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SYMMETRIC PENTAGONAL INTUITIONISTIC FUZZY NUMBER

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A B S T R A C T **RESEARCH ARTICLE**

Abstract: We define symmetric pentagonal intuitionistic fuzzy numbers (SPIFNs) and illustrate them with geometric representation. We discuss some of the properties of SPIFNs. We also propose important algebraic operations demonstrating them with a numerical example.

1. Introduction

Fuzzy set theory was introduced by Lotfi A. Zadeh in 1965 [6]. Since then fuzzy mathematics has been applied in various fields like decision making, pattern recognition, information processing and various analysis methodologies. Fuzzy numbers are one of the most used fuzzy tools in fuzzy applications. They are used to quantify certain concepts like close to five, nearly five, approximately five etc. Fuzzy numbers are basically fuzzy sets satisfying the properties of normality, convexity and piece-wise continuity. Lotfi A. Zadeh introduced the concept of fuzzy numbers in 1975 [7, 8, 9]. Triangular fuzzy numbers, trapezoidal fuzzy numbers and pentagonal fuzzy numbers are most popular forms of fuzzy numbers used in various applications. In 1994 Bustine, H., and Burillo, P., proposed the definition of intuitionistic fuzzy numbers [2]. In was based on the definition of intuitionistic fuzzy sets introduced by Atanassov in 1983[1]. Intuitionistic fuzzy numbers are very useful in capturing imprecise data. The triangular fuzzy numbers, trapezoidal fuzzy numbers have been extended to respective intuitionistic fuzzy numbers.

In the year 2014 T. Pathinathan and K. Ponnivalavan introduced pentagonal fuzzy number [4]. This number was introduced mainly to capture the variations that occur in the curve at different levels which triangular and trapezoidal fuzzy numbers fail to do so. Ponnivalavan, K. and Pathinathan, T. introduced intuitionistic pentagonal fuzzy numbers [5] in order to accommodate the value of non-membership in case of pentagonal fuzzy numbers. In 2012 R., Parvathi and C. Malathi introduced Symmetric trapezoidal intuitionistic fuzzy numbers [6]. In this paper we propose symmetric pentagonal intuitionistic fuzzy numbers. The shape of the membership functions of symmetric fuzzy numbers are simpler and more regular, which lead to less complex calculations and more natural interpretations. Therefore, introduction of SPIFNs can help us for less complex calculations and more natural interpretations.

In the second section, preliminary definitions are discussed. We also propose representation of symmetric pentagonal fuzzy numbers in this section. In the third section, we define symmetric pentagonal intuitionistic fuzzy numbers with geometric representation. We also discuss some of the properties of the number. Some of the algebraic operations are discussed with a numerical example in the fourth section.

2. Preliminaries

2.1. Fuzzy Number [3]: A fuzzy number is a fuzzy set $\tilde{A}$ on the real line $\Re$, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions:

1. Normality i.e. there exists an element x_0 such that $\mu_{\tilde{A}}(x_0) = 1$
2. Piece-wise continuity i.e.
$\forall \varepsilon > 0, \exists \delta > 0 \; such\,that \; \mu_{\tilde{A}}(x) - \mu_{\tilde{A}}(x_0) < \varepsilon$ $whenever \left| x - x_0 \right| < \delta$
3. convexity i.e.
$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \forall x_1, x_2 \in \Re$ and $\lambda \in [0,1]$

2.2. Intuitionistic fuzzy number [2] : An intuitionistic fuzzy number is an intuitionistic fuzzy set $\tilde{A}_I$ on the real line $\Re$ satisfying the following condition in addition to that of three conditions of fuzzy number:

1. Concavity for the non-membership function i.e.
2. $\nu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq \max(\nu_{\tilde{A}}(x_1), \nu_{\tilde{A}}(x_2)) \forall x_1, x_2 \in \Re$ and $\lambda \in [0,1]$

2.3. Pentagonal Fuzzy Number [4]: Pentagonal fuzzy number is a fuzzy set denoted as $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ and whose membership function is defined as [4]

$$\mu_{\tilde{A}_P}(x)=\begin{cases}\dfrac{(x-a_1)}{(a_2-a_1)} & for\, a_1\le x\le a_2\\ \dfrac{(x-a_2)}{(a_3-a_2)} & for\, a_2\le x\le a_3\\ 1 & for\, x=a_3\\ \dfrac{(a_4-x)}{(a_4-a_3)} & for\, a_3\le x\le a_4\\ \dfrac{(a_5-x)}{(a_5-a_4)} & for\, a_4\le x\le a_5\\ 0 & otherwise\end{cases}$$

2.4. Pentagonal Intuitionistic Fuzzy Number [5]

Pentagonal intuitionistic fuzzy number of a intuitionistic fuzzy set $\tilde{A}_I$ is denoted as

$\tilde{A}_{PI}$ = $(a_1, a_2, a_3, a_4, a_5; a'_1, a'_2, a'_3, a'_4, a'_5)$ and whose membership and non-membership function are defined as

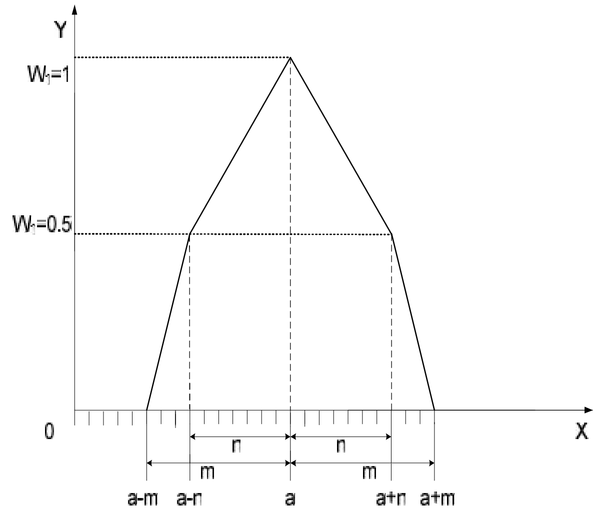
$$\mu_{\tilde{A}_{PI}}(x)=\begin{cases}0 & for\, x<a_1\\ \dfrac{(x-a_1)}{(a_2-a_1)} & for\, a_1\le x\le a_2\\ \dfrac{(x-a_2)}{(a_3-a_2)} & for\, a_2\le x\le a_3\\ 1 & for\, x=a_3\\ \dfrac{(a_4-x)}{(a_4-a_3)} & for\, a_3\le x\le a_4\\ \dfrac{(a_5-x)}{(a_5-a_4)} & for\, a_4\le x\le a_5\\ 0 & for\, x>a_5\end{cases}$$

$$\nu_{\tilde{A}_{PI}}(x)=\begin{cases}1 & for\, x<a'_1\\ \dfrac{(a'_2-x)}{(a'_2-a'_1)} & for\, a'_1\le x\le a'_2\\ \dfrac{(a'_3-x)}{(a'_3-a'_2)} & for\, a'_2\le x\le a'_3\\ 0 & for\, x=a'_3\\ \dfrac{(x-a'_3)}{(a'_4-a'_3)} & for\, a'_3\le x\le a'_4\\ \dfrac{(x-a'_4)}{(a'_5-a'_4)} & for\, a'_4\le x\le a'_5\\ 1 & for\, x>a'_5\end{cases}$$

2.5. Symmetric Pentagonal Fuzzy Number

Symmetric pentagonal fuzzy number is denoted as $\mu_{\tilde{A}_{SP}}=(a,n,n,m,m)$ and whose membership function is defined as

$$\mu_{\tilde{A}_{SP}}(x)=\begin{cases}0 & for\, x\le a-m\\ \dfrac{x-(a-m)}{m-n} & for\, a-m\le x\le a-n\\ \dfrac{x-(a-n)}{n} & for\, a-n\le x\le a\\ 1 & for\, x=a\\ \dfrac{(a+n)-x}{n} & for\, a\le x\le a+n\\ \dfrac{(a+m)-x}{m-n} & for\, a+n\le x\le a+m\\ 0 & for\, x\ge a+m\end{cases}$$



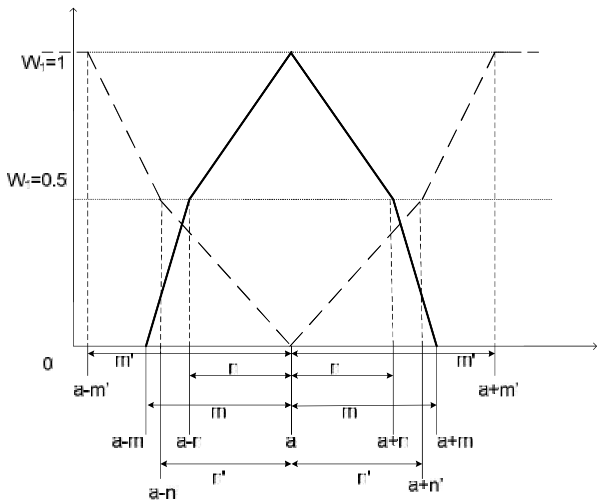
3. Symmetric Pentagonal Intuitionistic Fuzzy Number

Symmetric pentagonal intuitionistic fuzzy number is a intuitionistic fuzzy set $\tilde{A}_{SPI}$ = $(a_1, a_2, a_3, a_4, a_5; a'_1, a'_2, a'_3, a'_4, a'_5)$ membership and non-membership functions defined as:

$$\mu_{\tilde{A}_{SPI}}(x)=\begin{cases}\dfrac{x-(a-m)}{m-n} & for\, a-m\le x\le a-n\\ \dfrac{x-(a-n)}{n} & for\, a-n\le x\le a\\ 1 & for\, x=a\\ \dfrac{(a+n)-x}{n} & for\, a\le x\le a+n\\ \dfrac{(a+m)-x}{m-n} & for\, a+n\le x\le a+m\\ 0 & otherwise\end{cases}$$

$$³_{\tilde{A}_{SPI}}(x) = \begin{cases} \dfrac{(a-n')-x}{m'-n'} & for\ a-m' \le x \le a-n' \\ \dfrac{a-x}{n'} & for\ a-n' \le x \le a \\ 0 & for\ x = a \\ \dfrac{x-a}{n'} & for\ a \le x \le a+n' \\ \dfrac{x-(a+n')}{m'-n'} & for\ a+n' \le x \le a+m' \\ 1 & otherwise \end{cases}$$

3.1. Geometrical Representation of SPIFN



3.2. Properties of SPIFN

Property 1

If a SPIFN is of the form $\tilde{A}_{SPI} = [a, n, n, m, m; a, n', n', m', m']$ such that $m - n = m' - n'$ then $n \le n'\ and\ m \le m'$

$$for\ x \in [a-m, a]$$

$$«_{\tilde{A}_{SPI}}(x) + \frac{(a-n')-x}{m'-n'} \le 1\ as\ «_{\tilde{A}_{SPI}}(x) + ³_{\tilde{A}_{SPI}}(x) \le 1$$

$$\Rightarrow \frac{x-(a-m)}{m-n} + \frac{(a-n')-x}{m'-n'} \le 1 \qquad (1)$$

$$\Rightarrow \frac{m-n'}{m-n} \le 1$$

$$\Rightarrow m - n' \le m - n$$

$$\Rightarrow n \le n'$$

$$also\ from\ (1)$$

$$\frac{m-n'}{m'-n'} \le 1$$

$$\Rightarrow m - n' \le m' - n'$$

$$\Rightarrow m \le m'$$

Property 2

The membership function of $\tilde{C}_{SPI} = \tilde{A}_{SPI} + \tilde{B}_{SPI}$ is given by

The condition to transform SPIFN $\tilde{A}_{SPI} = [a, n, n, m, m; a, n', n', m', m' : n \le n'\ and\ m \le m']$ to a SPFN $\tilde{A}_{SP} = (a, n, n, m, m : n \le m)$ is that $n = n'\ and\ m = m'$ and $³_{\tilde{A}_{SPI}}(x)$ =1- $«_{\tilde{A}_{SPI}}(x)$

Property 3

The condition to transform SPIFN $\tilde{A}_{SPI} = [a, n, n, m, m; a, n', n', m', m' : n \le n'\ and\ m \le m']$ to a real number ~ is that $n = n' = 0\ and\ m = m' = 0$

3.3. Arithmetic operations on SPIFN

1. Addition:

Let $\tilde{A}_{SPI} = [a, n_1, n_1, m_1, m_1; a, n'_1, n'_1, m'_1, m'_1]$ and

$\tilde{B}_{SPI} = [b, n_2, n_2, m_2, m_2; b, n'_2, n'_2, m'_2, m'_2]$ be two SPIFN

$\tilde{C}_{SPI} = \tilde{A}_{SPI} + \tilde{B}_{SPI}$ is also SPIFN. It is given by

$\tilde{C}_{SPI}$ = [a+b, n_1+n_2, n_1+n_2, m_1+m_2, m_1+m_2; a+b, $n'_1+n'_2$, $n'_1+n'_2$, $m'_1+m'_2$, $m'_1+m'_2$]

Proof:

The Membership and non-membership function of $\tilde{C}_{SPI} = \tilde{A}_{SPI} + \tilde{B}_{SPI}$ can be established by

$(\sim, \ )$-cut method. ~ -cut for membership function of $\tilde{A}_{SPI}$ is [a+m_1(~ -1) - ~ n_1, ~ n_1+(a-n_1),(a+n_1) - ~ n_1, m_1(1-~)+~ n_1+a] for all~ in [0,1]. ~ -cut for membership function of $\tilde{B}_{SPI}$

is [b+m_2(~ -1) - ~ n_2, ~ n_2+(b-n_2),(b+n_2) - ~ n_2, m_2(1-~)+~ n_2+a] for all~ in [0,1]

Let u ∈ [a+m_1(~ -1) - ~ n_1, ~ n_1+(a-n_1),(a+n_1) - ~ n_1, m_1(1-~)+~ n_1+a] and

v∈ [b+m_2(~ -1) - ~ n_2, ~ n_2+(b+n_2),(a+n_2) - ~ n_2, m_2(1-~)+~ n_2+b] then

w=u+vI [(a+b)-(m_1+m_2)+ ~ {(m_1+m_2)-(n_1+n_2)}, ~ (n_1+n_2)+{(a+b)-(n_1+n_2)},

{(a+b)+(n_1+n_2)} - ~ (n_1+n_2),(a+b)+(m_1+m_2) - ~ {(m_1+m_2)-(n_1+n_2)}]

$$«_{\tilde{C}_{SPI}}(w)=\begin{cases}\dfrac{w-[(a+b)-(m_1+m_2)]}{(m_1+m_2)-(n_1+n_2)} & ;(a+b)-(m_1+m_2)\le x\le(a+b)-(n_1+n_2)\\ \dfrac{w-[(a+b)-(n_1+n_2)]}{n_1+n_2} & ;(a+b)-(n_1+n_2)\le x\le(a+b)\\ 1 & ;x=a+b\\ \dfrac{[(a+b)+(n_1+n_2)]-w}{n_1+n_2} & ;(a+b)\le x\le(a+b)+(n_1+n_2)\\ \dfrac{[(a+b)+(m_1+m_2)]-w}{(m_1+m_2)-(n_1+n_2)} & ;(a+b)+(n_1+n_2)\le x\le(a+b)+(m_1+m_2)\\ 0 & ;otherwise\end{cases}$$

-cut for non-membership function of $\underset{:}{A}_{SPI}$ is [(a-n'$_1$)- (m'$_1$-n'$_1$), a- n'$_1$, n$_1$ +a,

(m'$_1$-n'$_1$)+(a+m'$_1$)] for all in [0,1]. -cut for membership function of $\underset{:}{B}_{SPI}$ is

[(b-n'$_2$)- (m'$_2$-n'$_2$), b- n'$_2$, n$_2$ +b, (m'$_2$-n'$_2$)+(b+m'$_2$)]for all in [0,1]

Let u' I [(a-n'$_1$)- (m'$_1$-n'$_1$), a- n'$_1$, n'$_1$ +a, (m'$_1$-n'$_1$)+(a+m'$_1$)]and

v' I [(b-n'$_2$)- (m'$_2$-n'$_2$), b- n'$_2$, n$_2$ +b, (m'$_2$-n'$_2$)+(b+m'$_2$)] then

w'=u'+v' I [(a+b)-(n'$_1$+n'$_2$)- {(m'$_1$+m'$_2$)-(n'$_1$+n'$_2$)}, (a+b)- (n'$_1$+n'$_2$), (a+b)+ (n'$_1$+n'$_2$), (a+b)+(m'$_1$+m'$_2$)+ {(m'$_1$+m'$_2$)- (n'$_1$+n'$_2$)}]

The membership function of $\tilde{C}_{SPI}=\tilde{A}_{SPI}+\tilde{B}_{SPI}$ is given by

$$3_{\tilde{C}_{SPI}}(w')=\begin{cases}\dfrac{[(a+b)-(n'_1+n'_2)]-w'}{(m'_1+m'_2)-(n'_1+n'_2)} & ;(a+b)-(m'_1+m'_2)\le x\le(a+b)-(n'_1+n'_2)\\ \dfrac{(a+b)-w'}{n'_1+n'_2} & ;(a+b)-(n'_1+n'_2)\le x\le(a+b)\\ 0 & ;x=a+b\\ \dfrac{w'-(a+b)}{n'_1+n'_2} & ;(a+b)\le x\le(a+b)+(n'_1+n'_2)\\ \dfrac{w'-[(a+b)+(m'_1+m'_2)]}{(m'_1+m'_2)-(n'_1+n'_2)} & ;(a+b)+(n'_1+n'_2)\le x\le(a+b)+(m'_1+m'_2)\\ 1 & ;otherwise\end{cases}$$

2. Subtraction:

$\tilde{A}_{SPI}$ = [a, n$_1$, n$_1$, m$_1$, m$_1$; a, n'$_1$, n'$_1$, m'$_1$, m'$_1$]

$\tilde{B}_{SPI}$ = [b, n$_2$, n$_2$, m$_2$, m$_2$; b, n'$_2$, n'$_2$, m'$_2$, m'$_2$]

$\tilde{C}_{SPI}=\tilde{A}_{SPI}-\tilde{B}_{SPI}$ = [a-b, n$_1$+n$_2$, n$_1$+n$_2$, m$_1$+m$_2$, m$_1$+m$_2$; a-b, n'$_1$+n'$_2$, n'$_1$+n'$_2$, m'$_1$+m'$_2$, m'$_1$+m$_2$]

This could be proved in the same manner as in the case of addition.

3. Scalar Multiplication:

$\tilde{A}_{SPI}$ = [a, n$_1$, n$_1$, m$_1$, m$_1$; a, n'$_1$, n'$_1$, m'$_1$, m'$_1$]

$\sim\ \tilde{A}_{SPI}=\sim$ [a, n$_1$, n$_1$, m$_1$, m$_1$; a, n'$_1$, n'$_1$, m'$_1$, m'$_1$]

= [$\sim$ a, $\sim$ n$_1$, $\sim$ n$_1$, $\sim$ m$_1$, $\sim$ m$_1$; $\sim$ a, $\sim$ n'$_1$, $\sim$ n'$_1$, $\sim$ m'$_1$, $\sim$ m'$_1$]

All these results for algebraic operations could be proved by directly applying arithmetic operation rules already defined for the pentagonal intuitionistic fuzzy numbers. We need to substitute the values of the 5-tuples accordingly using the values of m, n, n' and m'.

3.3.1. Numerical Examples

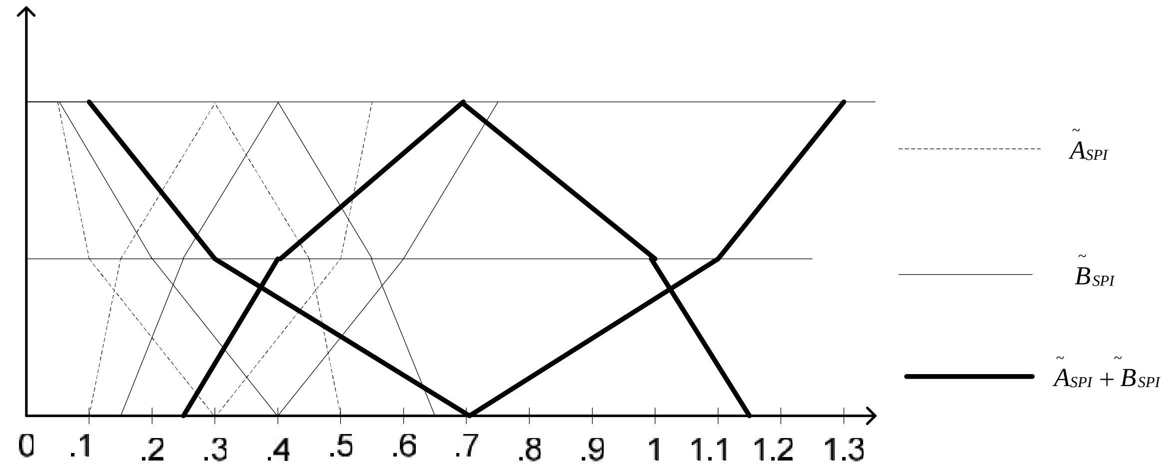
We consider two SPIFNs

$\tilde{A}_{SPI}$ = [.3,.15,.15,.2,.2; .3,.2,.2,.25,.25]

$\tilde{B}_{SPI}$ = [.4,.15,.15, 2.5,2.5; .4,.2,.2,.35,.35]

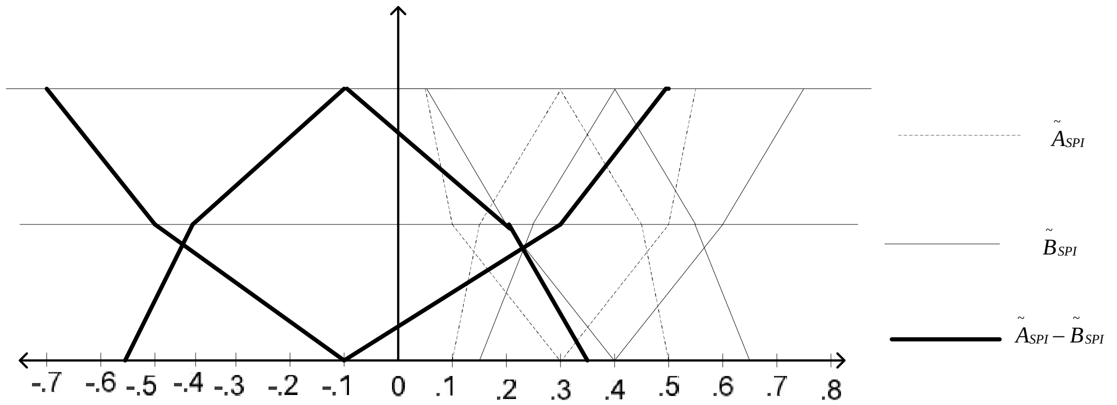
$\tilde{A}_{SPI} + \tilde{B}_{SPI}$ = [.3,.15,.15,.2,.2; .3,.2,.2,.25,.25]+ [.4,.15,.15, 2.5,2.5; .4,.2,.2,.35,.35]

= [.7,.3,.3,.45,.45; .7,.4,.4,.6,.6]



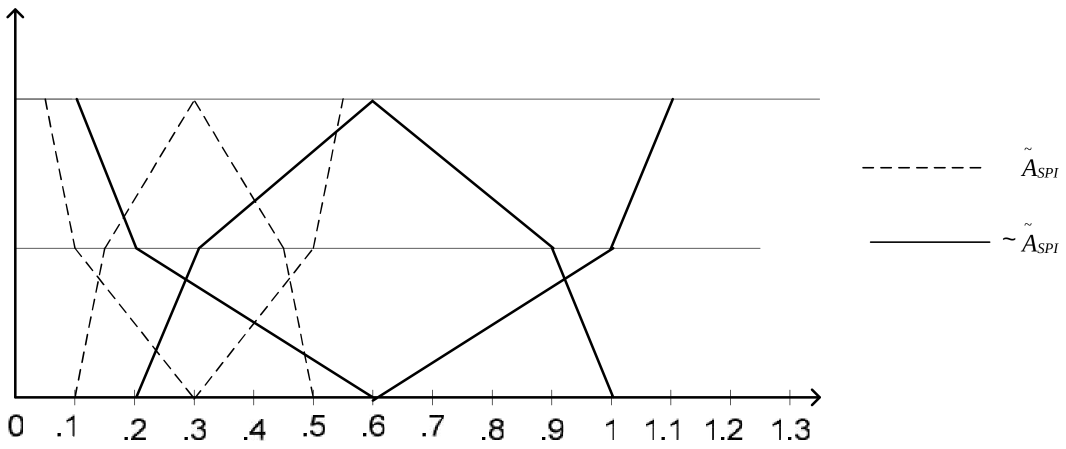
$\tilde{A}_{SPI} - \tilde{B}_{SPI}$ = [.3,.15,.15,.2,.2; .3,.2,.2,.25,.25]- [.4,.15,.15, 2.5,2.5; .4,.2,.2,.35,.35]

= [-.1, .3,.3,.45,.45; -.1,.4,.4,.6,.6]



~ $\tilde{A}_{SPI}$ =2 $\tilde{A}_{SPI}$ = 2[.3,.15,.15,.2,.2; .3,.2,.2,.25,.25]

= [.6,.3,.3,.4,.4; .6,.4,.4,.5,.5]



Conclusions

We have defined symmetric pentagonal intuitionistic fuzzy numbers with proper geometric representation. We have discussed some of the properties with proofs. We have also proposed algebraic operations illustrating them with an example.

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