International Journal of Current Advanced Research
ISSN: O: 2319-6475, ISSN: P: 2319-6505, Impact Factor: SJIF: 6.614
Available Online at www.journalijcar.org
Volume: 7| Issue: 1| Special Issue January: 2018 | Page No.1-9
DOI: http://dx.doi.org/10.24327/IJCAR

TWO TYPES OF GENERALIZED ULAM - HYERS STABILITY OF A ADDITIVE FUNCTIONAL EQUATION ORIGINATING FROM N OBSERVATIONS OF AN ARITHMETIC MEAN IN INTUITIONISTIC FUZZY BANACH SPACES

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A B S T R A C T

RESEARCH ARTICLE

The main aim of this paper is to observe generalized Ulam- Hyers stability of a additive functional equation which is originating from N observations of an arithmetic mean in Intuitionistic Fuzzy Banach Spaces with the help of Hyers Type and Fixed Point Type

1. Introduction

In [33], Ulam projected the general Ulam stability problem: When is it true that by slightly changing the hypotheses of a theorem one can still assert that the thesis of the theorem remains true or approximately true? In [18], Hyers gave the first affirmative answer to the question of Ulam for additive functional equations on Banach spaces. Hyers result has since then seen many significant generalizations, both in terms of the control condition used to define the concept of approximate solution [3, 16, 27, 28].

Now, we are celebrating the platinum jubilee of stability of functional equations. The solution and stability of various functional equation in various normed spaces were introduced and discussed in [1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 20, 21, 23, 24] and reference cited there in.

Definition 1.1 Arithmetic Mean (A.M.): Arithmetic mean is the total of all the items divided by their total number of items

$$A.M. = \frac{X_1 + X_2 + \cdots + X_N}{N}.$$

The main aim of this paper is to observe generalized Ulam-Hyers stability of a additive functional equation

$$f\left(\frac{\sum_{k=1}^{N} x_k}{N}\right) = \frac{1}{N}\sum_{k=1}^{N} f(x_k) \qquad (1.1)$$

originating from N observations of an arithmetic mean in Intuitionistic Fuzzy Banach Space with $N \geq 2$ with the help of Hyers Type and Fixed Point Type.

Now, we present the following theorem due to B. Margolis and J.B. Diaz [22] for the fixed point theory.

Theorem 1.2 [22] Suppose that for a complete generalized metric space (Ω, d) and a strictly contractive mapping $T : \Omega \to \Omega$ with Lipschitz constant L. Then, for each given $x \in \Omega$, either

$$d(T^n x, T^{n+1} x) = \infty, \quad \forall \quad n \geq 0,$$

or there exists a natural number n_0 such that the properties hold:

- $d(T^n x, T^{n+1} x) < \infty$ for all $n \geq n_0$;
- The sequence $(T^n x)$ is convergent to a fixed to a fixed point y^* of T ;
- y^* is the unique fixed point of T in the set $\Delta = \{y \in \Omega : d(T^{n_0} x, y) < \infty\}$;
- $d(y^*, y) \leq \frac{1}{1-L} d(y, Ty)$ for all $y \in \Delta$.

2. Preliminaries on Intuitionistic Fuzzy Banach Spaces

In this section, using the idea of Intuitionistic fuzzy metric spaces introduced by J.H. Park [25] and R. Saadati and J.H. Park [30, 31], we define the new notion of intuitionistic fuzzy Banach spaces with the help of the notion of continuous $t-$ representable (see [17]).

Lemma 2.1 [15] Consider the set L^* and the order relation $\leq_{L^*}$ defined by:

$$L^* = \{(x_1, x_2) : (x_1, x_2) \in [0,1]^2 \text{ and } x_1 + x_2 \leq 1\},$$
$$(x_1, x_2) \leq_{L^*} (y_1, y_2) \Leftrightarrow x_1 \leq y_1, x_2 \geq y_2, \ \forall (x_1, x_2), (y_1, y_2) \in L^*$$

Then $(L^*, \leq_{L^*})$ is a complete lattice.

Definition 2.2 [12] An intuitionistic fuzzy set $A_{\zeta,\eta}$ in a universal set U is an object

$$A_{\mu,\nu} = \{(\mu_A(u), \nu_A(u)) | u \in U\}$$

for all $u \in U$, $\mu_A(u) \in [0,1]$ and $\nu_A(u) \in [0,1]$ are called the membership degree and the non-membership degree, respectively, of u in $A_{\mu,\nu}$ and, furthermore, they satisfy $\mu_A(u) + \nu_A(u) \le 1$.

We denote its units by $0_{L^*} = (0,1)$ and $1_{L^*} = (1,0)$. Classically, a triangular norm $* = T$ on $[0,1]$ is defined as an increasing, commutative, associative mapping $T : [0,1]^2 \to [0,1]$ satisfying $T(1, x) = 1 * x = x$ for all $x \in [0,1]$. A triangular conorm $S = \blacklozenge$ is defined as an increasing, commutative, associative mapping $S : [0,1]^2 \to [0,1]$ satisfying $S(0, x) = 0 \blacklozenge x = x$ for all $x \in [0,1]$.

Using the lattice $(L^*, \le_{L^*})$, these definitions can be straightforwardly extended.

Definition 2.3 [12] A triangular norm ($t-$norm) on L^* is a mapping $T : (L^*)^2 \to L^*$ satisfying the following conditions:

- $(\forall \in L^*)(T(x, 1_{L^*}) = x)$ (boundary condition);
- $(\forall (x, y) \in (L^*)^2)(T(x, y) = T(y, x))$ (commutativity);
- $(\forall (x, y, z) \in (L^*)^3)(T(x, T(y, z)) = T(T(x, y), z))$ (associativity);
- $(\forall (x, x', y, y') \in (L^*)^4)(x \le_{L^*} x' \text{ and } y \le_{L^*} y' \Rightarrow T(x, y) \le_{L^*} T(x', y'))$ (monotonicity).

If $(L^*, \le_{L^*}, T)$ is an Abelian topological monoid with unit 1_{L^*}, then L^* is said to be a continuous $t-$norm.

Definition 2.4. [12] A continuous t-norms T on L* is said to be continuous t-representable if there exist a continuous t-norm $*$ and a continuous $t-$conorm $\blacklozenge$ on $[0,1]$ such that, for all $x = (x_1, x_2), y = (y_1, y_2) \in L^*$,

$$T(x, y) = (x_1 * y_1, x_2 \blacklozenge y_2).$$

For example,

$$T(a,b) = (a_1 b_1, \min\{a_2 + b_2, 1\}) \quad and$$
$$M(a,b) = (\min\{a_1, b_1\}, \max\{a_2, b_2\})$$

for all $a = (a_1, a_2), b = (b_1, b_2) \in L^*$ are continuous $t-$representable.

Now, we define a sequence T^n recursively by $T^1 = T$ and

$$T^n\left(x^{(1)}, \ldots, x^{(n+1)}\right) = T\left(T^{n-1}\left(x^{(1)}, \ldots, x^{(n)}\right), x^{(n+1)}\right),$$
$$\forall n \ge 2, x^{(i)} \in L^*.$$

Definition 2.5 [32] A negator on L^* is any decreasing mapping $N : L^* \to L^*$ satisfying $N : (0_{L^*}) = 1_{L^*}$ and $N(1_{L^*}) = 0_{L^*}$. If $N(N(x)) = x$ for all $x \in L^*$, then N is called an involutive negator. A negator on $[0,1]$ is a decreasing mapping $N : [0,1] \to [0,1]$ satisfying $N(0) = 1$ and $N(1) = 0$. N_s denotes the standard negator on $[0,1]$ defined by

$$N_s(x) = 1 - x, \ \forall x \in [0,1].$$

Definition 2.6 [32] Let μ and ν be membership and nonmembership degree of an intuitionistic fuzzy set from $X \times (0, +\infty)$ to $[0,1]$ such that $\mu_x(t) + \nu_x(t) \le 1$ for all $x \in X$ and all $t > 0$. The triple $(X, P_{\mu,\nu}, T)$ is said to be an intuitionistic fuzzy normed space (briefly IFN-space) if X is a vector space, T is a continuous $t-$representable and $P_{\mu,\nu}$ is a mapping $X \times (0, +\infty) \to L^*$ satisfying the following conditions: for all $x, y \in X$ and $t, s > 0$,

- $P_{\mu,\nu}(x, 0) = 0_{L^*}$;
- $P_{\mu,\nu}(x, t) = 1_{L^*}$ if and only if $x = 0$;
- $P_{\mu,\nu}(\lambda x, t) = P_{\mu,\nu}\left(x, \frac{t}{|\lambda|}\right)$ for all $\mu \ne 0$;
- $P_{\mu,\nu}(x + y, t + s) \ge_{L^*} T(P_{\mu,\nu}(x, t), P_{\mu,\nu}(y, s))$.

In this case, $P_{\mu,\nu}$ is called an intuitionistic fuzzy norm. Here, $P_{\mu,\nu}(x, t) = (\mu_x(t), \nu_x(t))$.

Example 2.7 [32] Let $(X, \|\cdot\|)$ be a normed space. Let $T(a,b) = (a, b, \min(a_2 + b_2, 1))$ for all $a = (a_1, a_2), b = (b_1, b_2) \in L^*$ and μ, ν be membership and non-membership degree of an intuitionistic fuzzy set defined by

$$P_{\mu,\nu}(x, t) = (\mu_x(t), \nu_x(t)) = \left(\frac{t}{t + \|x\|}, \frac{\|x\|}{t + \|x\|}\right), \forall \quad t \in R^+.$$

Then $(X, P_{\mu,\nu}, T)$ is an IFN-sapce.

Definition 2.8 [32] A sequence $\{x_n\}$ in an IFN-space $(X, P_{\mu,\nu}, T)$ is called a Cauchy sequence if, for any $\varepsilon > 0$ and $t > 0$, there exists $n_0 \in N$ such that

$$P_{\mu,\nu}(x_n - x_m, t) > L^*(N_s(\varepsilon), \varepsilon), \ \forall \quad n, m \ge n_0,$$

where N_s is the standard negator.

Definition 2.9 [32] The sequence $\{x_n\}$ is said to be convergent to a point $x \in X$ (denoted by $x_n \xrightarrow{P_{\mu,\nu}} x$) if $P_{\mu,\nu}(x_n - x, t) \to 1_{L^*}$ as $n \to \infty$ for every $t > 0$.

Definition 2.10 [32] An IFN-space $(X, P_{\mu,\nu}, T)$ is said to be complete if every Cauchy sequence in X is convergent to a point $x \in X$.

In order to establish the stability results let us consider the following:

- X - Linear space.
- $(Z, P'_{«,¬}, M)$ - IFN-space.
- $(Y, P'_{«,¬}, M)$ - Complete IFN-space (IFBS).
- $¥ \in \{1, -1\}$.
- Define a constant $\sim_i$ by

$$\sim_i = \begin{cases} N & if \quad i = 0; \\ \frac{1}{N} & if \quad i = 1. \end{cases} \tag{2.1}$$

3. Intuitionistic Fuzzy Banach Space: Stability Results: N Is An Even Integer

In this section, we test the generalized Ulam - Hyers stability of the additive functional equation (1) in Intuitionistic Fuzzy Banach Space by taking N is an Even Integer.

3.1 Hyers Type

Theorem 3.1 Assume $\mu : X^N \to Z$ be a function such that for some $0 < \left(\frac{a}{N}\right)^{¥} < 1$,

$$P'_{«,¬}\left(\mu\left(N^{q¥}x, 0, \cdots, 0\right), r\right) \geq_{L^*} P'_{«,¬}\left(a^{q¥}\ \mu\left(x, 0, \cdots, 0\right), r\right) \tag{3.1}$$

for all $x \in X$ and all $r > 0$ and

$$\lim_{q \to \infty} P'_{«,¬}\left(\mu\left(N^{¥q}x_1, \cdots, N^{¥q}x_N\right), N^{¥q}r\right) = 1_{L^*} \tag{3.2}$$

for all $x_1, \cdots, x_N \in X$ and $r > 0$. If $f : X \to Y$ is a mapping such that

$$P_{«,¬}\left(f\left(\frac{\sum_{k=1}^{N} x_k}{N}\right) - \frac{1}{N}\sum_{k=1}^{N} f(x_k), r\right) \geq_{L^*} P'_{«,¬}\left(\mu\left(x_1, \cdots, x_N\right), r\right) \tag{3.3}$$

for all $x_1, \cdots, x_N \in X$ and $r > 0$. Then the limit

$$P_{«,¬}\left(A(x) - \frac{f(N^{q¥}x)}{N^{q¥}}, r\right) \to 1_{L^*}, \quad as\ q \to \infty,\ r > 0$$

(3.4)

exists for all $x \in X$ and the mapping $A : X \to Y$ is the unique additive mapping such that

$$P_{«,¬}\left(f(x) - A(x), r\right) \geq_{L^*} P'_{«,¬}\left(\mu\left(x, 0, \cdots, 0\right), \frac{|N - a|\, r}{N}\right) \tag{3.5}$$

for all $x \in X$ and all $r > 0$.

Proof. Case 1 : $¥ = 1$.

Switching $(x_1, x_2, \cdots, x_N)$ by $(x, 0, \cdots, 0)$ in (3.3), we get

$$P_{«,¬}\left(f\left(\frac{x}{N}\right) - \frac{1}{N} f(x), r\right) \geq_{L^*} P'_{«,\nu}\left(\mu\left(x, 0, \cdots, 0\right), r\right) \tag{3.6}$$

for all $x \in X$ and $r > 0$. Setting x by Nx in (3.6), we obtain

$$P_{«,¬}\left(f(x) - \frac{f(N\,x)}{N}, r\right) \geq_{L^*} P'_{«,\nu}\left(\mu\left(N\,x, 0, \cdots, 0\right), r\right)$$

(3.7)

for all $x \in X$ and $r > 0$. Replacing x by $N^q x$ in (3.7) and using (3.1), (IFNS3), we arrive

$$P_{«,¬}\left(f(N^q x) - \frac{f(N^{q+1}x)}{N}, r\right) \geq_{L^*} P'_{«,¬}\left(\mu\left(x, 0, \cdots, 0\right), \frac{r}{a^q}\right) \tag{3.8}$$

for all $x \in X$ and $r > 0$. It is easy to verify from (3.8), that

$$P_{«,¬}\left(\frac{f(N^q x)}{N^q} - \frac{f(N^{q+1}x)}{N^{q+1}}, \frac{r}{N^q}\right) \geq_{L^*} P'_{«,¬}\left(\mu\left(x, 0, \cdots, 0\right), \frac{r}{a^q}\right) \tag{3.9}$$

holds for all $x \in X$ and all $r > 0$. Replacing r by $a^q r$ in (3.9), we have

$$P_{«,¬}\left(\frac{f(N^q x)}{N^q} - \frac{f(N^{q+1}x)}{N^{q+1}}, \left[\frac{a}{N}\right]^q \cdot r\right) \geq_{L^*} P'_{«,¬}\left(\mu\left(x, 0, \cdots, 0\right), r\right) \tag{3.10}$$

for all $x \in X$ and all $r > 0$. It is easy to see that

$$\frac{f(N^q x)}{N^q} - f(x) = \sum_{j=0}^{q-1} \frac{f(N^{j+1}x)}{N^{(j+1)}} - \frac{f(N^j x)}{N^j} \tag{3.11}$$

for all $x \in X$. From equations (3.10) and (3.11), we get

$$\begin{aligned} P_{«,¬}\left(\frac{f(N^q x)}{N^q} - f(x), \sum_{j=0}^{q-1}\left[\frac{a}{N}\right]^j \cdot r\right) &\geq_{L^*} \\ M_{j=0}^{q-1} P'_{«,¬}\left(\sum_{j=0}^{q-1} \frac{f(N^{j+1}x)}{N^{(j+1)}} - \frac{f(N^j x)}{N^j}, \sum_{j=0}^{q-1}\left[\frac{a}{N}\right]^j \cdot r\right) & \\ \geq_{L^*} M_{j=0}^{q-1} P'_{«,¬}\left(\mu\left(x, 0, \cdots, 0\right), r\right) & \\ \geq_{L^*} P'_{«,¬}\left(\mu\left(x, 0, \cdots, 0\right), r\right) & \end{aligned} \tag{3.12}$$

for all $x \in X$ and all $r > 0$. Replacing x by $N^p x$ in (3.12) and using (3.1), (IFNS3), we obtain

$$P_{«,¬}\left(\frac{f(N^{q+p}x)}{N^{q+p}} - \frac{f(N^p x)}{N^p}, \frac{1}{N^p}\sum_{j=0}^{q-1}\left[\frac{a}{N}\right]^j \cdot r\right) \geq_{L^*} P'_{«,¬}\left(\mu(x, 0, \cdots, 0), \frac{r}{a^p}\right)$$

(3.13)

for all $x \in X$ and all $r > 0$ and all $p, q \geq 0$. Replacing r

by $a^p\, r$ in (3.13), we arrive

$$P_{«,¬}\left(\frac{f(N^{q+p}x)}{N^{q+p}}-\frac{f(N^p x)}{N^p},\sum_{j=0}^{q-1}\left[\frac{a}{N}\right]^{j+p}\cdot r\right) \geq_{L^*} P'_{«,¬}\left(\mu(x,0,\cdots,0),r\right) \quad (3.14)$$

for all $x\in X$ and all $r>0$ and all $p,q\geq 0$. It follows from (3.14) that

$$P_{«,¬}\left(\frac{f(N^{q+p}x)}{N^{q+p}}-\frac{f(N^p x)}{N^p},r\right) \geq_{L^*} P'_{«,¬}\left(\mu(x,0,\cdots,0),\frac{r}{\sum_{j=p}^{q-1}\left[\frac{a}{N}\right]^{j}}\right) \quad (3.15)$$

for all $x\in X$ and all $r>0$ and all $p,q\geq 0$.

By data, we have $\left\{\frac{f(N^q x)}{N^q}\right\}$ is a Cauchy sequence in $\left(Y,P_{«,¬},M\right)$. Since $\left(Y,P_{«,¬},M\right)$ is a complete IFN-space this sequence convergent to some point $A(x)\in Y$. So one can define the mapping $A:X\to Y$ by

$$P_{«,¬}\left(A(x)-\frac{f(N^q x)}{N^q},r\right)\to 1_L^*,\ as\ q\to\infty, r>0 \quad (3.16)$$

for all $x\in X$. Letting $p=0$ in (3.15), we get

$$P_{«,¬}\left(\frac{f(N^q x)}{N^q}-f(x),r\right) \geq_{L^*} P'_{«,¬}\left(\mu(x,0,\cdots,0),\frac{r}{\sum_{j=0}^{q-1}\left[\frac{a}{N}\right]^{j}\cdot r}\right) \quad (3.17)$$

for all $x\in X$ and all $r>0$. Now for every ¢ >0 and from (3.17), we obtain

$$P_{«,¬}\left(A(x)-f(x),r+¢\right) \geq_{L^*} M\left(\begin{array}{l} P_{«,¬}\left(A(x)-\frac{f\left(N^q x\right)}{N^q},¢\right), \\ P_{«,¬}\left(f(x)-\frac{f\left(N^q x\right)}{N^q},r\right)\end{array}\right)$$

$$\geq_{L^*} M\left(\begin{array}{l} P_{«,¬}\left(A(x)-\frac{f\left(N^q x\right)}{N^q},¢\right), \\ P'_{«,¬}\left(\mu\left(x,0,\cdots,0\right),\frac{r}{\sum_{j=0}^{q-1}\left[\frac{a}{N}\right]^{j}\cdot r}\right)\end{array}\right) \quad (3.18)$$

for all $x\in X$ and all $r>0$. Taking the limit as $n\to\infty$ in (3.18) and using (3.16), we have

$$P_{«,¬}\left(A(x)-f(x),r+¢\right) \geq_{L^*} P'_{«,¬}\left(\mu\left(x,0,\cdots,0\right),\frac{(N-a)\,r}{N}\right) \quad (3.19)$$

for all $x\in X$ and all $r>0$ and ¢ >0. Since ¢ is arbitrary, by taking ¢ $\to 0$ in (3.19), we arrive

$$P_{«,¬}\left(A(x)-f(x),r\right) \geq_{L^*} P'_{«,¬}\left(\mu\left(x,0,\cdots,0\right),\frac{(N-a)\,r}{N}\right) \quad (3.20)$$

for all $x\in X$ and all $r>0$. To prove A satisfies the (1.1), replacing $(x_1,\cdots,x_N)$ by $(N^q\, x_1,\cdots,N^q\, x_N)$ in (3.3) respectively, we get

$$P_{«,¬}\left(\frac{1}{N^q}\left(f\left(\frac{\sum_{k=1}^{N}N^q\, x_k}{N}\right)-\frac{1}{N}\sum_{k=1}^{N}f(N^q\, x_k)\right),r\right) \geq_{L^*} P'_{«,¬}\left(\mu(N^q\, x_1,\cdots,N^q\, x_k),N^q r\right) \quad (3.21)$$

for all $x_1,\cdots,x_N\in X$ and $r>0$. Now,

$$P_{«,¬}\left(A\left(\frac{\sum_{k=1}^{N}x_k}{N}\right)-\frac{1}{N}\sum_{k=1}^{N}A(x_k),r\right)$$

$$\geq_{L^*} M^2\left\{\begin{array}{l} P_{«,¬}\left(A\left(\frac{\sum_{k=1}^{N}x_k}{N}\right)-\frac{1}{N^q}\,f\left(\frac{\sum_{k=1}^{N}N^q\, x_k}{N}\right),\frac{r}{3}\right), \\ P_{«,¬}\left(-\frac{1}{N}\sum_{k=1}^{N}A(x_k)+\frac{1}{N^q}\frac{1}{N}\sum_{k=1}^{N}f(N^q\, x_k),\frac{r}{3}\right), \\ P_{«,¬}\left(\frac{1}{N^q}\left(f\left(\frac{\sum_{k=1}^{N}N^q\, x_k}{N}\right)-\frac{1}{N}\sum_{k=1}^{N}f(N^q\, x_k)\right),\frac{r}{3}\right)\end{array}\right\} \quad (3.22)$$

for all $x_1,\cdots,x_N\in X$ and all $r>0$. Letting $q\to\infty$ in (3.22) and using (3.21),(3.2), we obtain

$$P_{«,¬}\left(A\left(\frac{\sum_{k=1}^{N}x_k}{N}\right)-\frac{1}{N}\sum_{k=1}^{N}A(x_k),r\right) \geq_{L^*} M^2\left(1_{L^*},1_{L^*},1_{L^*}\right)$$

(3.23)

for all $x_1,\cdots,x_N\in X$ and all $r>0$. Using (IFNS2) in (3.23), we arrive

$$A\left(\frac{\sum_{k=1}^{N} x_k}{N}\right) = \frac{1}{N}\sum_{k=1}^{N} A(x_k)$$

for all $x_1, \cdots, x_N \in X$. Hence A satisfies the additive functional equation (1.1). In order to prove $A(x)$ is unique, let $A'(x)$ be another additive functional equation satisfying (1.1) and (3.5). Hence,

$$P_{\text{«,¬}}(A(x) - A'(x), r) = P_{\text{«,¬}}\left(\frac{A(N^q x)}{N^q} - \frac{A'(N^q x)}{N^q}, r\right)$$

$$\geq_{L^*} M\left(\begin{array}{l} P_{\text{«,¬}}\left(\frac{A(N^q x)}{N^q} - \frac{f(N^q x)}{N^q}, \frac{r}{2}\right), \\ P_{\text{«,¬}}\left(\frac{f(N^q x)}{N^q} - \frac{A'(N^q x)}{N^q}, \frac{r}{2}\right)\end{array}\right)$$

$$\geq_{L^*} P'_{\text{«,¬}}\left(\mu(N^q x, 0, \cdots, 0), \frac{r(N-a)}{2N}\right)$$

$$= P'_{\text{«,¬}}\left(\mu(x, 0, \cdots, 0), \frac{r(N-a)}{2Na^q}\right)$$

for all $x \in X$ and all $r > 0$. Since $\lim_{q\to\infty} \frac{r(q-a)}{2Na^q} = \infty$, we obtain $\lim_{q\to\infty} P'_{\text{«,¬}}\left(\mu(x,0,\cdots,0), \frac{r(N-a)}{2Na^q}\right) = 1_{L^*}$. Thus $P_{\text{«,¬}}(A(x) - A'(x), r) = 1_{L^*}$ for all $x \in X$ and all $r > 0$, hence $A(x) = A'(x)$. Therefore $A(x)$ is unique. Thus, the theorem holds for ¥ $= 1$. For ¥ $= -1$, we can prove the result by a similar method. This completes the proof of the theorem.

The subsequent corollaries are immediate consequence of Theorem 3.1 concerning the stabilities of (1.1).

Corollary 3.2 If $f : X \to Y$ is a mapping satisfying the functional inequality

$$P_{\text{«,¬}}\left(f\left(\frac{\sum_{k=1}^{N} x_k}{N}\right) - \frac{1}{N}\sum_{k=1}^{N} f(x_k), r\right) \geq_{L^*} P'_{\text{«,¬}}(°, r) \tag{3.24}$$

for all $x_1, \cdots, x_N \in X$ and all $r > 0$ and $° > 0$ be a constant, then there exists a unique additive mapping $A : X \to Y$ such that

$$P_{\text{«,¬}}(f(x) - A(x), r) \geq_{L^*} P'_{\text{«,¬}}\left(°, \frac{r|N-1|}{N}\right) \tag{3.25}$$

for all $x \in X$ and $r > 0$.

Corollary 3.3 If $f : X \to Y$ is a mapping satisfying the functional inequality

$$P_{\text{«,¬}}\left(f\left(\frac{\sum_{k=1}^{N} x_k}{N}\right) - \frac{1}{N}\sum_{k=1}^{N} f(x_k), r\right) \tag{3.26}$$

$$\geq_{L^*} P'_{\text{«,¬}}\left(° \sum_{k=1}^{N} |x_k|^t, r\right)$$

for all $x_1, \cdots, x_N \in X$ and all $r > 0$ and $°, t$ are positive constants with $t \neq 1$, then there exists a unique additive mapping $A : X \to Y$ such that

$$P_{\text{«,¬}}(f(x) - A(x), r) \geq_{L^*} P'_{\text{«,¬}}\left(° |x|^t, \frac{r|N - N^t|}{N}\right) \tag{3.27}$$

for all $x \in X$ and $r > 0$.

3.2 Fixed Point Type

Theorem 3.4 Assume $\mu : X^N \to Z$ be a function satisfying the condition

$$\lim_{q\to\infty} P'_{\text{«,¬}}\left(\mu(\sim_i^q x_1, \cdots, \sim_i^q x_N), \sim_i^q r\right) = 1_{L^*} \tag{3.28}$$

for all $x_1, \cdots, x_N \in X$ and $r > 0$. If $f : X \to Y$ is a mapping such that

$$P_{\text{«,¬}}\left(f\left(\frac{\sum_{k=1}^{N} x_k}{N}\right) - \frac{1}{N}\sum_{k=1}^{N} f(x_k), r\right) \geq_{L^*} \tag{3.29}$$

$$P'_{\text{«,¬}}\left(\mu(x_1, \cdots, x_N), r\right)$$

for all $x_1, \cdots, x_N \in X$ and $r > 0$. If there exist a constant $L = L(i)$ such that

$$\Omega(x) = N\,\mu\left(\frac{x}{N}, 0, \cdots, 0\right) \tag{3.30}$$

with the property

$$P'_{\text{«,¬}}\left(\frac{1}{\sim_i}\Omega(\sim_i x), r\right) = P'_{\text{«,¬}}\left(L\,\Omega(x), r\right) \tag{3.31}$$

for all $x \in X$ and all $r > 0$. Then there exists a the mapping $A : X \to Y$ which is a unique additive mapping such that

$$P_{\text{«,¬}}(f(x) - A(x), r) \geq_{L^*} P'_{\text{«,¬}}\left(\frac{L^{1-i}}{1-L}\Omega(x), r\right) \tag{3.32}$$

for all $x \in X$ and all $r > 0$.

Proof. Consider the set

$$\Psi = \{ f_1 / f_1 : X \to Y,\ f_1(0) = 0\}$$

and introduce the generalized metric on Ψ,

$$d(f_1, f_2) = \inf\{¦ \in (0, \infty):$$

$$P_{\text{«,¬}}(f_1(x) - f_2(x), r) \geq_{L^*} P'_{\text{«,¬}}(¦\,\Omega(x), r) \tag{3.33}$$

for all $x \in X$ and all $r > 0$. It is easy to see that (33) is complete with respect to the defined metric. Define $J : \Psi \to \Psi$ by

$J\ f_1(x) = \frac{1}{\sim_i} f_1(\sim_i x)$, for all $x \in X$. Now, from (33) and $f_1, f_2 \in \Psi$, we arrive J is a strictly contractive mapping on Ψ with Lipschitz constant L (see [22]). It follows from (3.7), (3.33) and (1) for the case $i = 0$, we reach

$$P_{«,¬}\left(f(x) - Jf(x), r\right) \geq_{L^*} P'_{«,¬}\left(L\Omega(x), r\right) \quad (3.34)$$

for all $x \in X$ and $r > 0$. Again replacing $x = \frac{x}{N}$ in (3.7) and it follows from (3.33), (3.1) for the case $i = 1$, we again reach

$$P_{«,¬}\left(Jf(x) - f(x), r\right) \geq_{L^*} P'_{«,¬}\left(\Omega(x), r\right) \quad (3.35)$$

for all $x \in X$ and $r > 0$. Thus, from (3.34) and (3.35), we arrive

$$P_{«,¬}\left(Jf(x) - f(x), r\right) \geq_{L^*} P'_{«,¬}\left(L^{1-i}\Omega(x), r\right) \quad (3.36)$$

for all $x \in X$ and $r > 0$. Hence property (FP1) of Theorem 1.2 holds. It follows from property (FP2) of Theorem 1.2 that there exists a fixed point A of J in Ψ such that

$$\frac{f(\sim_i^q x)}{\sim^q} \xrightarrow{P_{«,¬}} A(x) \qquad as \qquad q \to \infty \quad (3.37)$$

for all $x \in X$ and $r > 0$. In order to show that A satisfies (1), the proof is similar lines to tat of Thorem 3.1 By property (FP3) of Theorem 1.2 , A is the unique fixed point of J in the set

$$\Delta = \{A \in \Psi : d(f, A) < \infty\},$$

such that

$$P_{«,¬}\left(f(x) - A(x), r\right) \geq_{L^*} P'_{«,¬}\left(¦\ \Omega(x), r\right)$$

for all $x \in X$ and $r > 0$. Finally by property (FP4) of Theorem 1.2 , we obtain our desired inequality (3.32). This completes the proof of the theorem.

The following corollary is an immediate consequence of Theorem 3.4 concerning the stability of (1.1).

Corollary 3.5 If $f : X \to Y$ is a mapping satisfying the functional inequality

$$P_{«,¬}\left(f\left(\frac{\sum_{k=1}^{N} x_k}{N}\right) - \frac{1}{N}\sum_{k=1}^{N} f(x_k), r \right) \quad (3.38)$$

$$\geq_{L^*} \begin{cases} P'_{«,¬}\left(\circ, r\right); \\ P'_{«,¬}\left(\circ \sum_{k=1}^{N} |x_k|^t, r\right); \end{cases}$$

for all $x_1, \cdots, x_N \in X$ and all $r > 0$ and $\circ, t$ are positive constants with $t \neq 1$, then there exists a unique additive mapping $A : X \to Y$ such that

$$P_{«,¬}\left(f(x) - A(x), r\right)$$

$$\geq_{L^*} \begin{cases} P'_{«,¬}\left(N\ \circ, r\,|N-1|\right); \\ P'_{«,¬}\left(N\ \circ\,|x|^t, r\,|N - N^t|\right); \end{cases} \quad (3.39)$$

for all $x \in X$ and $r > 0$.

Proof. Let $\mu(x_1, \cdots, x_n) = \begin{cases} \circ; \\ \circ \sum_{k=1}^{N} \|x_k\|^t; \end{cases}$ (3.40)

for all $x_1, \cdots, x_N \in X$. Now

$$P'_{«,¬}\left(\mu(\sim_i^n x_1, \sim_i^n x_n), \sim_i^q r\right)$$

$$= \begin{cases} P'_{«,¬}\left(\circ, \sim_i^q r\right); \\ P'_{«,¬}\left(\circ \sum_{k=1}^{N} |\sim_i^q x_k|^t, \sim_i^q r\right); \end{cases}$$

$$= \begin{cases} \to 1_{L^*}\ as\ q \to \infty; \\ \to 1_{L^*}\ as\ q \to \infty. \end{cases}$$

Thus, (3.28) holds. It follows from (3.40), (3.30) and (3.31), we have

$$P'_{«,¬}\left(\Omega(x), r\right) = P'_{«,¬}\left(N\,\mu\left(\frac{x}{N}, 0, \cdots, 0\right), r\right)$$

$$= \begin{cases} P'_{«,¬}\left(N\ \circ, r\right); \\ P'_{«,¬}\left(N\ \circ\,\left|\frac{x}{N}\right|^t, r\right); \end{cases} = \begin{cases} P'_{«,¬}\left(N\ \circ, r\right); \\ P'_{«,¬}\left(\frac{N\ \circ}{N^t}|x|^t, r\right); \end{cases} \quad (3.41)$$

and

$$P'_{«,¬}\left(\frac{1}{\sim_i}\Omega(\sim_i x), r\right) = \begin{cases} P'_{«,¬}\left(\frac{1}{\sim_i} N\ \circ, r\right); \\ P'_{«,¬}\left(\frac{1}{\sim_i} N\ \circ\,|\sim_i x|^t, r\right); \end{cases}$$

$$= \begin{cases} P'_{«,¬}\left(\sim_i^{-1} N\ \circ, r\right); \\ P'_{«,¬}\left(\sim_i^{t-1} N\ \circ\,|x|^t, r\right); \end{cases} = \begin{cases} P'_{«,¬}\left(L\,\Omega(x), r\right); \\ P'_{«,¬}\left(L\,\Omega(x), r\right); \end{cases}$$

for all $x \in X$. Hence, in view of (3.42) and the inequality (3.32) we arrive our inequality (3.39). Hence the proof is complete.

4. Intuitionistic Fuzzy Banach Space: Stability Results: N is an Odd Integer

In this section, we judge the generalized Ulam - Hyers stability of the additive functional equation (1.1) in

Intuitionistic Fuzzy Banach Space by taking N is an odd Integer.

Theorem 4.1 Assume $\mu : X^N \to Z$ be a function such that for some $0 < \left(\frac{a}{N}\right)^{¥} < 1$,

$$P'_{«,¬}\left(\mu\left(N^{q¥}x, \underbrace{-N^{q¥}x,\cdots,-N^{q¥}x}_{\frac{N-1}{2}}, \underbrace{N^{q¥}x,\cdots,N^{q¥}x}_{\frac{N-1}{2}}\right), r\right)$$

$$\geq_{L^*} P'_{«,¬}\left(a^{q¥}\mu\left(x, \underbrace{-x,\cdots,-x}_{\frac{N-1}{2}}, \underbrace{x,\cdots,x}_{\frac{N-1}{2}}\right), r\right) \quad (4.1)$$

for all $x \in X$ and all $r > 0$ and

$$\lim_{q\to\infty} P'_{«,¬}\left(\mu\left(N^{¥q}x_1,\cdots,N^{¥q}x_N\right), N^{¥q}r\right) = 1_{L^*} \quad (4.2)$$

for all $x_1,\cdots,x_N \in X$ and $r > 0$. If $f : X \to Y$ is a mapping such that

$$P_{«,¬}\left(f\left(\frac{\sum_{k=1}^{N} x_k}{N}\right) - \frac{1}{N}\sum_{k=1}^{N} f(x_k), r\right) \quad (4.3)$$

$$\geq_{L^*} P'_{«,¬}\left(\mu\left(x_1, x_2\cdots, x_{N-1}, x_N\right), r\right)$$

for all $x_1,\cdots,x_N \in X$ and $r > 0$. Then the limit

$$P_{«,¬}\left(A(x) - \frac{f(N^q x)}{N^q}, r\right) \to 1^*_L, as\ q \to \infty, r > 0 \quad (4.4)$$

exists for all $x \in X$ and the mapping $A : X \to Y$ is the unique additive mapping such that

$$P_{«,¬}\left(f(x) - A(x), r\right)$$

$$\geq_{L^*} P'_{«,¬}\left(\mu\left((x, \underbrace{-x,\cdots,-x}_{\frac{N-1}{2}}, \underbrace{x,\cdots,x}_{\frac{N-1}{2}})\right), \frac{|N-a|r}{N}\right) \quad (4.5)$$

for all $x \in X$ and all $r > 0$.

Proof. Case 1 : $¥ = 1$.

Switching $(x_1, x_2, \cdots, x_N)$ by $(x, \underbrace{-x,\cdots,-x}_{\frac{N-1}{2}}, \underbrace{x,\cdots,x}_{\frac{N-1}{2}})$ in (4.3), we get

$$P_{«,¬}\left(f\left(\frac{x}{N}\right) - \frac{1}{N}f(x), r\right)$$

$$\geq_{L^*} P'_{«,v}\left(\mu\left(x, \underbrace{-x,\cdots,-x}_{\frac{N-1}{2}}, \underbrace{x,\cdots,x}_{\frac{N-1}{2}}\right), r\right)$$

(4.6)

for all $x \in X$ and $r > 0$. Setting x by Nx in (4.6), we obtain

$$P_{«,¬}\left(f(x) - \frac{f(N\,x)}{N}, r\right)$$

$$\geq_{L^*} P'_{«,v}\left(\mu\left(Nx, \underbrace{-Nx,\cdots,-Nx}_{\frac{N-1}{2}}, \underbrace{Nx,\cdots,Nx}_{\frac{N-1}{2}}\right), r\right) \quad (4.7)$$

for all $x \in X$ and $r > 0$. Replacing x by $N^q x$ in (4.7) and using (4.1), (IFNS3), we arrive

$$P_{«,¬}\left(f(N^q x) - \frac{f(N^{q+1}x)}{N}, r\right)$$

$$\geq_{L^*} P'_{«,¬}\left(\mu\left(x, 0, \cdots, 0\right), \frac{r}{a^q}\right) \quad (4.8)$$

for all $x \in X$ and $r > 0$. The rest of proof is similar ideas to that of Theorem 3.1. This completes the proof of the theorem.

The subsequent corollaries are immediate consequence of Theorem 4.1 concerning the stabilities of (1.1)

Corollary 4.2 If $f : X \to Y$ is a mapping satisfying the functional inequality

$$P_{«,¬}\left(f\left(\frac{\sum_{k=1}^{N} x_k}{N}\right) - \frac{1}{N}\sum_{k=1}^{N} f(x_k), r\right) \geq_{L^*} P'_{«,¬}\left(\circ, r\right) \quad (4.9)$$

for all $x_1,\cdots,x_N \in X$ and all $r > 0$ and $\circ > 0$ be a constant, then there exists a unique additive mapping $A : X \to Y$ such that

$$P_{«,¬}\left(f(x) - A(x), r\right) \geq_{L^*} P'_{«,¬}\left(\circ, \frac{r\,|N-1|}{N}\right) \quad (4.10)$$

for all $x \in X$ and $r > 0$.

Corollary 4.3 If $f : X \to Y$ is a mapping satisfying the functional inequality

$$P_{«,¬}\left(f\left(\frac{\sum_{k=1}^{N} x_k}{N}\right) - \frac{1}{N}\sum_{k=1}^{N} f(x_k), r\right) \geq_{L^*} P'_{«,¬}\left(\circ \sum_{k=1}^{N} |x_k|^t, r\right)$$

(4.11)

for all $x_1,\cdots,x_N \in X$ and all $r > 0$ and $\circ, t$ are positive constants with $t \neq 1$, then there exists a unique additive mapping $A : X \to Y$ such that

$$P_{«,¬}\left(f(x) - A(x), r\right) \geq_{L^*} P'_{«,¬}\left(\circ\, N|x|^t, \frac{r\,|N-N^t|}{N}\right)$$

(4.12)

for all $x \in X$ and $r > 0$.

4.1 Fixed Point Type

The proof of the following theorem and corollary is similar clues that of Theorem 3.4 and Corollary 3.5. Hence the details of the proof are omitted.

Theorem 4.4 Assume $\mu : X^N \to Z$ be a function satisfying the condition

$$\lim_{q\to\infty} P'_{«,¬}\left(\mu\left(\sim_i^q x_1,\cdots,\sim_i^q x_N\right),\sim_i^q r\right)=1_{L^*} \quad (4.13)$$

for all $x_1,\cdots,x_N \in X$ and $r>0$. If $f: X \to Y$ is a mapping such that

$$P_{«,¬}\left(f\left(\frac{\sum_{k=1}^{N} x_k}{N}\right)-\frac{1}{N}\sum_{k=1}^{N} f(x_k), r\right) \quad (4.14)$$

$$\geq_{L^*} P'_{«,¬}\left(\mu\left(x_1,\cdots,x_N\right),r\right)$$

for all $x_1,\cdots,x_N \in X$ and $r>0$. If there exist a constant $L=L(i)$ such that

$$\Omega(x) = N\,\mu\left(\mu\left((x,\underbrace{-x,\cdots,-x}_{\frac{N-1}{2}},\underbrace{x,\cdots,x}_{\frac{N-1}{2}})\right)\right) \quad (4.15)$$

with the property

$$P'_{«,¬}\left(\frac{1}{\sim_i}\Omega(\sim_i x),r\right) = P'_{«,¬}\left(L\,\Omega(x),r\right) \quad (4.16)$$

for all $x\in X$ and all $r>0$. Then there exists a the mapping $A: X \to Y$ which is a unique additive mapping such that

$$P_{«,¬}\left(f(x)-A(x),r\right) \geq_{L^*} P'_{«,¬}\left(\frac{L^{1-i}}{1-L}\Omega(x),r\right) \quad (4.17)$$

for all $x\in X$ and all $r>0$.

Corollary 4.5 If $f: X\to Y$ is a mapping satisfying the functional inequality

$$P_{«,¬}\left(f\left(\frac{\sum_{k=1}^{N} x_k}{N}\right)-\frac{1}{N}\sum_{k=1}^{N} f(x_k), r\right) \quad (4.18)$$

$$\geq_{L^*} \begin{cases} P'_{«,¬}\left(°,r\right); \\ P'_{«,¬}\left(°\sum_{k=1}^{N}|x_k|^t,r\right); \end{cases}$$

for all $x_1,\cdots,x_N \in X$ and all $r>0$ and $°, t$ are positive constants with $t\neq 1$, then there exists a unique additive mapping $A: X\to Y$ such that

$$P_{«,¬}\left(f(x)-A(x),r\right)$$

$$\geq_{L^*} \begin{cases} P'_{«,¬}\left(N\,°,r\,|\,N-1|\right); \\ P'_{«,¬}\left(N\,°\,|x|^t,r\,|\,N-N^t\,|\right); \end{cases} \quad (4.19)$$

for all $x\in X$ and $r>0$.

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