



**A STUDY ON OPERATORS OVER INTUITIONISTIC FUZZY SETS OF THIRD TYPE**

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**A B S T R A C T**

**RESEARCH ARTICLE**

In this paper, we define the operators  $J_{\alpha, \beta}(\cdot)$  and  $J_{\alpha, \beta}^*(\cdot)$  on Intuitionistic Fuzzy Sets of Third Type and establish some of their properties and relations.

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**Keywords:**

Intuitionistic Fuzzy Sets (IFS), Intuitionistic Fuzzy Sets of Second Type (IFSST), Intuitionistic Fuzzy Sets of Third Type (IFSTT).

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**1. Introduction**

To overcome the uncertainty and vagueness, the inherent in the real world, L. A. Zadeh<sup>[12]</sup> introduced the notion of fuzzy sets in 1965. In 1983, K. T. Atanassov<sup>[1]</sup> introduced the Intuitionistic Fuzzy Sets and their extensions. The authors further extended the Intuitionistic Fuzzy Sets, namely, Intuitionistic Fuzzy Sets of Third Type and studied some of their properties. In section 2, we recollect some basic definitions and in section 3, we define the new operators  $J_{\alpha, \beta}$  and  $J_{\alpha, \beta}^*$  on intuitionistic fuzzy sets of third type and establish some of their properties and relations. The paper is concluded in section 4.

**2. Preliminaries**

In this section, we give some definitions of IFS and their extensions.

**Definition 2.1[1]** Let  $X$  be a non-empty set. An Intuitionistic Fuzzy Set (IFS)  $A$  in  $X$  is defined as an object of the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \},$$

where  $\mu_A(x) : X \rightarrow [0,1]$  and  $\nu_A(x) : X \rightarrow [0,1]$  denote the degree of membership and non-membership functions of  $A$  respectively, and

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1,$$

foreach  $x \in X$ .

**Definition 2.2[1]** The degree of non-determinacy (uncertainty) of an element  $x \in X$  in the IFS  $A$  is defined by

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

**Definition 2.3[1]** Let  $X$  be the non-empty set. An Intuitionistic Fuzzy Set of Second Type (IFSST)  $A$  in  $X$  is defined as an object of the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \},$$

where  $\mu_A(x) : X \rightarrow [0,1]$  and  $\nu_A(x) : X \rightarrow [0,1]$  denote the degree of membership and non-membership functions of  $A$  respectively, and

$$0 \leq \mu_A^2(x) + \nu_A^2(x) \leq 1,$$

foreach  $x \in X$ .

**Definition 2.4[11]** Let  $X$  be the non-empty set. An Intuitionistic Fuzzy Set of Third Type (IFSTT)  $A$  in  $X$  is defined as an object of the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \},$$

where  $\mu_A(x) : X \rightarrow [0,1]$  and  $\nu_A(x) : X \rightarrow [0,1]$  denote the membership and non-membership functions of  $A$ , respectively, and

$$0 \leq \mu_A^3(x) + \nu_A^3(x) \leq 1,$$

for each  $x \in X$ .

**Definition 2.5[9]** The degree of non-determinacy (uncertainty) of an element  $x \in X$  in the IFSTT  $A$  is defined by

$$\pi_A(x) = \sqrt[3]{1 - \mu_A^3(x) - \nu_A^3(x)}.$$

**Remark:** In case of ordinary fuzzy sets,  $\pi_A(x) = 0$ , for every  $x \in X$ .

**Definition 2.6[10]** Let  $A$  and  $B$  be any two IFSTTs of the non-empty set  $X$  such that

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \},$$

$$B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X \}.$$

We define the following basic operations on  $A$  and  $B$ .

- (i)  $A \subset B$  iff  $\mu_A(x) \leq \mu_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$ , for all  $x \in X$
- (ii)  $A \supset B$  iff  $\mu_A(x) \geq \mu_B(x)$  and  $\nu_A(x) \leq \nu_B(x)$ , for all  $x \in X$
- (iii)  $A = B$  iff  $\mu_A(x) = \mu_B(x)$  and  $\nu_A(x) = \nu_B(x)$ , for all  $x \in X$

(iv)  $\bar{A} = \langle x, \mu_A x, v_A x \rangle : x \in X$ ,  
 where  $\bar{A}$  is the complement of  $A$ .

**Definition 2.7**[10] For every IFSTT  $A$ , we have the following two topological operators:

$$C(A) = \langle x, K, L \rangle : x \in X$$

where  $K = \max_{y \in X} \mu_A y$ ,  $L = \min_{y \in X} v_A y$   
 and  $I(A) = \langle x, k, l \rangle : x \in X$

where  $k = \min_{y \in X} \mu_A y$ ,  $l = \max_{y \in X} v_A y$ .

We call these operators a ‘‘Closure’’ and ‘‘Interior’’ of  $A$  over the universe  $X$  respectively.

**Definition 2.8**[7] Let  $\alpha, \beta \in [0, 1]$ . Given an IFSTT  $A$ , we have an operator

$$H_{\alpha, \beta}(A) = \langle x, \sqrt[\alpha]{\mu_A(x)}, \sqrt[\beta]{v_A(x)} + \beta \pi_A^3(x) \rangle : x \in X$$

**Definition 2.9**[7] Let  $\alpha, \beta \in [0, 1]$ . Given an IFSTT  $A$ , we have an operator

$$H_{\alpha, \beta}^*(A) = \langle x, \sqrt[\alpha]{\mu_A x}, \sqrt[\beta]{v_A^3 x + \beta(1 - \alpha \mu_A^3 x - v_A^3 x)} \rangle : x \in X$$

### 3. Operators on IFSTT

**Definition 3.1** Let  $\alpha, \beta \in [0, 1]$ . Given an IFSTT  $A$ , an operator  $J_{\alpha, \beta}$  is defined as

$$J_{\alpha, \beta} A = \langle x, \sqrt[\alpha]{\mu_A^3 x + \alpha(1 - \mu_A^3 x - \beta v_A^3 x)}, \sqrt[\beta]{v_A x} \rangle : x \in X.$$

**Definition 3.2** Let  $\alpha, \beta \in [0, 1]$ . Given an IFSTT  $A$ , an operator  $J_{\alpha, \beta}$  is defined as

$$J_{\alpha, \beta} A = \langle x, \sqrt[\alpha]{\mu_A^3 x + \alpha(1 - \mu_A^3 x - \beta v_A^3 x)}, \sqrt[\beta]{v_A x} \rangle : x \in X.$$

**Remark:** Obviously,  $J_{\alpha, \beta} A$  and  $J_{\alpha, \beta} A$  are IFSTTs.

**Proposition 3.1** For every IFSTT  $A$  and for every real numbers  $\alpha, \beta \in [0, 1]$  we have

- (i)  $J_{\alpha, \beta} C(A) = C(J_{\alpha, \beta} A)$
- (ii)  $J_{\alpha, \beta} I(A) = I(J_{\alpha, \beta} A)$
- (iii)  $J_{\alpha, \beta} C(A) = C(J_{\alpha, \beta} A)$
- (iv)  $J_{\alpha, \beta} I(A) = I(J_{\alpha, \beta} A)$ .

**Proof.** (i)  $J_{\alpha, \beta} C(A)$

$$\begin{aligned} &= J_{\alpha, \beta}(\langle x, \max_{y \in X} \mu_A y, \min_{y \in X} v_A y \rangle : x \in X) \\ &= \langle x, \sqrt[\alpha]{\max_{y \in X} \mu_A y^3 + \alpha(1 - \max_{y \in X} \mu_A y^3 - \min_{y \in X} v_A y^3)}, \sqrt[\beta]{\min_{y \in X} v_A y} \rangle : x \in X \end{aligned}$$

$$\begin{aligned} &= \langle x, \sqrt[\alpha]{\max_{y \in X} \mu_A y^3 + \alpha(1 - \max_{y \in X} \mu_A y^3 - \min_{y \in X} v_A y^3)}, \sqrt[\beta]{\min_{y \in X} v_A y} \rangle : x \in X \\ &= \langle x, \sqrt[\alpha]{\max_{y \in X} \mu_A^3 y + \alpha(1 - \mu_A^3 y - v_A^3 y)}, \sqrt[\beta]{\min_{y \in X} v_A y} \rangle : x \in X \\ &= \langle x, \max_{y \in X} \mu_A^3 y + \alpha(1 - \mu_A^3 y - v_A^3 y), \sqrt[\beta]{\min_{y \in X} v_A y} \rangle : x \in X \end{aligned}$$

$$\begin{aligned} &= C \langle x, \sqrt[\alpha]{\mu_A^3 x + \alpha \pi_A^3 x}, \sqrt[\beta]{v_A x} \rangle : x \in X \\ &= C J_{\alpha, \beta} A \end{aligned}$$

(ii)  $J_{\alpha, \beta} I(A)$

$$\begin{aligned} &= J_{\alpha, \beta} \langle x, \min_{y \in X} \mu_A y, \max_{y \in X} v_A y \rangle : x \in X \\ &= \langle x, \sqrt[\alpha]{\min_{y \in X} \mu_A y^3 + \alpha(1 - \min_{y \in X} \mu_A y^3 - \max_{y \in X} v_A y^3)}, \sqrt[\beta]{\max_{y \in X} v_A y} \rangle : x \in X \end{aligned}$$

$$\begin{aligned} &= \langle x, \sqrt[\alpha]{\min_{y \in X} \mu_A y^3 + \alpha(1 - \min_{y \in X} \mu_A y^3 - \max_{y \in X} v_A y^3)}, \sqrt[\beta]{\max_{y \in X} v_A y} \rangle : x \in X \\ &= \langle x, \min_{y \in X} \mu_A^3 y + \alpha(1 - \mu_A^3 y - v_A^3 y), \sqrt[\beta]{\max_{y \in X} v_A y} \rangle : x \in X \end{aligned}$$

$$\begin{aligned} &= I \langle x, \sqrt[\alpha]{\mu_A^3 x + \alpha \pi_A^3 x}, \sqrt[\beta]{v_A x} \rangle : x \in X \\ &= I J_{\alpha, \beta} A \end{aligned}$$

(iii)  $J_{\alpha, \beta} C(A)$

$$\begin{aligned} &= J_{\alpha, \beta} \langle x, \max_{y \in X} \mu_A y, \min_{y \in X} v_A y \rangle : x \in X \\ &= \langle x, \sqrt[\alpha]{\max_{y \in X} \mu_A y^3 + \alpha(1 - \max_{y \in X} \mu_A y^3 - \beta \min_{y \in X} v_A y^3)}, \sqrt[\beta]{\min_{y \in X} v_A y} \rangle : x \in X \end{aligned}$$

$$\begin{aligned} &= \langle x, \sqrt[\alpha]{\max_{y \in X} \mu_A y^3 + \alpha(1 - \max_{y \in X} \mu_A y^3 - \beta \min_{y \in X} v_A y^3)}, \sqrt[\beta]{\min_{y \in X} v_A y} \rangle : x \in X \\ &= \langle x, \max_{y \in X} \mu_A^3 y + \alpha(1 - \mu_A^3 y - \beta v_A^3 y), \sqrt[\beta]{\min_{y \in X} v_A y} \rangle : x \in X \end{aligned}$$

$$\begin{aligned}
 & \frac{{}^3\bar{\beta} \min_{y-X} v_A y : x \ X}{{}^3\mu_A^3 y + \alpha(1 - \mu_A^3 y - \beta v_A^3 y)}, \\
 = & \frac{{}^3\bar{\beta} v_A y : x \ X}{{}^3\mu_A^3 x + \alpha(1 - \mu_A^3 x - \beta v_A^3 x)}, \\
 = & C \ I_{\alpha,\beta} A
 \end{aligned}$$

(iv)  $I_{\alpha,\beta} I(A)$

$$\begin{aligned}
 & I_{\alpha,\beta} \frac{x, \min_{y-X} \mu_A y, \max_{y-X} v_A y : x \ X}{{}^3\mu_A^3 y + \alpha(1 - \min_{y-X} \mu_A y - \beta \max_{y-X} v_A y)}, \\
 = & \frac{{}^3\bar{\beta} \max_{y-X} v_A y : x \ X}{{}^3\mu_A^3 y + \alpha(1 - \min_{y-X} \mu_A y - \beta \min_{y-X} v_A y)}, \\
 = & \frac{{}^3\bar{\beta} \max_{y-X} v_A y : x \ X}{{}^3\mu_A^3 y + \alpha(1 - \mu_A^3 y - \beta v_A^3 y)}, \\
 = & \frac{{}^3\bar{\beta} \max_{y-X} v_A y : x \ X}{{}^3\mu_A^3 x + \alpha(1 - \mu_A^3 x - \beta v_A^3 x)}, \\
 = & I \ I_{\alpha,\beta} A.
 \end{aligned}$$

**Proposition 3.2** Let  $\bar{E} = x, 1, 0 : x \ X$ ,  $\bar{O} = x, 0, 1 : x \ X$  and  $\bar{U} = x, 0, 0 : x \ X$ . Then we have

- (i)  $C \bar{E} = \bar{E}$
- (ii)  $C \bar{O} = \bar{O}$
- (iii)  $C \bar{U} = \bar{U}$
- (iv)  $I \bar{E} = \bar{E}$
- (v)  $I \bar{O} = \bar{O}$
- (vi)  $I \bar{U} = \bar{U}$ .

**Proof**

(i)  $C \bar{E} = C \ x, 1, 0 : x \ X$

$$\begin{aligned}
 & = x, \max_{y-X} 1, \min_{y-X} 0 : x \ X \\
 & = x, 1, 0 : x \ X \\
 & = \bar{E}
 \end{aligned}$$

(ii)  $C \bar{O} = C \ x, 0, 1 : x \ X$

$$\begin{aligned}
 & = x, \max_{y-X} 0, \min_{y-X} 1 : x \ X \\
 & = x, 0, 1 : x \ X \\
 & = \bar{O}
 \end{aligned}$$

(iii)  $C \bar{U} = C \ x, 0, 0 : x \ X$

$$\begin{aligned}
 & = x, \max_{y-X} 0, \min_{y-X} 0 : x \ X \\
 & = x, 0, 0 : x \ X \\
 & = \bar{U}
 \end{aligned}$$

(iv)  $I \bar{E} = I \ x, 1, 0 : x \ X$

$$\begin{aligned}
 & = x, \min_{y-X} 1, \max_{y-X} 0 : x \ X \\
 & = x, 1, 0 : x \ X \\
 & = \bar{E}
 \end{aligned}$$

(v)  $I \bar{O} = I \ x, 0, 1 : x \ X$

$$\begin{aligned}
 & = x, \min_{y-X} 0, \max_{y-X} 1 : x \ X \\
 & = x, 0, 1 : x \ X \\
 & = \bar{O}
 \end{aligned}$$

(vi)  $I \bar{U} = I \ x, 0, 0 : x \ X$

$$\begin{aligned}
 & = x, \min_{y-X} 0, \max_{y-X} 0 : x \ X \\
 & = x, 0, 0 : x \ X \\
 & = \bar{U}.
 \end{aligned}$$

This shows that the vertices of the triangle in the geometrical interpretation of an IFSTT is invariant.

**Theorem 3.1** For every IFSTT  $A$  and for every real numbers  $\alpha, \beta \in [0, 1]$  we have the following:

- (i)  $H_{\alpha,\beta} \bar{A} = I_{\beta,\alpha}(A)$
- (ii)  $J_{\alpha,\beta} \bar{A} = H_{\beta,\alpha}(A)$
- (iii)  $H_{\alpha,\beta} \bar{A} = I_{\beta,\alpha}(A)$
- (iv)  $J_{\alpha,\beta} \bar{A} = H_{\beta,\alpha}(A)$ .

**Proof.**

Let  $\bar{x} \in [0, 1]$ , for every given IFSTT  $A$ ,

(i)  $H_{\alpha,\beta} \bar{A}$

$$\begin{aligned}
 & = H_{\alpha,\beta} \bar{x}, \bar{\mu}_A \bar{x}, \bar{v}_A \bar{x} : \bar{x} \ \bar{X} \\
 & = \bar{H}_{\alpha,\beta} \bar{x}, \bar{v}_A \bar{x}, \bar{\mu}_A \bar{x} : \bar{x} \ \bar{X} \\
 & = x, \bar{\alpha} v_A x, \bar{\mu}_A^3 x + \beta \bar{\pi}_A^3 x : x \ X \\
 & = \{ x, \bar{\mu}_A^3 x + \beta \bar{\pi}_A^3 x, \bar{\alpha} v_A x : x \ X \} \\
 & = I_{\beta,\alpha}(A)
 \end{aligned}$$

(ii)  $J_{\alpha,\beta} \bar{A}$

$$\begin{aligned}
 & = J_{\alpha,\beta} \bar{x}, \bar{\mu}_A \bar{x}, \bar{v}_A \bar{x} : \bar{x} \ \bar{X}
 \end{aligned}$$

$$\begin{aligned}
 &= \overline{J_{\alpha,\beta}} \overline{x, v_A x, \mu_A x} : \overline{x} \overline{X} \\
 &= x, \overline{v_A^3 x + \alpha \overline{\mu_A^3(x)}, \overline{\beta \mu_A(x)}} : x \ X \\
 &= x, \overline{\beta \mu_A x, \overline{v_A^3 x + \alpha \overline{\mu_A^3(x)}}} : x \ X \\
 &= H_{\beta,\alpha}(A)
 \end{aligned}$$

(iii)  $H_{\alpha,\beta} \overline{A} = H_{\alpha,\beta} \overline{x, \mu_A x, v_A x} : \overline{x} \overline{X}$

$$\begin{aligned}
 &= \overline{H_{\alpha,\beta}} \overline{x, \mu_A x, v_A x} : \overline{x} \overline{X} \\
 &= x, \overline{\mu_A^3 x + \alpha (1 - \mu_A^3 x - \beta v_A^3(x)), \overline{\beta v_A(x)}} : x \ X \\
 &= x, \overline{\mu_A^3 x + \alpha (1 - \mu_A^3 x - \beta v_A^3(x)), \overline{\beta v_A(x)}} : x \ X \\
 &= J_{\beta,\alpha}(A)
 \end{aligned}$$

(iv)  $J_{\alpha,\beta} \overline{A}$

$$\begin{aligned}
 &= J_{\alpha,\beta} \overline{x, \mu_A x, v_A x} : \overline{x} \overline{X} \\
 &= \overline{J_{\alpha,\beta}} \overline{x, v_A x, \mu_A x} : \overline{x} \overline{X}
 \end{aligned}$$

$$\begin{aligned}
 &= x, \overline{\mu_A^3 x + \alpha (1 - \mu_A^3 x - \beta v_A^3(x)), \overline{\beta v_A(x)}} : x \ X \\
 &= x, \overline{\beta \mu_A(x), \overline{v_A^3 x + \alpha (1 - \mu_A^3 x - \beta v_A^3(x))}} : x \ X
 \end{aligned}$$

=  $H_{\beta,\alpha}(A)$

These equalities show that the operators  $H_{\alpha,\beta}$  and  $J_{\alpha,\beta}$ , and the operators  $H_{\alpha,\beta}(\cdot)$  and  $J_{\alpha,\beta}(\cdot)$  are dual.

### Conclusion

In this paper, we have introduced the new operators  $J_{\alpha,\beta}$  and  $J_{\alpha,\beta}$  on IFSTT and studied some of their properties. We have proved that the vertices of the triangle in the geometrical interpretation of an IFSTT is invariant and also the operators

$H_{\alpha,\beta}$  and  $J_{\alpha,\beta}$ , and the operators  $H_{\alpha,\beta}(\cdot)$  and  $J_{\alpha,\beta}(\cdot)$  are dual.

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