



BLOCK CHAIN FUZZY GRAPH

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A B S T R A C T

RESEARCH ARTICLE

In this paper, we have introduced the Block chain in fuzzy graphs named as Block chain Fuzzy graph. The relation between the fuzzy graphs and block chains are combined and established. Some of the basic Properties & concepts related to the Block chain Fuzzy graphs has also been presented.

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1. Introduction:

1.1 -Fuzzy Graph

Fuzzy Logic has developed into a large and deep subject. Zadeh [1],[4] addresses the terminology and stress the fuzzy graph are the generalization of the calculi of crisp graphs. Several other formulations of fuzzy graph problems have appeared in the literature. The first definition of fuzzy graph by Kaufmann [5] in 1975 who considered fuzzy relation on fuzzy sets and developed the theory of fuzzy graphs. During the same time Yeh and Banh[2] in 1975 also introduced fuzzy graph independently and studied various connectedness concepts. The degree of the vertex in fuzzy graph was discussed by Nagoorgani and Radha[3].

1.2 - Bitcoin

Bitcoin is a crypto currency and worldwide payment system[11]. It is the initial decentralized digital currency, as the system works without a central bank or single administrator[11]. The system is peer-to-peer and transactions take place between users directly through the use of cryptography, without an intermediary[11]. These transactions are proved by network nodes and recorded in a public distributed ledger called a block chain. Bitcoin was developed by an unknown person or group of people under the name Satoshi Nakamoto[11] and released as open-source software in 2009[11].

Bitcoins are created as a reward for a process known as mining. They can be exchanged for other currencies[11], products, and services. As of February 2015, over 100,000 merchants and vendors accepted bitcoin as payment[11]. Research submitted by the University of Cambridge evaluates that in 2017, there are 2.9 to 5.8 million unique users using a crypto currency wallet. Among that most of them are using Bitcoin[11].

1.3 – Block Chain

The first block chain was conceptualized in 2008 by an anonymous person or group recognized as Satoshi Nakamoto and implemented in 2009 as a core component of bitcoin where it serves as the public ledger for all transactions[10].

The discovery of the Blockchain[9] for the bitcoin [11] made it the first digital currency to solve the double spending problem without the need of a trusted authority or central server. As a distributed ledger, a block chain is typically managed by a peer-to-peer network collectively adhering to a protocol for validating new blocks. Once recorded, the data in given block cannot be changed retroactively without any adjustment of all subsequent blocks, which requires collision of the network majority. The first work on a cryptographically secured chain of blocks was described in 1991 by Stuart Haber and W. Scott Stornetta 1992. Bayer, Haber and Stornetta incorporated Merkle trees to the design, which improved its efficiency by allowing several documents to be collected into one block. By using a block chain, bitcoin became first digital currency to solve the double spending problem without requiring a trusted administrator and has been the inspiration for many additional applications. In this paper we are introducing Fuzzy Graphs using block chains.

2. Preliminaries:

Definition:2.1: Let V be a non empty set. A fuzzy graph is a pair of functions $G:(V, \mu)$ where μ is a fuzzy subset of V , μ is a symmetric fuzzy relation on V . (i. e) $\mu: V \times V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$ such that $\mu(u,v) = \min(\mu(u), \mu(v))$ for all u, v in V where \min stands for minimum. The Underlying crisp graph of the fuzzy graph $G:(V, \mu)$ is denoted as $G^*:(V, \mu^*)$ where $\mu^* = \{u \in V / \mu(u) > 0\}$, $\mu^* = \{(u,v) \in V \times V / \mu(u,v) > 0\}$.

Definition:2.2: A fuzzy graph $G:(V, \mu)$, with the underlying set V , the order of G is defined as $O(G)$ and it is denoted as $O(G) = \sum_{v \in V} \mu(v)$, where $v \in V$.

A fuzzy graph $G:(V, \mu)$, with the underlying set V , the size of G is defined as $S(G)$ and it is denoted as $S(G) = \{\mu(u,v) \mid u,v \in V\}$.

Definition:2.3: A fuzzy graph $G:(V, \mu)$, is said to be complete fuzzy graph if $\mu(u,v) = \min\{\mu(u), \mu(v)\}$ for all $u,v \in V$.

Definition:2.4: A fuzzy graph $G:(V, \mu)$, is said to be a regular Fuzzy graph if each vertex has same degree n , then G is said to be a regular fuzzy graph of degree n .

Definition:2.5: A fuzzy graph $G:(V, \mu)$ is said to be product complete fuzzy graph of G if $\mu(uv) = \mu(u) \times \mu(v) \forall u, v \in V$.

Definition:2.6: Let $G:(V, \mu)$ be a fuzzy graph. The degree of a vertex u in G defined by $d_G(u) = \sum_{v \in V} \mu(u,v)$ where $u,v \in V$.

Definition:2.7: Let $G:(V, \mu)$ be a fuzzy graph such that $\mu(u,v) = \frac{1}{2} \min\{\mu(u), \mu(v)\}$ for all $u,v \in V$. Then G is self complementary fuzzy graph.

Definition:2.8: Let V be a non empty set. A block chain fuzzy graph is a pair of functions $G:(V, \mu)$ where V is a fuzzy subset of V , μ is a symmetric fuzzy relation on V . (i.e.) $\mu: V \times V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$ such that $\mu(x,y) = \min\{\mu(x), \mu(y)\}$ for all u, v in V , with the following criterion.
 If $i \neq j$ then $[\mu(v_i, v_j) - \min\{\mu(v_i), \mu(v_j)\}] = 1$,
 If $i = j$ then $[\mu(v_i, v_j) - \max\{\mu(v_i), \mu(v_j)\}] = 1$,
 then $i=j$ then $[\mu(v_i, v_j) - \min\{\mu(v_i), \mu(v_j)\}] = 0$.

Definition:2.9: Let $G:(V, \mu)$ be a fuzzy graph such that $\mu(u,v) = \frac{1}{2} \min\{\mu(u), \mu(v)\}$ for all $u,v \in V$. Then G is self complementary block chain fuzzy graph.

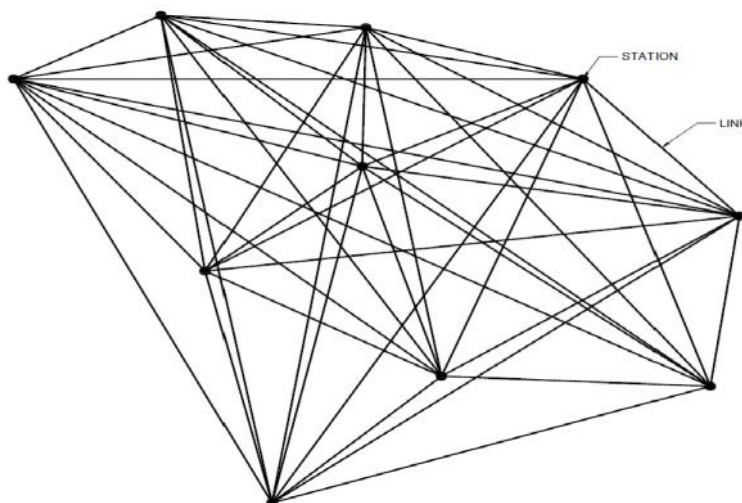


Fig-1: Distributed ledger in Block Chain

Illustration 2.10

Let us consider there are 10 persons in the blockchain and each of them doing transactions using bitcoin and each of them saving 30% and investing remaining 70% in bitcoin

- Person-1: investing 15 lakhs and doing 9 transactions
- Person-2: investing 10 lakhs and doing 9 transactions
- Person-3: investing 14 lakhs and doing 9 transactions
- Person-4: investing 7 lakhs and doing 9 transactions
- Person-5: investing 10.5 lakhs and doing 9 transactions
- Person-6: investing 9 lakhs and doing 9 transactions
- Person-7: investing 8 lakhs and doing 9 transactions
- Person-8: investing 5 lakhs and doing 9 transactions
- Person-9: investing 8.5 lakhs and doing 9 transactions
- Person-10: investing 5.5 lakhs and doing 9 transactions

Block Chain Fuzzy Graph

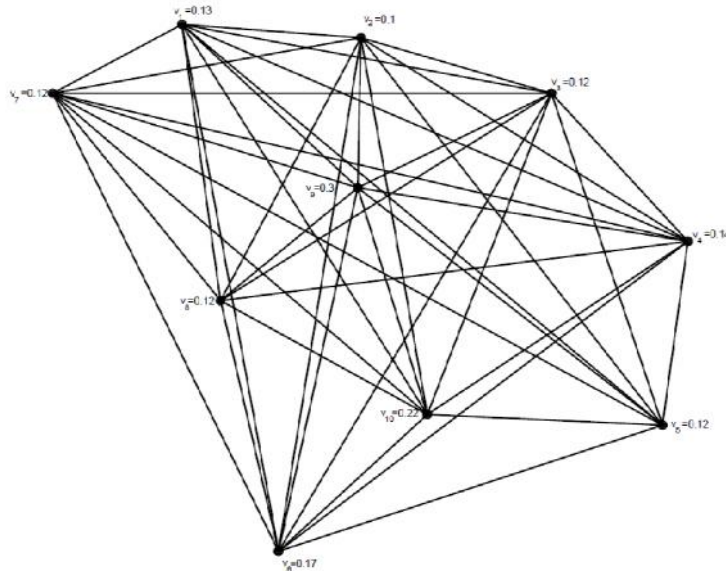


Fig-2: Blockchain fuzzy graph

For Example, consider that the Person-1 (v_1) has total amount of 15Lakhs in that 30% for his savings, remaining amount is invested as bitcoins, and he uses Blockchain for his cryptocurrencies (Bitcoins). The following are his transactions:-

- Transaction1: person-1 to person-2 = $0.1 \times 10,50000$ (v_1 to v_2) = 105000
- Transaction2: person-1 to person-3 = $0.1 \times 10,50000$ (v_1 to v_3) = 105000
- Transaction3: person-1 to person-4 = $0.12 \times 10,50000$ (v_1 to v_4) = 126000
- Transaction4: person-1 to person-5 = $0.12 \times 10,50000$ (v_1 to v_5) = 126000
- Transaction5: person-1 to person-6 = $0.09 \times 10,50000$ (v_1 to v_6) = 94500
- Transaction6: person-1 to person-7 = $0.12 \times 10,50000$ (v_1 to v_7) = 126000
- Transaction7: person-1 to person-8 = $0.13 \times 10,50000$ (v_1 to v_8) = 136500
- Transaction8: person-1 to person-9 = $0.1 \times 10,50000$ (v_1 to v_9) = 105000
- Transaction9: person-1 to person-10 = $0.12 \times 10,50000$ (v_1 to v_{10}) = 126000

Table-1: $\mu(v_i, v_j) = \min[\mu(v_i), \mu(v_j)]$

		0.13	0.1	0.12	0.14	0.12	0.17	0.12	0.25	0.3	0.22	(μ_i, μ_j)
		V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	
0.13	V1	0	0.1	0.1	0.12	0.12	0.09	0.12	0.13	0.1	0.12	1
0.1	V2	0.1	0	0.1	0.12	0.12	0.09	0.12	0.13	0.1	0.12	1
0.12	V3	0.1	0.1	0	0.09	0.16	0.14	0.1	0.09	0.1	0.12	1
0.14	V4	0.12	0.12	0.09	0	0.11	0.15	0.1	0.09	0.1	0.12	1
0.12	V5	0.12	0.12	0.16	0.11	0	0.05	0.2	0	0.02	0.04	1
0.17	V6	0.09	0.09	0.14	0.15	0.05	0	0.12	0.12	0.1	0.14	1
0.12	V7	0.12	0.12	0.1	0.1	0.2	0.12	0	0.12	0.1	0.02	1
0.25	V8	0.13	0.13	0.09	0.09	0	0.12	0.12	0	0.1	0.22	1
0.3	V9	0.1	0.1	0.1	0.1	0.2	0.1	0.1	0.1	0	0.1	1
0.22	V10	0.12	0.12	0.12	0.12	0.04	0.14	0.02	0.22	0.1	0	1
(μ_i, μ_j)		1	1	1	1	1	1	1	1	1	1	

Table-2: $\mu(v_i, v_j) = \max[\mu(v_i), \mu(v_j)]$

		0.13	0.1	0.12	0.14	0.12	0.17	0.12	0.25	0.3	0.22	(μ_i, μ_j)
		V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	
0.13	V1	0	0.12	0.09	0.13	0.09	0.13	0.1	0.13	0.12	0.09	1
0.1	V2	0.12	0	0.1	0.1	0.1	0.13	0.12	0.1	0.1	0.13	1
0.12	V3	0.09	0.1	0	0.1	0.13	0.09	0.12	0.13	0.11	0.13	1
0.14	V4	0.13	0.1	0.1	0	0.1	0.1	0.12	0.1	0.13	0.12	1
0.12	V5	0.09	0.1	0.13	0.1	0	0.16	0.12	0	0.15	0.15	1
0.17	V6	0.13	0.13	0.09	0.1	0.16	0	0.09	0.1	0.1	0.1	1
0.12	V7	0.1	0.12	0.12	0.12	0.12	0.09	0	0.12	0.11	0.11	1
0.25	V8	0.13	0.1	0.13	0.1	0	0.1	0.12	0	0.16	0.16	1
0.3	V9	0.12	0.1	0.11	0.13	0.15	0.1	0.11	0.16	0	0.02	1
0.22	V10	0.09	0.13	0.13	0.12	0.15	0.1	0.11	0.16	0.02	0	1
(μ_i, μ_j)		1	1	1	1	1	1	1	1	1	1	

Conclusion: From the above two tables we can observe that the sum of all the edges of particular vertices is equal to 1.

4. Properties of Blockchain Fuzzy Graph:

4.1 Degree of Fuzzy Graph

Let $G=(V, \mu)$ be a fuzzy graph, the degree of a vertices v_1 is

$$D_G(v_1) = \sum_{v_i \neq v_j \in X} \mu(v_i v_j)$$

Example:

In the Fig-2 the following vertices have these degree.
 $d(v_1)=9$; $d(v_2)=9$; $d(v_3)=9$; $d(v_4)=9$; $d(v_5)=8$; $d(v_6)=9$;
 $d(v_7)=9$; $d(v_8)=8$; $d(v_9)=9$; $d(v_{10})=9$

4.2 Maximum Degree of Fuzzy graph

Let $G=(V, \mu)$ be a fuzzy graph, the maximum degree of G is denoted as $D(G) = \max \{ D_G(v_i); v_i \in V \}$.

Example :

In the Fig-2, the following vertices have the maximum degree.

$d(v_1)$; $d(v_2)$; $d(v_3)$; $d(v_4)$; $d(v_6)$; $d(v_7)$; $d(v_8)$; $d(v_9)$;
 $d(v_{10})$

4.3 Order of Fuzzy Graph

Let $G=(V, \mu)$ be a fuzzy graph, the Order of fuzzy graph G is denoted as $f_0 G = \sum_{v_i \in V} \sigma(v_i)$

Example:

In the Fig-2, the order of fuzzy graph is 10.

4.4 Size of Fuzzy graph

Let $G=(V, \mu)$ be a Fuzzy Graph, the fuzzy size of the graph G is denoted as $f_s G = \sum_{v_i v_j \in X} \mu(v_i v_j)$

4.5 Product Fuzzy graph

If fuzzy graph $G = (V, \mu)$ is said to be a product blockchain fuzzy graph if $\mu(uv) = \mu(u) \times \mu(v)$ if $u \neq v$

Example :

In the fig-2

$\mu(v_1, v_4)=0.12$, $\mu(v_1)=0.13$; $\mu(v_4)=0.12$

therefore $\mu(v_1) \times \mu(v_4) = 0.0156 = \mu(v_1, v_4)$

5. Remarks:

- 1) Blockchain fuzzy graph cannot be a complete fuzzy graph, because it is not necessary that all can do equal number of transactions.
- 2) Blockchain fuzzy graph is not a regular fuzzy graph.

- 3) Blockchain doesn't satisfy the equivalence relations.
- 4) Blockchain fuzzy graph cannot be a complete product fuzzy graph.
- 5) We cannot find the complement in any Blockchain fuzzy graph.

Conclusion

In this paper we have introduced a new fuzzy graph called block chain fuzzy graph and illustrated with some calculations. Also, the related results have been studied and proved. The above discussed block chain fuzzy graph presented from the blockchain which is used for transactions of crypto currencies like Bitcoin, Ethereum etc., In the forthcoming paper on this block chain fuzzy graph we will extend the other valid concepts with more examples.

References:

- [1] "Fuzzy graphs and its applications to cognitive and decision process", Academic press, New York,(1975)by A.Rosen Field, Fuzzy Graphs in: L.A Zadeh,K.S Fu,K.Tanaka and M.Shimura, "
- [2] R.T.Yeh and S.Y.Banh, "Fuzzy relations, fuzzy graphs and their applications to clustering analysis", "In Fuzzy sets and their applications to cognitive and decision process".
- [3] L.A.Zadeh, K.S.Fu.M.Shimuraeds : Academic Press, New York, (1975).
- [4] Nagoor Gani. A and Radha. K, "On Regular Fuzzy Graphs", Journal of Physical Sciences, (2008).
- [5] Zadeh, L.A., "Fuzzy Sets", Information and Control, (1965).
- [6] Kaufmann, "A Introduction a la Theorie des sous-ensembles flous", Vol. 1, Masson Paris,(1973).
- [7] A. Nagoor Gani and J. Malarvizhi, "Some aspects of total fuzzy graph", International Journal on mathematics and computations Tiruchirappalli (2009).
- [8] T.Pathinathan, J.Jon Arockiaraj, J.Jesintha rosline. "Hesistancy Fuzzy graph", India Journal of science and technology.(2015)
- [9] Mourad oqla Massa'deh and Natheer k.Gharaibeh , "Some properties of Fuzzy graph". Advances in fuzzy mathematics (2011).
- [10] A gentle introduction to block chain technology article from www.bitsonblock.net published on 9 september (2015).
- [11] "Understanding of Block chain" from www.blockgeeks.com.
- [12] "Bitcoinand Block chain definitions" from www.WikiPedia.com.
