{d_{2k}x_{2k}+(-1)^{l}d_{2k+1}x_{2k+1}}{2}\right)\right]$$
$$-\sum_{k=0}^{n}\left[\sum_{l=1}^{2}\left[\left(\frac{d_{2k}}{2}\right)^{l}\left[h(x_{2k})+(-1)^{l}h(-x_{2k})\right]\right.\right.$$
$$\left.\left.+\left(\frac{d_{2k+1}}{2}\right)^{2}\left[h(x_{2k+1})+h(-x_{2k+1})\right]\right]\right]$$

for all $x_0,x_1,\cdots x_{2n},x_{2n+1}\in X$.

Theorem 3.1 Let ¢ $\in\{1,-1\}$. Let $\Lambda: X^{n}\rightarrow Z$ be a function such that for some $0<\left(\frac{a}{T}\right)^{¢}<1$,

$$P'_{«,¬}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1 times},T^{¢b}x,0\right),r\right) \leq_{L^*} P'_{«,¬}\left(a^{¢b}\ \Lambda\left(\underbrace{0,\cdots,0}_{2n-1 times},x,0\right),r\right) \quad (1)$$

for all $x\in X$ and all $r>0$ and

$$\lim_{b\rightarrow\infty}P'_{«,¬}\left(\Lambda\begin{pmatrix}T^{¢b}x_0,T^{¢b}x_1,T^{¢b}x_2,\\ \cdots,T^{¢b}x_{2n},T^{¢b}x_{2n+1}\end{pmatrix},T^{¢b}r\right)=1_{L^*} \quad (2)$$

for all $x_0,x_1,\cdots x_{2n},x_{2n+1}\in X$ and all $r>0$. Let $f_o: X\rightarrow Y$ be an odd function satisfies the inequality

$$P_{«,¬}\left(Dh_o(x_0,x_1,\cdots x_{2n},x_{2n+1}),r\right) \leq_{L^*} P'_{«,¬}\left(\Lambda(x_0,x_1,\cdots x_{2n},x_{2n+1}),r\right) \quad (3)$$

for all $x_0,x_1,\cdots x_{2n},x_{2n+1}\in X$ and all $r>0$. Then the limit

$$P_{«,¬}\left(A(x)-\frac{h_o(T^{b}x)}{T^{b}},r\right)\rightarrow 1_{L}^{*}, as\quad b\rightarrow\infty,\quad r>0 \quad (4)$$

exists for all $x\in X$ and the mapping $A: X\rightarrow Y$ is a unique Additive mapping satisfying (5) and

$$P_{«,¬}\left(h_o(x)-A(x),r\right) \leq_{L^*} P'_{«,¬}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1 times},x,0\right),|2T-2a|r\right) \quad (5)$$ for all $x\in X$ and all $r>0$.

Proof. Let ¢ $=1$. Since f_o is an odd function, replacing $(x_0,x_1,\cdots x_{2n},x_{2n+1})$ by $\left(\underbrace{0,\cdots,0}_{2n-1 times},x,0\right)$ in (3), we get

$$P_{«,¬}\left(2h_o\left(\frac{d_{2n}x}{2}\right)-d_{2n}h_o(x),r\right) \leq_{L^*} P'_{«,¬}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1 times},x,0\right),r\right) \quad (6)$$

where $T = \frac{d_{2n}}{2}$ for all $x \in X$ and all $r > 0$. Using $(IFN3)$ in (6), we obtain

$$P_{\text{«},\neg}\left(\frac{h_o(Tx)}{T} - h_o(x), \frac{r}{2T}\right) \leq_{L^*} P'_{\text{«},\neg}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times}, x, 0\right), r\right) \tag{7}$$

for all $x \in X$ and all $r > 0$. Replacing x by $T^b x$ in (7), we have

$$P_{\text{«},\neg}\left(\frac{h_o(T^{b+1}x)}{T} - h_o(T^b x), \frac{r}{2T}\right) \leq_{L^*} P'_{\text{«},\neg}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times}, T^b x, 0\right), r\right) \tag{8}$$

for all $x \in X$ and all $r > 0$. Using (1), $(IFN3)$ in (8), we arrive

$$P_{\text{«},\neg}\left(\frac{h_o(T^{b+1}x)}{T} - h_o(T^b x), \frac{r}{2T}\right) \leq_{L^*} P'_{\text{«},\neg}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times}, x, 0\right), \frac{r}{a^b}\right) \tag{9}$$

for all $x \in X$ and all $r > 0$. It is easy to verify from (9), that

$$P_{\text{«},\neg}\left(\frac{h_o(T^{b+1}v)}{T^{b+1}} - \frac{h_o(T^b x)}{T^b}, \frac{r}{2T \cdot T^b}\right) \leq_{L^*} P'_{\text{«},\neg}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times}, x, 0\right), \frac{r}{a^b}\right) \tag{10}$$

holds for all $x \in X$ and all $r > 0$. Replacing r by $a^b r$ in (10), we get

$$P_{\text{«},\neg}\left(\frac{h_o(T^{b+1}x)}{T^{b+1}} - \frac{h_o(T^b x)}{T^b}, \frac{a^b\, r}{2T \cdot T^b}\right) \leq_{L^*} P'_{\text{«},\neg}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times}, x, 0\right), r\right) \tag{11}$$

for all $x \in X$ and all $r > 0$. It is easy to see that

$$\frac{h_o(T^b x)}{T^b} - h_o(x) = \sum_{i=0}^{b-1} \frac{h_o(T^{i+1}x)}{T^{i+1}} - \frac{h_o(T^i x)}{T^i} \tag{12}$$

for all $x \in X$. From equations (11) and (12), we have

$$\begin{aligned} P_{\text{«},\neg}\left(\frac{h_o(T^b x)}{T^b} - h_o(x), \sum_{i=0}^{b-1}\frac{a^i\, r}{2T \cdot T^b}\right) &\leq_{L^*} T_{i=0}^{b-1}\left(P'_{\text{«},\neg}\left(\sum_{i=0}^{b-1}\frac{h_o(T^{i+1}x)}{T^{i+1}} - \frac{h_o(T^i x)}{T^i}, \sum_{i=0}^{b-1}\frac{a^i\, r}{2T \cdot T^b}\right)\right) \\ &\leq_{L^*} T_{i=0}^{b-1}\left\{P'_{\text{«},\neg}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times}, x, 0\right), r\right)\right\} \\ &\leq_{L^*} P'_{\text{«},\neg}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times}, x, 0\right), r\right) \end{aligned} \tag{13}$$

for all $x \in X$ and all $r > 0$. Replacing x by $T^c x$ in (??) and using (1), $(IFN3)$, we obtain

$$P_{\text{«},\neg}\left(\frac{h_o(T^{b+c}x)}{T^{b+c}} - \frac{h_o(T^c x)}{T^c}, \sum_{i=0}^{b-1}\frac{a^i\, r}{2T \cdot T^{i+c}}\right) \leq_{L^*} P'_{\text{«},\neg}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times}, x, 0\right), \frac{r}{a^c}\right) \tag{14}$$

for all $x \in X$ and all $r > 0$ and all $b, c \geq 0$. Replacing r by $a^c r$ in (14), we get

$$P_{\text{«},\neg}\left(\frac{h_o(T^{b+c}x)}{T^{b+c}} - \frac{h_o(T^c x)}{T^c}, \sum_{i=0}^{b+c-1}\frac{a^i\, r}{2T \cdot T^i}\right) \leq_{L^*} P'_{\text{«},\neg}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times}, x, 0\right), r\right) \tag{15}$$

for all $x \in X$ and all $r > 0$ and all $b, c \geq 0$. It follows from (15), that

$$P_{\text{«},\neg}\left(\frac{h_o(T^{b+c}x)}{T^{b+c}} - \frac{h_o(T^c x)}{T^c}, r\right) \leq_{L^*} P'_{\text{«},\neg}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times}, x, 0\right), \frac{T}{\sum_{i=0}^{b+c-1}\frac{a^i}{2T \cdot T^i}}\right) \tag{16}$$

holds for all $x \in X$ and all $r > 0$ and all $b, c \geq 0$. Since $0 < a < T$ and $\sum_{i=0}^{b}\left(\frac{a}{T}\right)^i < \infty$. Thus $\left\{\frac{h_o(T^b x)}{T^b}\right\}$ is a Cauchy sequence in $\left(Y, P_{\text{«},\neg}, T\right)$. Since $\left(Y, P_{\text{«},\neg}, T\right)$ is a complete IFN-space this sequence convergent to some point $A(v) \in Y$. So, one can define the mapping $A : X \to Y$ by

$$P_{\text{«},\neg}\left(A(x) - \frac{h_o(T^b x)}{T^b}, r\right) \to 1_L^*, \quad as \quad b \to \infty, \quad r > 0 \tag{17}$$

for all $x \in X$. Letting $c = 0$ in (16), we get

$$P_{\text{«},\neg}\left(\frac{h_o(T^b x)}{T^b} - h_o(x), r\right) \leq_{L^*} P'_{\text{«},\neg}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times}, x, 0\right), \frac{T}{\sum_{i=0}^{b-1}\frac{a^i}{2T \cdot T^i}}\right) \tag{18}$$

for all $x \in X$ and all $r > 0$. Now for every £ > 0 and from (18), we have

$$\begin{aligned} P_{\text{«},\neg}\left(A(x) - h_o(x), r + £\right) &\leq_{L^*} T\begin{pmatrix} P'_{\text{«},\neg}\left(A(x) - \frac{h_o\left(T^b x\right)}{T^b}, £\right), \\ P'_{\text{«},\neg}\left(h_o(x) - \frac{h_o\left(T^b x\right)}{T^b}, r\right) \end{pmatrix} \\ &\leq_{L^*} T\begin{pmatrix} P'_{\text{«},\neg}\left(A(x) - \frac{h_o(Tx)}{T^b}, £\right), \\ P'_{\text{«},\neg}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times}, x, 0\right), \frac{r}{\sum_{i=0}^{b-1}\frac{a^i}{2T \cdot T^i}}\right) \end{pmatrix} \end{aligned} \tag{19}$$

for all $x \in X$ and all $r > 0$. Taking the limit as $b \to \infty$ in (19), we get

$$P_{«,¬}\left(A(x)-h_o(x),r+£\right)$$
$$\leq_{L^*} T\left(1_{L^*},P'_{«,¬}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times},x,0\right),(2T-2a)r\right)\right) \quad (20)$$
$$\leq_{L^*} P'_{«,¬}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times},x,0\right),(2T-2a)r\right)$$

for all $x\in X$ and all $r>0$ and $£>0$. Since $£$ is arbitrary, by taking $£\to 0$ in (20), we obtain

$$P_{«,¬}\left(A(x)-h_o(x),r\right)$$
$$\leq_{L^*} P'_{«,¬}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times},x,0\right),(2T-2a)r\right) \quad (21)$$

for all $x\in X$ and all $r>0$. To prove A satisfies (5), replacing $(x_0,x_1,\cdots x_{2n},x_{2n+1})$ by $(T^bx_0,T^bx_1,\cdots T^bx_{2n},T^bx_{2n+1})$ in(3) respectively, we obtain

$$P_{«,¬}\left(\frac{1}{T}Dh_o(T^bx_0,T^bx_1,\cdots T^bx_{2n},T^bx_{2n+1}),r\right)$$
$$\leq_{L^*} P'_{«,¬}\left(\pm(T^bx_0,T^bx_1,\cdots T^bx_{2n},T^bx_{2n+1}),T^br\right) \quad (22)$$

for all $x_0,x_1,\cdots x_{2n},x_{2n+1}\in X$ and all $r>0$. Now,

$$P_{«,¬}\left(\sum_{k=0}^{n}\left[\sum_{l=1}^{2}h\left(\frac{d_{2k}x_{2k}+(-1)^l d_{2k+1}x_{2k+1}}{2}\right)\right]\right.$$
$$\left.=\sum_{k=0}^{n}\left[\begin{array}{l}\sum_{l=1}^{2}\left[\left(\frac{d_{2k}}{2}\right)^l\left[h(x_{2k})+(-1)^l h(-x_{2k})\right]\right]\\+\left(\frac{d_{2k+1}}{2}\right)^2\left[h(x_{2k+1})+h(-x_{2k+1})\right]\end{array}\right],r\right)$$
$$\leq_{L^*}\left\{P'_{«,¬}\left(\sum_{k=0}^{n}\left[\sum_{l=1}^{2}h\left(\frac{d_{2k}x_{2k}+(-1)^l d_{2k+1}x_{2k+1}}{2}\right)\right]\right.\right.$$
$$\left.-\frac{1}{T^b}\sum_{k=0}^{n}\left[T^b\sum_{l=1}^{2}h\left(\frac{d_{2k}x_{2k}+(-1)^l d_{2k+1}x_{2k+1}}{2}\right)\right],\frac{r}{4}\right),$$
$$P'_{«,¬}\left(\sum_{k=0}^{n}\left[\sum_{l=1}^{2}\left[\left(\frac{d_{2k}}{2}\right)^l\left[h(x_{2k})+(-1)^l h(-x_{2k})\right]\right]\right]\right.$$
$$\left.-\frac{1}{T^b}\sum_{k=0}^{n}\left[T^b\sum_{l=1}^{2}\left[\left(\frac{d_{2k}}{2}\right)^l\left[h(x_{2k})+(-1)^l h(-x_{2k})\right]\right]\right],\frac{r}{4}\right),$$
$$P'_{«,¬}\left(\sum_{k=0}^{n}\left[\left(\frac{d_{2k+1}}{2}\right)^2\left[h(x_{2k+1})+h(-x_{2k+1})\right]\right]\right.$$
$$\left.+\frac{1}{T^b}\sum_{k=0}^{n}\left[T^b\left(\frac{d_{2k+1}}{2}\right)^2\left[h(x_{2k+1})+h(-x_{2k+1})\right]\right],\frac{r}{4}\right),$$
$$P_{«,¬}\left(\frac{1}{T^b}\sum_{k=0}^{n}\left[T^b\sum_{l=1}^{2}h\left(\frac{d_{2k}x_{2k}+(-1)^l d_{2k+1}x_{2k+1}}{2}\right)\right]\right.$$
$$\left.\left.-\sum_{k=0}^{n}\frac{1}{T^b}\left[\begin{array}{l}T^b\sum_{l=1}^{2}\left[\left(\frac{d_{2k}}{2}\right)^l\left[h(x_{2k})+(-1)^l h(-x_{2k})\right]\right]\\-\left(\frac{d_{2k+1}}{2}\right)^2\left[h(x_{2k+1})+h(-x_{2k+1})\right]\end{array}\right],r\right)\right\}$$

for all $x_0,x_1,\cdots x_{2n},x_{2n+1}\in X$ and all $r>0$. Letting $b\to\infty$ in the above and using (22),(2), we arrive

$$P_{«,¬}\left(\sum_{k=0}^{n}\left[\sum_{l=1}^{2}h\left(\frac{d_{2k}x_{2k}+(-1)^l d_{2k+1}x_{2k+1}}{2}\right)\right]\right.$$
$$\left.=\sum_{k=0}^{n}\left[\begin{array}{l}\sum_{l=1}^{2}\left[\left(\frac{d_{2k}}{2}\right)^l\left[h(x_{2k})+(-1)^l h(-x_{2k})\right]\right]\\+\left(\frac{d_{2k+1}}{2}\right)^2\left[h(x_{2k+1})+h(-x_{2k+1})\right]\end{array}\right],r\right) \quad (23)$$
$$\leq_{L^*} T\left(\begin{array}{l}1_{L^*},1_{L^*},1_{L^*},1_{L^*},P'_{«,¬}\\\left(\Lambda(T^bx_0,T^bx_1,\cdots T^bx_{2n},T^bx_{2n+1}),T^br\right)\end{array}\right)$$

$$\leq_{L^*} P'_{«,¬}\left(\Lambda(T^bx_0,T^bx_1,\cdots T^bx_{2n},T^bx_{2n+1}),T^br\right) \quad (24)$$

for all $x_0,x_1,\cdots x_{2n},x_{2n+1}\in X$ and all $r>0$. Letting $b\to\infty$ in (24) and using (2), $(IFN2)$, we arrive

$$\sum_{k=0}^{n}\left[\sum_{l=1}^{2}A\left(\frac{d_{2k}x_{2k}+(-1)^l d_{2k+1}x_{2k+1}}{2}\right)\right]$$
$$=\sum_{k=0}^{n}\left[\begin{array}{l}\sum_{l=1}^{2}\left[\left(\frac{d_{2k}}{2}\right)^l\left[A(x_{2k})+(-1)^l A(-x_{2k})\right]\right]\\+\left(\frac{d_{2k+1}}{2}\right)^2\left[A(x_{2k+1})+A(-x_{2k+1})\right]\end{array}\right]$$

for all $x_0,x_1,\cdots x_{2n},x_{2n+1}\in X$. Hence A satisfies the functional equation (5). In order to prove $A(x)$ is unique, let $A'(x)$ be another Additive functional equation satisfying (5) .Hence,

$$P_{«,¬}(A(x)-A'(x),r)$$
$$\leq_{L^*} T\left(\begin{array}{l}P'_{«,¬}\left(A(T^bx)-\frac{f_o(T^bx)}{T^b},\frac{T^br}{2}\right),\\P'_{«,¬}\left(\frac{h_o(T^bx)}{T^b}-A'(T^bx),\frac{T^br}{2}\right)\end{array}\right)$$
$$\leq_{L^*} P'_{«,¬}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times},T^bx,0\right),\frac{T^b}{2}(2T-2a)r\right)$$
$$\leq_{L^*} P'_{«,¬}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times},x,0\right),\frac{T^b}{2}(2T-2a)r\right)$$

for all $x\in X$ and all $r>0$. Since $\lim_{b\to\infty}\frac{T^b}{2}(2T-2a)=\infty,$ we obtain

$$\lim_{n\to\infty}P'_{«,¬}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1\,times},x,0\right),\frac{T^b}{2}(2T-2a)r\right)=1_{L^*}.$$

Thus

$$P_{«,¬}(A(x)-A'(x),r)=1_{L^*}$$

for all $x \in X$ and all $r > 0$, hence $A(x) = A'(x)$. Therefore $A(x)$ is unique.

For $\mathfrak{c} = -1$, we can prove the similar stability result. This completes the proof of the theorem.

The following corollary is an immediate consequence of Theorem 3.1, regarding the stability of(5)

Corollary 3.2 Suppose that an odd function $h_o : X \to Y$ satisfies the inequality

$$P_{\alpha,\neg}\left(Dh(x_0, x_1, \cdots x_{2n}, x_{2n+1}), r\right) \leq_{L^*} \begin{cases} P'_{\alpha,\neg}\left(\mathrm{a}, r\right), \\ P'_{\alpha,\neg}\left(\mathrm{a}\sum_{i=0}^{2n+1} \|x_i\|^s, r\right), \\ P'_{\alpha,\neg}\left(\mathrm{a}\left(\prod_{i=0}^{2n+1}\|x_i\|^s + \sum_{i=1}^{2n+1}\|x_i\|^{2n+1s}\right), r\right), \end{cases} \quad (25)$$

for all $x_0, x_1, \cdots x_{2n}, x_{2n+1} \in X$ and all $r > 0$, where a, s are constants with $\mathrm{a} > 0$. Then there exists a unique Additive mapping $A : X \to Y$ such that

$$P_{\alpha,\neg}\left(h_o(x) - A(x), r\right) \leq_{L^*} \begin{cases} P'_{\alpha,\neg}\left(\mathrm{a}, 2|T - T^0|r\right), \\ P'_{\alpha,\neg}\left(\mathrm{a}\|x\|^s, 2|T - T^s|r\right), & s \neq 1; \\ P'_{\alpha,\neg}\left(\mathrm{a}\|x\|^{(2n+1)s}, 2|T - T^{(2n+1)s}|r\right), & s \neq \frac{1}{2n+1}; \end{cases} \quad (26)$$

for all $x \in X$ and all $r > 0$.

Proof. Replacing

$$\Lambda(x_0, x_1, \cdots x_{2n}, x_{2n+1}) = \begin{cases} \mathrm{a}, \\ \mathrm{a}\left(\|x_0\|^s + \|x_1\|^s + \cdots + \|x_{2n}\|^s + \|x_{2n+1}\|^s\right), \\ \mathrm{a}\left\{\|x_0\|^s\|x_1\|^s \cdots \|x_{2n}\|^s\|x_{2n+1}\|^s \right. \\ \left. + \left(\|x_0\|^{(2n+1)s} + \|x_1\|^{(2n+1)s} + \cdots + \|x_{2n}\|^{ns} + \|x_{2n+1}\|^{(2n+1)s}\right)\right\}, \end{cases}$$

then the corollary is followed from Theorem 3.1. If we define

$$a = \begin{cases} T^0, \\ T^s, \\ T^{(2n+1)s}. \end{cases} \qquad \text{where } T = \frac{d_{2n}}{2}.$$

The proof of the following Theorem and Corollary is similar tracing to that of Theorem 3.1 and Corollary 3.2, when h_e is even. Hence the details of the proof is omitted.

Theorem 3.3 Let $\mathfrak{c} \in \{1, -1\}$. Let $\Lambda : X^n \to Z$ be a function such that for some $0 < \left(\frac{a}{T^2}\right)^{\mathfrak{c}} < 1$,

$$P'_{\alpha,\neg}\left(\Lambda\left(\underbrace{0, \cdots, 0}_{2n-1\,times}, T^{\mathfrak{c}b}x, 0\right), r\right) \leq_{L^*} P'_{\alpha,\neg}\left(a^{\mathfrak{c}b}\ \Lambda\left(\underbrace{0, \cdots, 0}_{2n-1\,times}, x, 0\right), r\right) \quad (27)$$

for all $x \in X$ and all $r > 0$ and

$$\lim_{b\to\infty} P'_{\alpha,\neg}\left(\Lambda\left(T^{\mathfrak{c}b}x_0, T^{\mathfrak{c}b}x_1, T^{\mathfrak{c}b}x_2, \cdots, T^{\mathfrak{c}b}x_{2n}, T^{\mathfrak{c}b}x_{2n+1}\right), T^{\mathfrak{c}b}r\right) = 1_{L^*} \quad (28)$$

for all $x_0, x_1, \cdots x_{2n}, x_{2n+1} \in X$ and all $r > 0$. Let $h_e : X \to Y$ be an even function satisfies the inequality

$$P_{\alpha,\neg}\left(Dh_e(x_0, x_1, \cdots x_{2n}, x_{2n+1}), r\right) \leq_{L^*} P'_{\alpha,\neg}\left(\Lambda\left(x_0, x_1, \cdots x_{2n}, x_{2n+1}\right), r\right) \quad (29)$$

for all $x_0, x_1, \cdots x_{2n}, x_{2n+1} \in X$ and all $r > 0$. Then the limit

$$P_{\alpha,\neg}\left(Q(x) - \frac{h_e(T^{2b}x)}{T^{2b}}, r\right) \to 1_L^*, as \ \ b \to \infty, \ \ r > 0 \quad (30)$$

exists for all $x \in X$ and the mapping $Q : X \to Y$ is a unique quadratic mapping satisfying (5) and

$$P_{\alpha,\neg}\left(h_o(x) - Q(x), r\right) \leq_{L^*} P'_{\alpha,\neg}\left(\Lambda\left(\underbrace{0, \cdots, 0}_{2n-1\,times}, x, 0\right), |(2T)^2 - (2a)^2|\,2r\right) \quad (31)$$

for all $x \in X$ and all $r > 0$.

Corollary 3.4 Suppose that an even function $h_e : X \to Y$ satisfies the inequality

$$P_{\alpha,\neg}\left(Dh(x_0, x_1, \cdots x_{2n}, x_{2n+1}), r\right) \leq_{L^*} \begin{cases} P'_{\alpha,\neg}\left(\mathrm{a}, r\right), \\ P'_{\alpha,\neg}\left(\mathrm{a}\sum_{i=0}^{2n+1} \|x_i\|^s, r\right), \\ P'_{\alpha,\neg}\left(\mathrm{a}\left(\prod_{i=0}^{2n+1}\|x_i\|^s + \sum_{i=1}^{2n+1}\|x_i\|^{2n+1s}\right), r\right), \end{cases} \quad (32)$$

for all $x_0, x_1, \cdots x_{2n}, x_{2n+1} \in X$ and all $r > 0$, where a, s are constants with $\mathrm{a} > 0$. Then there exists a unique quartic mapping $Q : X \to Y$ such that

$$P_{«,¬}\left(h_e(x)-Q(x),r\right)$$
$$\leq_{L^*}\begin{cases} P'_{«,¬}\left(ª,2|T^2-T^0|r\right), & \\ P'_{«,¬}\left(ª\,\|x\|^s,2|T^2-T^s|r\right), & s\neq 2; \\ P'_{«,¬}\left(ª\,\|x\|^{(2n+1)s},2|T^2-T^{(2n+1)s}|r\right), & s\neq \frac{2}{2n+1}; \end{cases} \quad (33)$$

for all $x\in X$ and all $r>0$.

Theorem 3.5 Let $¢=\pm 1$ be fixed and let $\Lambda: X^n\to Z$ be a mapping such that for some d with $0<\left(\frac{a}{T}\right)^{¢}<1$ and $0<\left(\frac{a}{T^2}\right)^{¢}<1$ satisfying (1),(2),(27) and (28). Suppose that a function $h: X\to Y$ satisfies the inequality

$$P_{«,¬}\left(Dh(x_0,x_1,\cdots x_{2n},x_{2n+1}),r\right) \leq_{L^*} P'_{«,¬}\left(\Lambda\left(x_0,x_1,\cdots x_{2n},x_{2n+1}\right),r\right) \quad (34)$$

for all $x_0,x_1,\cdots x_{2n},x_{2n+1}\in X$ and all $r>0$. Then there exists a unique additive mapping $A: X\to Y$ and unique quadratic mapping $Q: X\to Y$ satisfying (5) and

$$P_{«,¬}\left(h(x)-A(x)-Q(x),r\right) \leq_{L^*} P^3_{«,¬}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1 times},x,0\right),r\right) \quad (35)$$

where

$$P^3_{«,¬}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1 times},x,0\right),r\right) = T\left\{\begin{matrix} P^1_{«,¬}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1 times},x,0\right),|2T-2a|r\right), \\ P^2_{«,¬}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1 times},x,0\right),|(2T)^2-(2a)^2|r\right)\end{matrix}\right\} \quad (36)$$

for all $x\in X$ and all $r>0$.

Proof. Let $h_a(x)=\frac{h_o(x)-h_o(-x)}{2}$ for all $x\in X$. Then $h_a(0)=0$ and $h_a(-x)=-h_a(x)$ for all $x\in X$. Hence

$$\begin{aligned} &P_{«,¬}\left(D\,h_a(x_0,x_1,\cdots x_{2n},x_{2n+1}),r\right) \\ &\leq_{L^*} T\left\{P'_{«,¬}\left(D\,h_o(x_0,x_1,\cdots x_{2n},x_{2n+1}),r\right),P'_{«,¬}\left(D\,h_o(-x_0,-x_1,\cdots -x_{2n},-x_{2n+1}),r\right)\right\} \\ &\leq_{L^*} T\left\{P'_{«,¬}\left(\Lambda(x_0,x_1,\cdots x_{2n},x_{2n+1}),r\right),P'_{«,¬}\left(\Lambda(-x_0,-x_1,\cdots -x_{2n},-x_{2n+1}),r\right)\right\} \end{aligned} \quad (37)$$

for all $x_0,x_1,\cdots x_{2n},x_{2n+1}\in X$ and all $r>0$. By Theorem 3.1 there exists a unique additive mapping $A: X\to Y$ such that

$$P_{«,¬}\left(h_o(x)-A(x),r\right) \leq_{L^*} P^1_{«,¬}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1 times},x,0\right),|2T-2a|2r\right) \quad (38)$$

for all $x\in X$ and all $r>0$, where

$$P^1_{«,¬}(\Lambda(x_0,x_1,\cdots x_{2n},x_{2n+1}),r) = T\left\{\begin{matrix} P'_{«,¬}\left(\Lambda(x_0,x_1,\cdots x_{2n},x_{2n+1}),r\right), \\ P'_{«,¬}\left(\Lambda(x_0,x_1,\cdots x_{2n},x_{2n+1}),r\right)\end{matrix}\right\} \quad (39)$$

for all $x_0,x_1,\cdots x_{2n},x_{2n+1}\in X$ and all $r>0$.

Also, let $h_q(x)=\frac{h_e(x)+h_e(-x)}{2}$ for all $x\in X$. Then $h_q(0)=0$ and $h_q(-x)=h_q(x)$ for all $x\in X$. Hence

$$\begin{aligned} &P_{«,¬}\left(D\,h_q(x_0,x_1,\cdots x_{2n},x_{2n+1}),r\right) \\ &\leq_{L^*} T\left\{P'_{«,¬}\left(D\,h_e(x_0,x_1,\cdots x_{2n},x_{2n+1}),r\right),P'_{«,¬}\left(D\,h_e(-x_0,-x_1,\cdots -x_{2n},-x_{2n+1}),r\right)\right\} \\ &\leq_{L^*} T\left\{P'_{«,¬}\left(\Lambda(x_0,x_1,\cdots x_{2n},x_{2n+1}),r\right),P'_{«,¬}\left(\Lambda(-x_0,-x_1,\cdots -x_{2n},-x_{2n+1}),r\right)\right\} \end{aligned} \quad (40)$$

for all $x_0,x_1,\cdots x_{2n},x_{2n+1}\in X$ and all $r>0$. By Theorem 3.3, there exists a unique quadratic mapping $Q: X\to Y$ such that

$$P_{«,¬}\left(h_e(x)-Q(x),r\right) \leq_{L^*} P^2_{«,¬}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1 times},x,0\right),2|(2T)^2-(2a)^2|r\right) \quad (41)$$

for all $x\in X$ and all $r>0$, where

$$P^2_{«,¬}(\Lambda(x_0,x_1,\cdots x_{2n},x_{2n+1}),r) = T\left\{\begin{matrix} P'_{«,¬}\left(\Lambda(x_0,x_1,\cdots x_{2n},x_{2n+1}),r\right), \\ P'_{«,¬}\left(\Lambda(x_0,x_1,\cdots x_{2n},x_{2n+1}),r\right)\end{matrix}\right\} \quad (42)$$

for all $x_0, x_1, \cdots x_{2n}, x_{2n+1} \in X$ and all $r > 0$. Define

$$h(x) = h_a(x) + h_q(x) \quad (43)$$

for all $x \in X$. From (35),(38) and (39), we arrive

$$\begin{aligned}
&P_{\alpha,\beta}\left(h(x) - A(x) - Q(x), r\right)\\
&= P_{\alpha,\beta}\left(h_a(x) + f_q(x) - A(x) - Q(x), r\right)\\
&\le_{L^*} T\left\{P_{\alpha,\beta}\left(h_a(x) - A(x), \frac{r}{2}\right), P_{\alpha,\beta}\left(h_q(x) - Q(x), \frac{r}{2}\right)\right\}\\
&\le_{L^*} T\left\{P^1_{\alpha,\beta}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1 times}, x, 0\right), |2T - 2a| r\right)\right. \quad \text{where}\\
&\left., P^2_{\alpha,\beta}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1 times}, x, 0\right), |(2T)^2 - (2a)^2| r\right)\right\}\\
&= P^3_{\alpha,\beta}\left(\Lambda\left(\underbrace{0,\cdots,0}_{n-1 times}, v\right), r\right)
\end{aligned}$$

$$P^3_{\alpha,\beta}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1 times}, x, 0\right), r\right) = T\left\{\begin{array}{l} P^1_{\alpha,\beta}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1 times}, x, 0\right), |2T - 2a| r\right), \\ P^2_{\alpha,\beta}\left(\Lambda\left(\underbrace{0,\cdots,0}_{2n-1 times}, x, 0\right), |(2T)^2 - (2a)^2| r\right)\end{array}\right\} \quad (44)$$

for all $x \in X$ and all $r > 0$. Hence the theorem is proved.

The following corollary is the immediate consequence of corollaries 3.2, 3.4 and Theorem 3.5 concerning the stability for the functional equation (5).

Corollary 3.6 Suppose that a function $h: X \to Y$ satisfies the inequality

$$P_{\alpha,\beta}\left(Dh(x_0, x_1, \cdots x_{2n}, x_{2n+1}), r\right) \le_{L^*} \begin{cases} P'_{\alpha,\beta}\left(a, r\right), \\ P'_{\alpha,\beta}\left(a \sum_{i=1}^{2n+1} \|x_i\|^s, r\right), \\ P'_{\alpha,\beta}\left(a\left(\prod_{i=1}^{2n+1} \|x_i\|^s + \sum_{i=1}^{2n+1} \|x_i\|^{(2n+1)s}\right), r\right), \end{cases} \quad (45)$$

for all $x_0, x_1, \cdots x_{2n}, x_{2n+1} \in X$ and all $r > 0$, where a, s are constants with $a > 0$. Then there exists a unique additive mapping $A: X \to Y$ and a unique quadratic mapping $Q: X \to Y$ such that

$$P_{\alpha,\beta}\left(h(x) - A(x) - Q(x), r\right) \ge_{L^*} \begin{cases} T\{P'_{\alpha,\beta}\left(a, 2|T - T^0| r\right), \\ \quad P'_{\alpha,\beta}\left(a, 2|T^2 - T^0| r\right)\} \\ T\{P'_{\alpha,\beta}\left(a\|x\|^s, 2|T - T^s| r\right), \\ \quad P'_{\alpha,\beta}\left(a\|x\|^s, 2|T^2 - T^s| r\right)\}, & s \ne 1, 2; \\ T\{P'_{\alpha,\beta}\left(a\|x\|^{(2n+1)s}, 2|T - T^{(2n+1)s}| r\right), \\ \quad P'_{\alpha,\beta}\left(a\|x\|^{(2n+1)s}, 2|T^2 - T^{(2n+1)s}| r\right)\}, & s \ne \frac{1}{2n+1}, \frac{2}{2n+1}; \end{cases}$$

(46)

for all $x \in X$ and all $r > 0$.

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