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TESTING EXPONENTIALITY AGAINST HARMONIC NEW BETTER THAN USED IN EXPECTATION AGEING CLASSES

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A B S T R A C T **ESEARCH ARTICLE**

In this paper, we present a test statistic for testing exponentiality against Harmonic New Better than Used in Expectation (HNBUE) ageing class based on the Excess Wealth Transform. Considering a sample of size n=10(5)30(10)100, Pitman Asymptotic Relative Efficiency is employed to assess the performance of the test. For HNBUE classes of life distributions, our tests are better than, or well comparable with, other available tests. Numerical simulations are carried out and the critical values of the proposed test are presented.

AMS Subject Classification: 60K10.

1. Introduction

The exponential density is probably one of the most widely used distributions in practice. Important application areas of the exponential distribution include survival analysis and reliability theory.

Some of the aging classes are developed Bhattacharjee and Sethuraman [1990], Bhattacharjee et al.[2000], Cao and Wang [1991], Franco et al. [2001], Kaur et al. [1994], Li et al.[2000], Muller and Stoyan [2002], and Shaked and Shanthikumar [1994]. Most of these authors address probabilistic properties of the ageing classes they study. Abouammoh et al. [2000] introduced the NRBU, RNBU, NRBUE, HNRBUE classes of life distributions and studied the relation between them. Abouammoh and Khalique (1998) investigated a test statistic of NRBU based on Total Time on Test (TTT)-transform empirically.

More applications can be found in Kochar and Li [2002] and Shaked, Ahmad, Kayid and Li [2005]. Basu and Ebrahimi (1985) proposed two tests for testing exponentiality against HNBUE (HNWUE) alternatives, both constructed by using the definitions of the TTT-transform and the scaled TTT-transform. These definitions were also used by Klefsj ö (1983) for testing against IFR or IFRA alternatives (Section 4.3, p. 77). One-sided tests for HNBUE alternatives were developed by Kochar and Deshpande (1985) and Aly (1992), while two-sided tests for HNBUE (HNWUE) alternatives were developed by Klefsj ö (1983). Singh and Kochar (1986), Hendi, Al-Nachawati and Alwasel (1998) and Klar (2000). The TTT-statistic, is consistent against the larger class of HNBUE alternatives. Hollander and Proschan (1975) have already proven that the TTT-statistic is consistent against NBUE alternativcs, and not only against IFR alternatives. Klefsjo (1983).

The rest of the paper is organized as follows : In section 2, we present some preliminaries of these ageing classes of life distributions. In section 3, we present the concept of excess wealth transform and some theorems. In section 4, Testing against exponential alternatives, in section 5, Monte carlo Method. In section 6, Power of different alternatives. In section 7, PAE, in section Finally, conclusion is given in section 8.

2. Definitions and Some Related Concepts

In reliability theory, ageing life is usually characterized by a nonnegative random variable $X \geq 0$ with cumulative distribution function (cdf) $F(\cdot)$ and survival function $\overline{F}(\cdot) = 1 - F(\cdot)$. For any random variable X, let

$X_t = [X - t \mid X > t], \quad t \in \{x : F(x) < 1\}$

denote a random variable whose distribution is the same as the conditional distribution of $X - t$ given $X > t$, where X is the lifetime of a device, X_t can be regarded as the residual lifetime of the device at time t, given that the device has survived upto time t. Its survival function is

$$\overline{F}_t(x) = \frac{\overline{F}(t+x)}{\overline{F}(t)}, \quad \overline{F}(t) > 0,$$

A life distribution $F(\cdot)$ or its survival function $\overline{F}(\cdot)$ with support $S = \{t : \overline{F}(t) > 0\}$ and finite mean $« = \int_0^\infty \overline{F}(t)\, dt$ is said to be Harmonic New Better than Used in Expectation (HNBUE), if

$$\int_u^\infty \overline{F}(u)\, du \leq « \ \exp(-u/«),$$

for all $u \geq 0$.

3. Excess Wealth Transform

In this section, we present the concept of Excess Wealth Transform and some known definitions of certain aging classes.

Let $F(\cdot)$ be a life distribution with finite mean «

The Excess Wealth Transform H_F^{-1} of $F(\cdot)$ is defined by

$$H_F^{-1}(p) = \int_{F^{-1}(p)}^{\infty} \overline{F}(s)\ ds \ \ for\ \ 0 \le p \le 1, (3.0)$$

where $F^{-1}(p) = \inf\{x : F(x) > p\}$. Since the mean of $F(\cdot)$ is given by

$$« = H_F^{-1}(0) = \int_0^{\infty} \overline{F}(s)\ ds$$

the transform

$$\cdot_F(p) = \frac{H_F^{-1}(p)}{H_F^{-1}(0)} = \frac{1}{«} \int_{F^{-1}(p)}^{\infty} \overline{F}(s)\ ds \ \ for\ \ 0 \le p \le 1,$$

is scale invariant and is called scaled excess wealth transform.

Let F be a lifetime distribution function Then F is HNBUE if the following holds:

$$\frac{«}{«+1} - \frac{1}{H_F^{-1}(0)} \int_0^{\infty} H_F^{-1}(F(u))\ dF(u) \ge 0$$

Proof. Suppose that F is HNBUE, then

$$\int_u^{\infty} \overline{F}(x)\ dx \le « \ \exp(-u/«),$$

For $u \ge 0$, integrating with respect to $F(u)$ where $F(u) = 1 - \exp(-u)$ on both side over $[0, \infty)$ we have

$$\int_0^{\infty} H_F^{-1}(F(u))\ dF(u) \le « \int_0^{\infty} \exp(-u/«)\ dF(u).$$

$$\frac{1}{H^{-1}(0)} \int_0^{\infty} H_F^{-1}(F(u))\ dF(u) \le \int_0^{\infty} \exp(-u/«) \exp(-u)\ du$$

On simplification we obtain

$$\frac{«}{«+1} - \frac{1}{H_F^{-1}(0)} \int_0^{\infty} H_F^{-1}(F(u))\ dF(u) \ge 0$$

This completes the proof of the theorem.

4. Testing Against HNBUE Alternatives

Suppose that $X_1, X_2, \cdots, X_n$ represent a random sample from a population with distribution $F(\cdot)$. We wish to test the null hypothesis

- $H_0 : F = 1 - \exp(-ª\ x)$ for $x \ge 0$, and ª unknown.
- $H_1 : F$ is HNBUE, but not exponential.

Using Theorem (3.1), we use the following $\Delta(F_n)$ as a measure of departure from H_0 in favor of H_1, where

$$\Delta(F_n) = \frac{«}{«+1} - \int_0^{\infty} \frac{H_F^{-1}(F_n(t))}{H_F^{-1}(0)}\ dF_n(t) \quad (5.1)$$

Specifically, $X_{(1)}, X_{(2)}, \cdots, X_{(n)}$ denotes the order statistics based on the random sample of size n from F (assume that $X_{(0)} = 0$). Therefore, by straightforward computation

$$\Delta(F_n) = \frac{n\overline{X}}{n\overline{X}+1} - \frac{1}{n} \frac{\sum_{j=2}^{n} (j-1)\ D_j}{\sum_{j=1}^{n} D_j},$$

where $D_j = (n - j + 1)\ (X_{(j)} - X_{(j-1)})$ for $j = 1, 2, \cdots, n,$ are the corresponding normalized spacings.

Table 1: Estimated critical values for the statistic $\Delta(F_n)$

n / ~	0.005	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99	0.995
10	0.2175	0.2394	0.269	0.298	0.331	0.569	0.600	0.628	0.657	0.680
15	0.2745	0.2910	0.323	0.345	0.374	0.561	0.587	0.610	0.640	0.654
20	0.3160	0.3301	0.352	0.370	0.395	0.557	0.580	0.600	0.622	0.637
25	0.3318	0.3498	0.368	0.386	0.407	0.554	0.574	0.592	0.612	0.625
30	0.3467	0.3634	0.381	0.397	0.416	0.551	0.567	0.583	0.602	0.616
40	0.3706	0.3845	0.401	0.413	0.430	0.547	0.564	0.576	0.596	0.608
50	0.3869	0.3953	0.410	0.422	0.437	0.541	0.556	0.567	0.583	0.597
60	0.3936	0.4033	0.417	0.430	0.444	0.539	0.552	0.565	0.579	0.589
70	0.4043	0.4140	0.426	0.437	0.449	0.537	0.549	0.559	0.570	0.578
80	0.4135	0.4213	0.433	0.442	0.453	0.536	0.547	0.557	0.569	0.579
90	0.4137	0.4228	0.434	0.444	0.455	0.534	0.544	0.553	0.564	0.573
100	0.4215	0.4290	0.439	0.447	0.459	0.531	0.542	0.551	0.561	0.568

5. Monte Carlo Results

In practice, small sample sizes often arises. Critical values of the $\Delta(F_n)$ statistic are computed for 10000 random samples by Monte Carlo simulation for different significance levels and form sample sizes n=10(5)30(10)100, as given Table 1, using simulation techniques. Since the statistic is scale invariant, we can take the scale parameter ª to be unity, without loss of generality, while performing the simulation.

Table 2 Power of the $\Delta(F_n)$ test computed on 10000 sample of size n and shape parameter of ¯ for the Weibull distribution with significance level of 0.05

n / ¯	1.2	1.3	1.4	1.6	1.75	1.8	2	3
10	0.999	0.999	1.000	1.000	1.000	1.000	1	1
15	1.000	1.000	1.000	1.000	1.000	1.000	1	1
20	1.000	1.000	1.000	1.000	1.000	1.000	1	1
25	1.000	1.000	1.000	1.000	1.000	1.000	1	1
30	1.000	1.000	1.000	1.000	1.000	1.000	1	1
40	1.000	1.000	1.000	1.000	1.000	1.000	1	1
50	1.000	1.000	1.000	1.000	1.000	1.000	1	1
60	1.000	1.000	1.000	1.000	1.000	1.000	1	1
70	0.002	0.014	0.068	0.197	0.766	0.984	1	1
80	1.000	1.000	1.000	1.000	1.000	1.000	1	1
90	1.000	1.000	1.000	1.000	1.000	1.000	1	1
100	1.000	1.000	1.000	1.000	1.000	1.000	1	1

Table 3 Power of the $\Delta(F_n)$ test computed on 10000 sample of size n and shape parameter of ¯ for the Gamma distribution with significance level of 0.05

n/¯	1.2	1.3	1.4	1.6	1.75	1.8	2	3
10	0.913	0.947	0.965	0.976	0.992	0.998	1.000	1.000
15	0.998	0.999	1.000	1.000	1.000	1.000	1.000	1.000
20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
70	0.000	0.001	0.004	0.012	0.077	0.246	0.743	0.96
80	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

6. Power of different alternatives

The power of the test under discussion is evaluated by Monte Carlo simulation. Power estimates are calculated as the proportion of 10000 Monte Carlo samples that resulted in the rejection of H_0 at significance level ~ = 5% for the alternative distributions considered. The standard errors of the estimated probabilities are less than or equal to $\sqrt{(0.25)/10000}$;0.005. We check whether these particular realizations of the test statistic reject the null hypothesis of exponentiality. The whole procedure is then repeated ten thousand times and the proportion of times the proposed statistic takes the correct decision of rejecting the null hypothesis H_0 is observed. Thus, the power of the test for different alternatives are estimated.

7. Pitman Asymptotic Efficiency (PAE)

The Pitman asymptotic efficiency of the class HNBUE was calculated using the Weibull and Gamma distributions The PAE is considered $\Delta(F_n)$ at 5% percentile and for commonly used distribution in reliability modeling, these distribution are

- The Weilbull distribution with density

$$\overline{F}_1(x) = ¯\ x^{¯-1} \exp(-x^{¯});\quad ¯ \ge 1,\ x \ge 0.$$

- The Gamma distribution with density

$$\bar{F}_2(x) = \frac{1}{\Gamma(\ )} x^{\ -1} \exp(-x);\quad \geq 1, x \geq 0.$$

In the simulation study, we compute the mean value of $\Delta(F_n)$ 10000 replications were performed for each distribution. Simulation results are summerized in Table 2 and Table 3. We make the following observations.

In Weibull distribution,

- if <1, Weibull distribution is in HNWUE.
- if >1, Weibull distribution is in HNBUE.

In Gamma distribution,

- if <1, Gamma distribution is in HNWUE.
- if >1, Gamma distribution is in HNBUE.

In this case the PAEs are respectively equal to 0.793 and 0.687. The corresponding values of the NBUE test of Hollander and Proschan (1975) are 0.866 and 0.289. Clearly our test is better and also much simpler to do. Ahmad and Mugdadi (2004) gave a test for the NBUC class which is smaller than one and reported PAEs values of 0.894 and 0.167. Thus our test is better and works for a much larger class, the HNBUE, when compared with NBUE or NBUC classes.

8. Conclusion

We have presented test a statistic for testing exponentiality against Harmonic New Better than Used in Expectation (HNBUE) ageing class based on the Excess Wealth Transform. Considering a sample of size $n = 10(5)30(10\ )100,$ Pitman Asymptotic Relative Efficiency is employed to assess the performance of the test. For HNBUE classes of life distributions, our tests are better than, or well comparable with, other available tests. Numerical simulations are carried out and the critical values of the proposed test is presented, and we conclude that findings our test is better and works for a much larger class, the HNBUE.

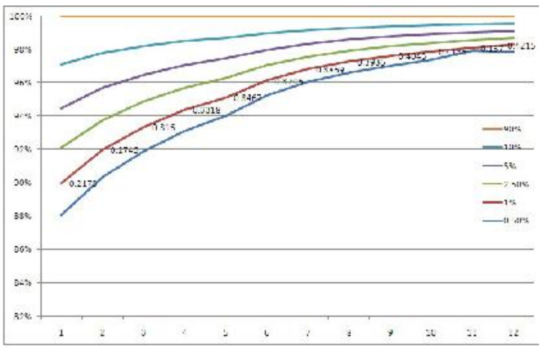

Figure 1: The relation between sample size and critical values $\Delta(F_n)$

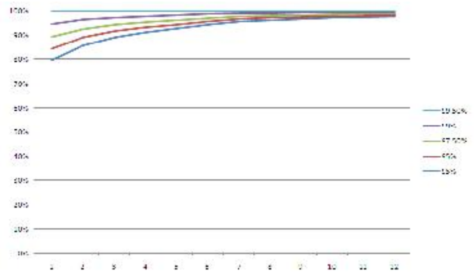

Figure 2: The relation between sample size and critical values $\Delta(F_n)$

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