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BI-MAGIC LABELING ON INTERVAL-VALUED INTUITIONISTIC FUZZY GRAPHS

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A B S T R A C T **RESEARCH ARTICLE**

Intuitionistic fuzzy graph is a highly growing research area dealing with real life applications. In this paper, we introduced the concept of interval valued intuitionistic fuzzy graphs and defined Bi-Magic labeling of interval valued intuitionistic fuzzy graphs. Here, we discussed the significance of Bi-Magic labeling in interval-valued intuitionistic fuzzy graphs. We also analyzed some of the properties and structures and implemented them in the operations of interval-valued intuitionistic fuzzy Bi-Magic labeling graphs. We have also investigated the existence of some bounds over the size and shape of the interval-valued intuitionistic fuzzy graphs based on δ-neighborhood and confined the membership values of the vertices and edges of them.

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Keywords:

Intuitionistic fuzzy graphs, Interval valued intuitionistic fuzzy graphs (IVIFG), Bi-Magic labeling of interval valued intuitionistic fuzzy graph

1. Introduction

Graph theory has established its significance in many realtime problems.Recent applications in graph theory is quite interesting and analyzing in many fields. It has got numerous applications in Operations Research, System analysis, Network routing, Transportation and many more. To analyse any complete information, we make intensive use of graphs and their properties. For working on partial information or incomplete information or to handle the systems containing the elements of uncertainty, we understand that fuzzylogic and its involvement in graph theory is applied. In 1975, Rosenfeld [15] discussed the concept of fuzzy graphs whose ideas are implemented by Kauffman [11] in 1973. The fuzzy relation between fuzzy sets were also considered by Rosenfeld who developed the structure of fuzzy graphs, obtaining various analogous results of several graph theoretical concepts. Bhattacharya gave some remarks of fuzzy graphs. The complement of fuzzy graphs was introduced by Mordeson [12]. Atanassov introduced the concept of intuitionistic fuzzy relation and intuitionistic fuzzy graphs [7,8, 26, 27]. Talebi and Rashmanlou [28] studied the properties of isomorphism and complement of interval - valued fuzzy graphs. Rashmanlou et al., defined the complete interval-valued fuzzy graphs and Intuitionistic fuzzy with categorical properties, some properties of highly irregular interval – valued fuzzy graphs , more results on highly irregular bipolar fuzzy graphs and antipodal interval valued fuzzy graphs [16-21]. Samanta and Pal defined fuzzy k-competition and p-competition graphs, introduced fuzzy tolerance graphs, bipolar fuzzy hypergraphs and investigated several properties on them [22-25]. K.AmeenalBibi and M.Devi studied about the concepts of Bi-Magic labeling on Interval-valued fuzzy graphs. In this article, we define the interval-valued intuitionistic fuzzy graph(IVIFG) and analyses its labeling properties. We implement the ideas of Bi-Magic labeling to IVIFG and defined some structures of IVIFG. We also discussed some properties of IVIFS and found the lower and upper bounds based on δ-neighborhood. For other notations and terminologies in this paper, refer to [5,9,14].

2. Preliminaries:

In this section we introduced some definitions which are essential for the subsequent sections.

Definition 2.1

By an interval valued fuzzy graph of a graph G, we mean a pair G^*=(A,B) where $A = [\mu_\alpha^-, \mu_\alpha^+]$ is an interval valued fuzzy set on V and $B = [\mu_\beta^-, \mu_\beta^+]$ is an interval valued fuzzy relation on E such that $\mu_\beta^-(xy) \leq \min(\mu_\alpha^-(x), \mu_\alpha^-(y))$

$\mu_\beta^+(xy) \leq \min(\mu_\alpha^+(x), \mu_\alpha^+(y))$ for all x,y ∈ E.

Definition 2.2

A fuzzy labeling graph admits Bi-magic labeling if the sum of membership values of vertices and edges incident at the vertices are k_1 and k_2 where k_1 and k_2 are constants.

A fuzzy labeling graph which admits a Bi-magic labeling is called a Fuzzy Bi-magic graph.

Definition 2.3

A graph G^*=(A,B) is said to be an interval valued fuzzy graph if $\mu_\alpha^-, \mu_\alpha^+, \mu_\beta^-, \mu_\beta^+ \in [0,1]$ are distinct for all vertices and edges where μ_α^- *and* μ_β^- are all the lower limits of the interval membership of vertices and edges and μ_α^+ *and* μ_β^+ are the upper limits of the interval membership values of vertices and edges respectively.

Definition 2.4

By an interval-valued intuitionistic fuzzy graph of a graph *G* ,we mean, a pair G^* =(A,B) where A= $[(\mu_\alpha^-, \mu_\alpha^+), (\gamma_\alpha^-, \gamma_\alpha^+)]$and $\mu_\beta: V \times V \to [0,1]$ and $\gamma_\beta: V \times V \to [0,1]$ are bijectivessuch that membership and non-membership values of vertices and edges are distinct and $\mu_\beta\ uv \le min\ \mu_\alpha\ u\ ,\mu_\alpha\ v$ and $\gamma_\beta\ uv \le max\ \gamma_\alpha\ u\ ,\gamma_\alpha\ v$ and $0 \le \mu_\beta\ uv + \gamma_\beta\ uv \le 1$ for every(u,v) ∈ E(G).

Definition: 2.5

An Intuitionistic Fuzzy Labeling Graph (IFLG) is said to be an Intuitionistic Fuzzy Bi-magic Labeling (IFBL) if the degree of membership values of vertices and edges $\{\mu_\alpha(u) + \mu_\beta\ uv + \mu_\alpha(v)\}$ incident at the vertices are k_1 and k_2 and the degree of non- membership values of vertices and edges $\{ \gamma_\alpha(u) + \gamma_\beta\ uv + \gamma_\alpha(v)\}$ incident at the vertices are l_1 and l_2 where k_1, k_2, l_1 and l_2 are constants and denoted by $Bm_0(G)$where$Bm_0\ G = Bm_\mu\ G\ ,Bm_\gamma\ G$.

Definition 2.6

A graphG^*= (*A*,*B*) is said to bean interval-valued intuitionistic fuzzy labeling graph, if $(\mu_\alpha^-, \mu_\alpha^+, \mu_\beta^-, \mu_\beta^+) \in [0,1]$ and $(\gamma_\alpha^-, \gamma_\alpha^+, \gamma_\beta^-, \gamma_\beta^+) \in [0,1]$ are all distinct for each of the vertices and edges , where μ_α^-, γ_α^- are the lower limits of an Interval valued Intuitionistic fuzzy membership and non-membership value of vertices and $\mu_\alpha^+, \gamma_\alpha^+$ are the upper limit of an Interval valued Intuitionistic fuzzy membership and non-membership value of vertices. Similarly , $\mu_\beta^-, \gamma_\beta^-$, μ_β^+ , γ_β^+ are the lower and upper limits of an Interval valued Intuitionistic fuzzy membership and non-membership value of edges.

Definition 2.7

An interval $[\mu - \delta, \mu + \delta]$ is said to be aδ-neighbourhood of any membership and non-membership value for any δ satisfying the following conditions.

(i)$\delta \ge$ min $\{\mu_v\ v_i\ ,\gamma_v\ v_i\ ,\mu_e\ e_{ij}\ ,\gamma_e\ e_{ij}\ \}$

(ii) $\delta \le 1 -$ max $\{\mu_v\ v_i\ ,\gamma_v\ v_i\ ,\mu_e\ e_{ij}\ ,\gamma_e\ e_{ij}\ \}$

(iii) $\delta \le$ d$(\mu\ x\ ,\mu\ y\ )$Where d$(\mu\ x\ ,\mu\ y\ )=|\mu\ x\ - \mu(y)|$ and

$\delta \le$ d$(\gamma\ x\ ,\gamma\ y\ )$Where d$(\gamma\ x\ ,\gamma\ y\ )=|\gamma\ x\ - \gamma(y)|$ where $\mu\ x\ ,\mu\ y\ and\ \gamma\ x\ ,\gamma\ y$ are the membership and non-membership values of vertices or edges.

(iv) M+δ ≤ 1 and m-δ ≥ 0 where M= max$\{\mu_v\ v_i\ ,\gamma_v\ v_i\ ,\mu_e\ e_{ij}\ ,\gamma_e\ e_{ij}\ \}$ and

m= min $\{\mu_v\ v_i\ ,\gamma_v\ v_i\ ,\mu_e\ e_{ij}\ ,\gamma_e\ e_{ij}\ \}$.

Proposition 2.8.[14] Any intuitionistic fuzzy graph can be converted into interval-valued intuitionistic fuzzylabeling graph.

Proof:

We know that intuitionistic fuzzy graph is not an intuitionistic fuzzy labeling graph. Thus for labelingany intuitionistic fuzzy graph we consider the interval-valued intuitionistic membership values of all vertices and edges in such a way that the graph obtained is labeled.

For this, we take the *δ*-neighborhood corresponding to each vertex and edge. Here we claim that it gives anintuitionistic interval-valued labeling graph. For any intuitionistic fuzzy graph, there are only three possibilities:

(i) All the vertices and edges have the same membership and non-membershipvalues.

(ii) Few vertices and edges will have the same membership and non-membership values.

(iii) All the vertices and edges have the distinct membership and non-membership values.

Case (i):

If all the vertices and edges have the same membershipand non-membership values, then the sum of the number of vertices and edges is *n*. Then we take *n* distinct *δ*-neighborhood as defined above, assigned to all the vertices and edges. So that we get an interval-valued intuitionistic fuzzy labeling graph.

Case (ii):

If only few vertices and edges have the same membership and non-membership values, then we first make a list of all the membership values corresponding to the vertices or edges, then we take a set of non-membership values and assign one neighborhood $\delta = \delta_1$ forthem and strike off those values which are repeated. Again we take another set of membership values from theremaining elements of the list and assign another neighborhood $\delta = \delta_2$ to them and again we strike off the repetitive values from the list. Continue this process till the last repetitive element in the list is removed .Thus corresponding toeach δ_i, i = 1, 2,... we obtained distinct *δ*-neighborhoodinterval, assigning it to the corresponding vertices andedges, arriving at an interval-valued intuitionistic fuzzy graph satisfying the condition of interval-valuedintuitionistic fuzzy labeling.

Case(iii):

If all the vertices and edges have the distinct membership and non-membership values, then there will be only one *δ* and we get distinct *δ*-neighborhood corresponding toeach vertexand edge.

Hence in each case, we found that an intuitionistic fuzzy graph could be converted into an interval-valued intuitionistic fuzzylabeling graph.

3. Interval valued Intuitionistic Fuzzy Bi-Magic labeling graph:

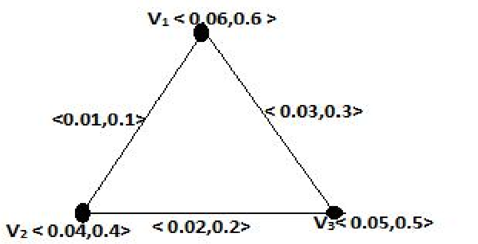
Definition 3.1

An interval valued Intuitionistic Fuzzy labeling graph is said to be an interval valued Intuitionistic fuzzy Bi-Magic graph if the sum of lower membership and non-membership values (ie)., $(\mu_\alpha^-(x) + \mu_\beta^-(xy) + \mu_\alpha^-(y))$ and $(\gamma_\alpha^-(x) + \gamma_\beta^-(xy) + \gamma_\alpha^-(y))$ respectively of vertices and edges incident at the vertices are the constants k_1 and l_1.

Similarly, if the sum of upper membership and non-membership values (ie).,$(\mu_\alpha^+(x) + \mu_\beta^+(xy) + \mu_\alpha^+(y))$ and $(\gamma_\alpha^+(x) + \gamma_\beta^+(xy) + \gamma_\alpha^+(y))$ respectively of vertices and edges incident at the vertices are the constants k_2 and l_2.

This is denoted by $Bm_o(G)= \langle Bm_\mu^-(G), Bm_\mu^+(G), Bm_\gamma^-(G), Bm_\gamma^+(G)\rangle$.

Example 3.2



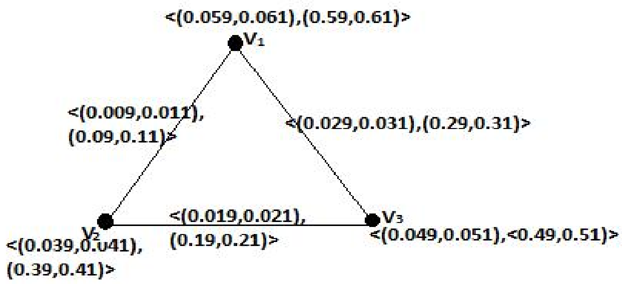


Figure1

Intuitionistic Fuzzy Bi-Magic graph

Interval valed Intuitionistic Fuzzy Bi-Magic graph

In this example, we obtained an interval valued intuitionistic fuzzy Bi-Magic labeling for a cycle graph with three vertices and three edges given in Fig.1.

In this graph,

$\mu_\alpha^-(x) + \mu_\beta^-(xy) + \mu_\alpha^-(y)$ = 0.137 and 0.107 = $Bm_\mu^-(G)$ for all v_i *and* v_j $\mu_\alpha^+(x) + \mu_\beta^+(xy) + \mu_\alpha^+(y)$ = 0.143 and 0.113 = $Bm_\mu^+(G)$ for all v_i *and* v_j

$\gamma_\alpha^-(x) + \gamma_\beta^-(xy) + \gamma_\alpha^-(y)$ = 1.37 and 1.07 = $Bm_\gamma^-(G)$ for all v_i *and* v_j

$\gamma_\alpha^+(x) + \gamma_\beta^+(xy) + \gamma_\alpha^+(y)$ = 1.43 and 1.13= $Bm_\gamma^+(G)$ for all v_i *and* v_j.

Therefore the above cycle graph admits fuzzy Bi-Magic labeling.

Theorem: 3.3

Every Intuitionistic fuzzy Bi-Magic graph can be converted into an Interval valued Intuitionistic fuzzy Bi-Magic graph.

Proof:

In Intuitionistic fuzzy Bi-Magic graph, all the vertices and edges are assignedwiththe distinct membership and non-membership values. Let the membership and non-membership value of vertices are $\mu_\alpha(v_i)$, $\gamma_\alpha(v_i)$ and edges are $\mu_\beta(e_{ij})$, $\gamma_\beta(e_{ij})$ which are incident with the vertices v_i *and* v_j. Let the sum of membership and non-membership values for each pair of vertices and corresponding edges are k_1, k_2 and l_1,l_2 respectively.

(ie)., $\mu_\alpha(v_i)$ +$\mu_\beta(e_{ij})$ + $\mu_\alpha(v_i)$ = k_1 and k_2

$\gamma_\alpha(v_i)$ +$\gamma_\beta(e_{ij})$ + $\gamma_\alpha(v_i)$ = l_1 and l_2.

Then we choose any δ which satisfy the conditions in (2.7). Now replace the membership and non-membership values of each vertex with $|\mu_\alpha(v_i) - \delta, \mu_\alpha(v_i) + \delta|and|\gamma_\alpha(v_i) - \delta, \gamma_\alpha(v_i) + \delta|$ and each edge by $\mu_\beta(e_{ij}) - \delta, \mu_\beta(e_{ij}) + \delta$ and $\gamma_\beta(e_{ij}) - \delta, \gamma_\beta(e_{ij}) + \delta$.

As Intuitionistic fuzzy Bi-Magic labeling admits all the distinct membership and non-membership values for each vertex and edge. we choose δ ,satisfying all the conditions in (2.7), we always get disjoint intervals because intervals are symmetric about δ. In this way, the obtained graph becomes an Interval valued Intuitionistic fuzzy Bi-Magic graph.

Theorem 3.4

Every cycle graph with odd number of vertices isalways an interval-valued intuitionistic fuzzy Bi-magic graph.

Proof

Let G be a cycle with odd number of vertices. Let$v_1, v_2, v_3, \ldots v_n$ and $v_1v_2, v_2v_3, \ldots v_nv_1$ be the vertices and edges of C_n. Let δ ∈ [0,1] such that one can choose $\delta_1 = 0.001$ and $\delta_2 = 0.01$ for lower and upper limit respectively for $n = 3$ and we can choose $\delta_1 = 0.0001$ and $\delta_2 = 0.001$ for lower and upper limit respectively for $n > 4$ and the membership intervals are defined as follows:

$$\mu_\alpha^-(v_{2i}) = (2n - 4 - i)\delta_2 - \delta_1 \quad ,1 \le i \le \frac{n-3}{2}$$

$$\mu_\alpha^-(v_{2i}) = 2(2n - 4 - i)\delta_2 - \delta_1 \quad ,i \le \frac{n-1}{2}$$

$$\mu_\alpha^+(v_{2i}) = (2n - 4 - i)\delta_2 + \delta_1 \quad ,1 \le i \le \frac{n-3}{2}$$

$$\mu_\alpha^+(v_{2i}) = 2(2n - 4 - i)\delta_2 + \delta_1 \quad ,i \le \frac{n-1}{2}$$

$$\mu_\alpha^-(v_{2i-1}) = Max\left(\mu_\alpha^-(v_{2i}) / 1 \le i \le \frac{n-1}{2}\right) - i\delta_2 \quad for\ 1 \le i \le \frac{n+1}{2}$$

$\mu_\alpha^+\ v_{2i-1}\ = Max\ \ \mu_\alpha^+\ v_{2i}\ /1 \le i \le \frac{n-1}{2}\ - i\delta_2 \quad for\ 1 \le i \le \frac{n+1}{2}$

$\mu_\beta^-\ v_1, v_n\ = \frac{1}{2} Max\ \mu_\alpha^-\ v_i\ /1 \le i \le n$

$\mu_\beta^+\ v_1, v_n\ = \frac{1}{2} Max\ \mu_\alpha^+\ v_i\ /1 \le i \le n$

$\mu_\beta^-\ v_{n-i+1}, v_{n-i}\ = \mu_\beta^-\ v_1, v_n\ - i\delta_2 \quad for\ 1 \le i \le n-1$

$\mu_\beta^+\ v_{n-i+1}, v_{n-i}\ = \mu_\beta^+\ v_1, v_n\ - i\delta_2 \quad for\ 1 \le i \le n-1$

Similarly, we can choose $\delta_1 = 0.01$ and $\delta_2 = 0.1$ for lower and upper limit respectively for $n = 3$ and we can choose $\delta_1 = 0.001$ and $\delta_2 = 0.01$ for lower and upper limits respectively for $n > 4$ and the non-membership intervals are defined as follows:

$\gamma_\alpha^-\ v_{2i}\ = \ 2n-4-i\ \delta_2 - \delta_1 \quad ,1 \le i \le \frac{n-3}{2}$

$\gamma_\alpha^-\ v_{2i}\ = 2\ 2n-4-i\ \delta_2 - \delta_1 \quad ,i \le \frac{n-1}{2}$

$\gamma_\alpha^+\ v_{2i}\ = \ 2n-4-i\ \delta_2 + \delta_1 \quad ,1 \le i \le \frac{n-3}{2}$

$\gamma_\alpha^+\ v_{2i}\ = 2\ 2n-4-i\ \delta_2 + \delta_1 \quad ,i \le \frac{n-1}{2}$

$\gamma_\alpha^-\ v_{2i-1}\ = Max\ \gamma_\alpha^-\ v_{2i}\ /1 \le i \le \frac{n-1}{2}\ - i\delta_2 \quad for\ 1 \le i \le \frac{n+1}{2}$

$\gamma_\alpha^+\ v_{2i-1}\ = Max\ \gamma_\alpha^+\ v_{2i}\ /1 \le i \le \frac{n-1}{2}\ - i\delta_2 \quad for\ 1 \le i \le \frac{n+1}{2}$

$\gamma_\beta^-\ v_1, v_n\ = \frac{1}{2} Max\ \gamma_\alpha^-\ v_i\ /1 \le i \le n$

$\gamma_\beta^+\ v_1, v_n\ = \frac{1}{2} Max\ \gamma_\alpha^+\ v_i\ /1 \le i \le n\ .$

$\gamma_\beta^-\ v_{n-i+1}, v_{n-i}\ = \gamma_\beta^-\ v_1, v_n\ - i\delta_2 \quad for\ 1 \le i \le n-1$

$\gamma_\beta^+\ v_{n-i+1}, v_{n-i}\ = \gamma_\beta^+\ v_1, v_n\ - i\delta_2 \quad for\ 1 \le i \le n-1$

Here, we investigated the results for an interval valued Intuitionistic fuzzy Bi-Magic cycle for n=7.

Case (i) :for i is even

Then i=2z for any positive integer z

For each edge v_i, v_{i+1}

$Bm_\mu^-\ C_7\ = \mu_\alpha^-\ v_i\ + \mu_\beta^-(v_i, v_{i+1}) + \mu_\alpha^-\ v_{i+1}$

$= \mu_\alpha^-\ v_{2z}\ + \mu_\beta^-(v_{2z}, v_{2z+1}) + \mu_\alpha^-\ v_{2z+1}$

$= \frac{1}{2} Max\ \mu_\alpha^-\ v_i\ /1 \le i \le n + Max\ \ \mu_\alpha^-\ v_{2i}\ /1 \le i \le \frac{n-1}{2} +$

$n-5\ \delta_2 - \delta_1.$

If i=2z for any positive integer z, $z \le \frac{n-1}{2}$

$Bm_\mu^-\ C_7\ = \mu_\alpha^-\ v_{2z}\ + \mu_\beta^-(v_{2z}, v_{2z+1}) + \mu_\alpha^-\ v_{2z+1}$

$= \frac{1}{2} Max\ \mu_\alpha^-\ v_i\ /1 \le i \le n + Max\ \ \mu_\alpha^-\ v_{2i}\ /1 \le i \le \frac{n-1}{2} +$

$3n-12\ \delta_2 - \delta_1.$

$Bm_\mu^+\ C_7\ = \mu_\alpha^+\ v_{2z}\ + \mu_\beta^+(v_{2z}, v_{2z+1}) + \mu_\alpha^+\ v_{2z+1}$

$=\frac{1}{2} Max\ \mu_\alpha^+\ v_i\ /1 \le i \le n + Max\ \ \mu_\alpha^+\ v_{2i}\ /1 \le i \le \frac{n-1}{2} +$

$n-5\ \delta_2 + \delta_1.$

If i=2z for any positive integer z, $z \le \frac{n-1}{2}$

$Bm_\mu^+\ C_7\ = \mu_\alpha^+\ v_{2z}\ + \mu_\beta^+(v_{2z}, v_{2z+1}) + \mu_\alpha^+\ v_{2z+1}$

$=\frac{1}{2} Max\ \mu_\alpha^+\ v_i\ /1 \le i \le n + Max\ \ \mu_\alpha^+\ v_{2i}\ /1 \le i \le \frac{n-1}{2} +$

$3n-12\ \delta_2 + \delta_1.$

Similarly,

$Bm_\gamma^-\ C_7\ = \frac{1}{2} Max\ \gamma_\alpha^-\ v_i\ /1 \le i \le n + Max\ \gamma_\alpha^-\ v_{2i}\ /1 \le i \le \frac{n-1}{2} +$

$n-5\ \delta_2 - \delta_1.$

If i=2z for any positive integer z, $z \le \frac{n-1}{2}$

$Bm_\gamma^-\ C_7\ = \frac{1}{2} Max\ \gamma_\alpha^-\ v_i\ /1 \le i \le n + Max\ \gamma_\alpha^-\ v_{2i}\ /1 \le i \le \frac{n-1}{2} +$

$3n-12\ \delta_2 - \delta_1.$

$Bm_\gamma^+\ C_7\ = \frac{1}{2} Max\ \gamma_\alpha^+\ v_i\ /1 \le i \le n + Max\ \gamma_\alpha^+\ v_{2i}\ /1 \le i \le \frac{n-1}{2} +$

$n-5\ \delta_2 + \delta_1.$

If i=2z for any positive integer z, $z \le \frac{n-1}{2}$

$Bm_\gamma^+\ C_7\ = \frac{1}{2} Max\ \gamma_\alpha^+\ v_i\ /1 \le i \le n + Max\ \gamma_\alpha^+\ v_{2i}\ /1 \le i \le \frac{n-1}{2} +$

$3n-12\ \delta_2 + \delta_1.$

Case (ii): for i is odd

Then i=2z+1 for any positive integer z

For each edge v_i, v_{i+1}

$Bm_\mu^-\ C_7\ = \mu_A^-\ v_{2z+1}\ + \mu_B^-(v_{+12z}, v_{2z+2}) + \mu_A^-\ v_{2z+2}$

$=\frac{1}{2} Max\ \mu_\alpha^-\ v_i\ /1 \le i \le n + Max\ \ \mu_\alpha^-\ v_{2i}\ /1 \le i \le \frac{n-1}{2} +$

$n-5\ \delta_2 - \delta_1.$

$Bm_\mu^-\ C_7\ = \frac{1}{2} Max\ \mu_\alpha^-\ v_i\ /1 \le i \le n + Max\ \ \mu_\alpha^-\ v_{2i}\ /1 \le i \le \frac{n-1}{2} +$

$3n-12\ \delta_2 - \delta_1$for $z \le \frac{n-1}{2}$

$Bm_\mu^+\ C_7\ = \frac{1}{2} Max\ \mu_\alpha^+\ v_i\ /1 \le i \le n + Max\ \ \mu_\alpha^+\ v_{2i}\ /1 \le i \le \frac{n-1}{2} +$

$n-5\ \delta_2 + \delta_1.$

$Bm_\mu^+\ C_7\ = \mu_\alpha^+\ v_{2z}\ + \mu_\beta^+(v_{2z}, v_{2z+1}) + \mu_\alpha^+\ v_{2z+1}$

$=\frac{1}{2} Max\{\mu_\alpha^+(v_i)/1 \le i \le n\} + Max\{\mu_\alpha^+(v_{2i})/1 \le i \le \frac{n-1}{2}\} +$

$(3n-12)\delta_2 + \delta_1$ for $z \le \frac{n-1}{2}$

Similarly, we can prove the result for the non-membership Bi-Magic values $Bm_\gamma^-(C_7)$ and $Bm_\gamma^+(C_7)$ for the cycle graph C_7.

Hence from the above cases we can say that the odd cycle is always an interval-valued intuitionistic fuzzy Bi-magic graph.
Proof:

It is not always true for an even cycle because when we apply this process for an even cycle, some vertices receive certain such interval for the membership and non- membership values which violate the condition of Bi-magic labeling on them.

Theorem: 3.5

For any $n \ge 4$, an Intuitionistic fuzzy labeled Star graph $S_{1,n}$ is always an Interval valued Intuitionistic fuzzy Bi-Magic graph.

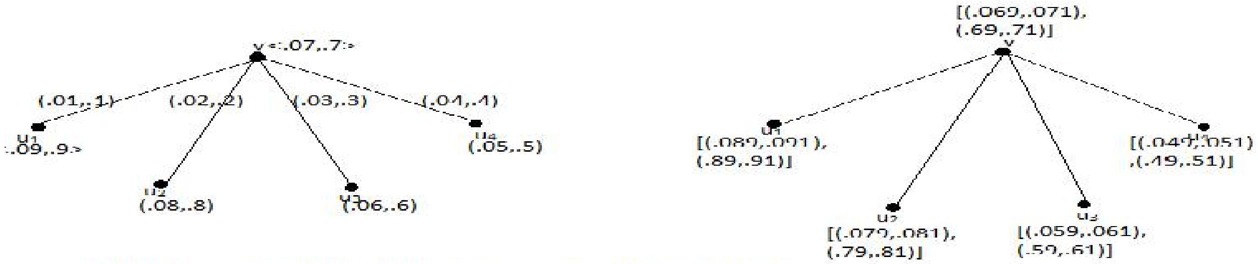


Figure 2 Intuitionistic Fuzzy Star and Corresponding Interval valued Intuitionistic Fuzzy star graph

Let $S_{1,n}$ be the Star graph with $v, u_1, u_2, \ldots u_n$ as vertices and $vu_1, vu_2, \ldots, vu_n$ as edges.

Let δ □ [0,1] such that one can choose $\delta_1 = 0.001$ and $\delta_2 = 0.01$ for lower and upper limits respectively for $n \ge 4$ and the membership intervals are defined as follows:

$\mu_\alpha^-(v) = \frac{\mu_\alpha^-(u_i)}{n} - (n+1)\delta_1 \quad for\ 1 \le i \le n$

$\mu_\alpha^+(v) = \sum \frac{\mu_\alpha^+(u_i)}{n} - (n+1)\delta_1 \quad for\ 1 \le i \le n.$

$\mu_\alpha^-(u_i) = (2(n+1) - i)\delta_2 - \delta_1 \quad for\ i = 1,2,3$

$\mu_\alpha^+(u_i) = (2(n+1) - i)\delta_2 + \delta_1 \quad for\ i = 1,2,3$

$\mu_\alpha^-(u_i) = (2(n+1) - (i-1))\delta_2 - \delta_1 \quad for\ 4 \le i \le n$

$\mu_\alpha^+(u_i) = (2(n+1) - (i-1))\delta_2 + \delta_1 \quad for\ 4 \le i \le n$

$\mu_\beta^-(v,u_1) = Max(\mu_\alpha^-(v), \mu_\alpha^-(u_1)) - Min(\mu_\alpha^-(v), \mu_\alpha^-(u_1)) - 2\delta_2 - \delta_1 \quad for\ i = 1$

$\mu_\beta^-(v,u_2) = Max(\mu_\alpha^-(v), \mu_\alpha^-(u_2)) - Min(\mu_\alpha^-(v), \mu_\alpha^-(u_2)) - \delta_1 \quad for\ i = 2$

$\mu_\beta^-(v,u_3) = Max(\mu_\alpha^-(v), \mu_\alpha^-(u_3)) - Min(\mu_\alpha^-(v), \mu_\alpha^-(u_3)) + 2\delta_2 - \delta_1 \quad for\ i = 3$

$\mu_\beta^-(v,u_i) = Max(\mu_\alpha^-(v), \mu_\alpha^-(u_i)) - Min(\mu_\alpha^-(v), \mu_\alpha^-(u_i)) + 3\delta_2 - \delta_1 \quad for\ 4 \le i \le n$

$\mu_\beta^+(v,u_1) = Max(\mu_\alpha^+(v), \mu_\alpha^+(u_1)) - Min(\mu_\alpha^+(v), \mu_\alpha^+(u_1)) - 2\delta_2 + \delta_1 \quad for\ i = 1$

$\mu_\beta^+(v,u_2) = Max(\mu_\alpha^+(v), \mu_\alpha^+(u_2)) - Min(\mu_\alpha^+(v), \mu_\alpha^+(u_2)) + \delta_1 \quad for\ i = 2$

$\mu_\beta^+(v,u_3) = Max(\mu_\alpha^+(v), \mu_A^+(u_3)) - Min(\mu_\alpha^+(v), \mu_A^+(u_3)) + 2\delta_2 + \delta_1 \quad for\ i = 3$

$\mu_\beta^+(v,u_i) = Max(\mu_\alpha^+(v), \mu_\alpha^+(u_i)) - Min(\mu_\alpha^+(v), \mu_\alpha^+(u_i)) + 3\delta_2 + \delta_1 \quad for\ 4 \le i \le n$

Similarly, we can choose $\delta_1 = 0.01$ and $\delta_2 = 0.1$ for the lower and upper limit respectively for $n \ge 4$and define the non-membership intervals in the similar manner.
Then the constants for the membership values are k_1 and k_2 and for the non-membership values are l_1 and $l_2$for the Interval valued intuitionistic fuzzy Bi-Magic labelings. These constants are defined as follows:

To find k_1 and l_1:

Case (i) for i=1

$Bm_\mu^-(S_{1,n}) = \mu_\alpha^-(v) + \mu_\beta^-(v,u_1) + \mu_\alpha^-(u_1)$

$= \sum \frac{\mu_\alpha^-(u_i)}{n} - (n+1)\delta_1/1 \le i \le n +$
$Max(\mu_\alpha^-(v), \mu_\alpha^-(u_1)) - Min(\mu_\alpha^-(v), \mu_\alpha^-(u_1)) - (n+3)\delta_1 + (2n-1)\delta_2$

$Bm_\mu^+(S_{1,n}) = \mu_\alpha^+(v) + \mu_\beta^+(v,u_1) + \mu_\alpha^+(u_1)$

$= \sum \frac{\mu_\alpha^+(u_i)}{n} - (n+1)\delta_1/1 \le i \le n +$
$Max(\mu_\alpha^+(v), \mu_\alpha^+(u_1)) - Min(\mu_\alpha^+(v), \mu_\alpha^+(u_1)) - (n-1)\delta_1 + (2n-1)\delta_2$

$Bm_\gamma^-(S_{1,n}) = \gamma_\alpha^-(v) + \gamma_\beta^-(v,u_1) + \gamma_\alpha^-(u_1)$

$= \sum \frac{\gamma_\alpha^-(u_i)}{n} - (n+1)\delta_1/1 \le i \le n +$
$Max(\gamma_\alpha^-(v), \gamma_\alpha^-(u_1)) - Min(\gamma_\alpha^-(v), \gamma_\alpha^-(u_1)) - (n+3)\delta_1 + (2n-1)\delta_2$

$Bm_\gamma^+ \ S_{1,n} \ = \gamma_\alpha^+ \ v \ + \gamma_\beta^+(v,u_1) + \gamma_\alpha^+ \ u_1$

$= \sum \frac{\gamma_\alpha^+(u_i)}{n} - \ n+1 \ \delta_1/1 \le i \le n \ +$

$Max \ \gamma_\alpha^+ \ v \ , \gamma_\alpha^+ \ u_1 \ - Min \ \gamma_\alpha^+ \ v \ , \gamma_\alpha^+ \ u_1 \ - \ n-1 \ \delta_1 + (2n-1)\delta_2$

Case (ii) for i=2

$Bm_\mu^- \ S_{1,n} \ = \mu_\alpha^- \ v \ + \mu_\beta^-(v,u_2) + \mu_\alpha^- \ u_2$

$= \sum \frac{\mu_\alpha^-(u_i)}{n} - \ n+1 \ \delta_1/1 \le i \le n \ +$

$Max \ \mu_\alpha^- \ v \ , \mu_\alpha^- \ u_2 \ - Min \ \mu_\alpha^- \ v \ , \mu_\alpha^- \ u_2 \ - \ n+3 \ \delta_1 + 2n\delta_2$

$Bm_\mu^+ \ S_{1,n} \ = \mu_\alpha^+ \ v \ + \mu_\beta^+(v,u_2) + \mu_\alpha^+ \ u_2$

$= \sum \frac{\mu_\alpha^+(u_i)}{n} - \ n+1 \ \delta_1/1 \le i \le n \ +$

$Max \ \mu_\alpha^+ \ v \ , \mu_\alpha^+ \ u_2 \ - Min \ \mu_\alpha^+ \ v \ , \mu_\alpha^+ \ u_2 \ - \ n-1 \ \delta_1 + 2n\delta_2$

$Bm_\gamma^- \ S_{1,n} = \sum \frac{\gamma_\alpha^-(u_i)}{n} - \ n+1 \ \delta_1/1 \le i \le n \ +$

$Max \ \gamma_\alpha^- \ v \ , \gamma_\alpha^- \ u_2 \ - Min \ \gamma_\alpha^- \ v \ , \gamma_\alpha^- \ u_2 \ - \ n+3 \ \delta_1 + 2n\delta_2$

$Bm_\gamma^+ \ S_{1,n} = \sum \frac{\gamma_\alpha^+(u_i)}{n} - \ n+1 \ \delta_1/1 \le i \le n \ +$

$Max \ \gamma_\alpha^+ \ v \ , \gamma_\alpha^+ \ u_2 \ - Min \ \gamma_\alpha^+ \ v \ , \gamma_\alpha^+ \ u_2 \ - \ n-1 \ \delta_1 + 2n\delta_2$

Case (iii) for i=3

$Bm_\mu^- \ S_{1,n} \ = \mu_\alpha^- \ v \ + \mu_\beta^-(v,u_3) + \mu_\alpha^- \ u_3$

$= \sum \frac{\mu_\alpha^-(u_i)}{n} - \ n+1 \ \delta_1/1 \le i \le n \ +$

$Max \ \mu_\alpha^- \ v \ , \mu_\alpha^- \ u_3 \ - Min \ \mu_\alpha^- \ v \ , \mu_\alpha^- \ u_3 \ - \ 2\delta_1 + (2n+1)\delta_2$

$Bm_\mu^+ \ S_{1,n} \ = \mu_\alpha^+ \ v \ + \mu_\beta^+(v,u_3) + \mu_\alpha^+ \ u_3$

$= \sum \frac{\mu_\alpha^+(u_i)}{n} - \ n+1 \ \delta_1/1 \le i \le n \ +$

$Max \ \mu_\alpha^+ \ v \ , \mu_\alpha^+ \ u_3 \ - Min \ \mu_\alpha^+ \ v \ , \mu_\alpha^+ \ u_3 \ - \ 2\delta_1 + (2n+1)\delta_2$

$Bm_\gamma^- \ S_{1,n} = \sum \frac{\gamma_\alpha^-(u_i)}{n} - \ n+1 \ \delta_1/1 \le i \le n \ +$

$Max \ \gamma_\alpha^- \ v \ , \gamma_\alpha^- \ u_3 \ - Min \ \gamma_\alpha^- \ v \ , \gamma_\alpha^- \ u_3 \ - \ 2\delta_1 + (2n+1)\delta_2$

$Bm_\gamma^+ \ S_{1,n} = \sum \frac{\gamma_\alpha^+(u_i)}{n} - \ n+1 \ \delta_1/1 \le i \le n \ +$

$Max \ \gamma_\alpha^+ \ v \ , \gamma_\alpha^+ \ u_3 \ - Min \ \gamma_\alpha^+ \ v \ , \gamma_\alpha^+ \ u_3 \ - \ 2\delta_1 + (2n+1)\delta_2$

To find k_2 and l_2:

Case (iv) for $4 \le i \le n$

$Bm_\mu^- \ S_{1,n} \ = \mu_\alpha^- \ v \ + \mu_\beta^-(v,u_i) + \mu_\alpha^- \ u_i$

$= \sum \frac{\mu_\alpha^-(u_i)}{n} - \ n+1 \ \delta_1/1 \le i \le n \ +$

$Max \ \mu_\alpha^- \ v \ , \mu_\alpha^- \ u_i \ - Min \ \mu_\alpha^- \ v \ , \mu_\alpha^- \ u_i \ - 2\delta_1 + 10\delta_2$

$Bm_\mu^+ \ S_{1,n} \ = \mu_\alpha^+ \ v \ + \mu_\beta^+(v,u_i) + \mu_\alpha^+ \ u_i$

$= \sum \frac{\mu_\alpha^+(u_i)}{n} - \ n+1 \ \delta_1/1 \le i \le n \ +$

$Max \ \mu_\alpha^+ \ v \ , \mu_\alpha^+ \ u_i \ - Min \ \mu_\alpha^+ \ v \ , \mu_\alpha^+ \ u_i \ + 2\delta_1 + 10\delta_2$

$Bm_\gamma^- \ S_{1,n} = \sum \frac{\gamma_\alpha^-(u_i)}{n} - \ n+1 \ \delta_1/1 \le i \le n \ +$

$Max \ \gamma_\alpha^- \ v \ , \gamma_\alpha^- \ u_i \ - Min \ \gamma_\alpha^- \ v \ , \gamma_\alpha^- \ u_i \ - 2\delta_1 + 10\delta_2$

$Bm_\gamma^+ \ S_{1,n} = \sum \frac{\gamma_\alpha^+(u_i)}{n} - \ n+1 \ \delta_1/1 \le i \le n \ +$

$Max \ \gamma_\alpha^+ \ v \ , \gamma_\alpha^+ \ u_i \ - Min \ \gamma_\alpha^+ \ v \ , \gamma_\alpha^+ \ u_i \ + 2\delta_1 + 10\delta_2$

Hence from the above cases we can say that the Star graph is always an interval-valued intuitionistic fuzzy Bi-magic graph.

Theorem 3.6

In an interval-valued intuitionistic fuzzy Bi-Magic graph , the interval membership and non- membership values are mutually inclusive.

Proof:

The Intuitionistic fuzzy labeled graph assigns some membership and non- membership values for its vertices and edges which preserves bijective mapping. Then we can find some δ which can be added to the corresponding vertices and edges. Hence, we obtained the Bi-Magic sum for each pair of vertices and related edges. That Bi-Magic sum is taken as the upper limits for the interval. To get the lower limits of the interval, we just multiply all the obtained upper limits by 0.1.

Continuing this process, we find that the interval-valued intuitionistic fuzzy graph satisfies the conditions of Bi-Magic labeling. But the resultant interval need not be mutually disjoint as we choose the length of the interval as arbitraryselection.

Proposition 3.7

A Path graph ($4 \le n \le 5$) with an Intuitionistic fuzzy labeling of vertices and edges can be transformed into an interval-valued intuitionistic fuzzy Bi-Magic graph.

Proof:

In a Path graph ($4 \le n \le 5$), for vertex v_1 and edge v_1v_2 , we allocate δ=0.02 for the membership and non-membership values and for the rest of the vertices and edges of membership and non-membershipvalues , we allocate δ=0.01.

Adding these values of δ to the corresponding vertices and edges, we obtained two constant values that are assumed as an upper limit for the interval.

(ie)., $\mu_\alpha^+ \ v_i \ + \mu_\beta^+(v_i,v_j) + \mu_\alpha^+ \ v_j$ = k_1 and k_2

$\gamma_\alpha^+ \ v_i \ + \gamma_\beta^+(v_i,v_j) + \gamma_\alpha^+ \ v_j$ = l_1 and l_2 for all v_i, v_j □ V and on multiplying all the upper limits by 0.1, we get

$\mu_\alpha^- \ v_i \ + \mu_\beta^-(v_i,v_j) + \mu_\alpha^- \ v_j$ = k_1 and k_2

$\gamma_\alpha^- \ v_i \ + \gamma_\beta^-(v_i,v_j) + \gamma_\alpha^- \ v_j$ = l_1 and l_2 for all v_i, v_j □ V which satisfy the conditions of an interval-valued intuitionistic fuzzy Bi-Magic graph (The length of the interval is chosen arbitrarily).

Example 3.8

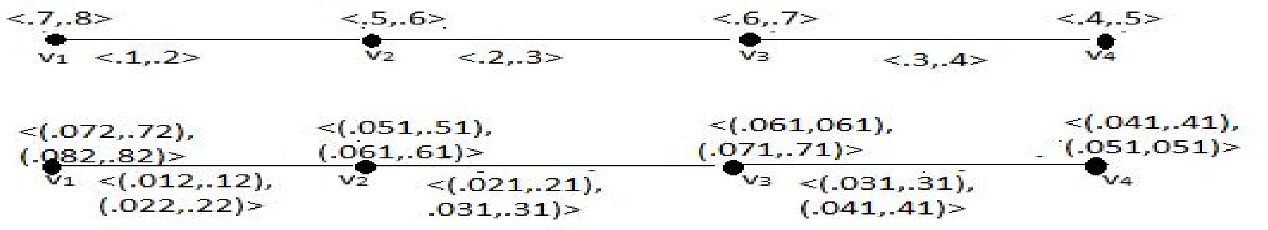


Figure 3 Intuitionistic Fuzzy Bi-Magic graph and corresponding Interval valued Intuitionistic Bi-Magic fuzzy graph

Consider the Path P_4 with intuitionistic fuzzy labeling which is not Bi-Magic. Assign δ=0.02 for vertex v_1 and edge v_1v_2 and assign δ=0.01 for the rest of the vertices and edges.

Then we obtain,

$$\mu_\alpha^+ \ v_1 \ + \mu_\beta^+(v_1, v_2) + \mu_\alpha^+ \ v_2 \ = 1.35$$

$\mu_\alpha^+ \ v_i \ + \mu_\beta^+(v_i, v_j) + \mu_\alpha^+ \ v_j \ = 1.33$ for i=2,3,4 and j=3,4.

$$\gamma_\alpha^+ \ v_1 \ + \gamma_\beta^+(v_1, v_2) + \gamma_\alpha^+ \ v_2 \ = 1.65$$

$\gamma_\alpha^+ \ v_i \ + \gamma_\beta^+(v_i, v_j) + \gamma_\alpha^+ \ v_j \ = 1.63$ for i=2,3,4 and j=3,4

This is a Bi-Magic labeling satisfying the conditions of interval-valued intuitionistic fuzzy graph.

Now, on multiplying all the upper limits by 0.1, we get the interval-valued intuitionistic fuzzy Bi-Magic graph, thus obtaining the lower limits of the membership and non-membership degrees.

Conclusion

In this Paper, we introduced the concept of an Interval valued intuitionistic fuzzy Bi-Magic labeling. Interval valued Intuitionistic fuzzy Bi-Magic labeling for Cycle ,Star and Path graphs have been discussed. The upper and lower limits of the membership and non-membership values are attained for this new type of labeling.

We further extended this study on some more special classes of graphs.

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