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Research Article

AN INVENTORY MODEL FOR DETERIORATING ITEMS WITH TIME DEPENDENT DEMAND USING TRIANGULAR TRIDENT FUZZY NUMBERS

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ABSTRACT

The aim of this research work is to find minimum cost and optimum time period in fuzzy inventory models. For this objective, the parameters in the inventory models are considered as triangular trident fuzzy numbers. Numerical example is worked out to explain the concept. Sensitivity analysis is also provided in this paper.

INTRODUCTION

Inventory is very essential in business such as manufacturing goods, selling goods etc. The inventory level should be maintained to avoid loss and increase the business profit. Uncertainties and imprecision is inherent in real inventory problems. This can be approached by probabilistic methods. But there are uncertainties that cannot be appropriately treated by usual probabilistic models. To define inventory optimization tasks in such environment and to interpret optimal solution, fuzzy set theory is considered as more convenient than probability theory. Many researchers have done their research work in fuzzy inventory models by considering the parameters as fuzzy number and defined various methods to get optimum cost. In this research work we get better optimum solution when compared to previous available methods.

The structure of this paper is as follows: Section 2 gives preliminaries that are essential for our work. Section 3 discusses an inventory model in crisp and fuzzy sense. Section 4 compares crisp model and fuzzy model (triangular trident fuzzy number) using a numerical example. Section 5 gives sensitivity analysis. Finally section 6 concludes this research paper.

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Preliminaries

Definition

Let X be a nonempty set. Then a fuzzy set A in X (ie., a fuzzy subset A of X) is characterized by a function of the form μ_A : X $\rightarrow$ [0,1]. Such a function μ_A is called the membership function and for each xϵX , $\mu_{\tilde{A}}(x)$ is the degree of membership of x (membership grade of x) in the fuzzy set A.

In other words, A fuzzy set $\tilde{A} = \{(x, \mu_A(x))/ x\varepsilon X\}$ where μ_A: X $\rightarrow$ [0,1]. F(X) denotes the collection of all fuzzy sets in X, called the fuzzy power set of X.

Definition

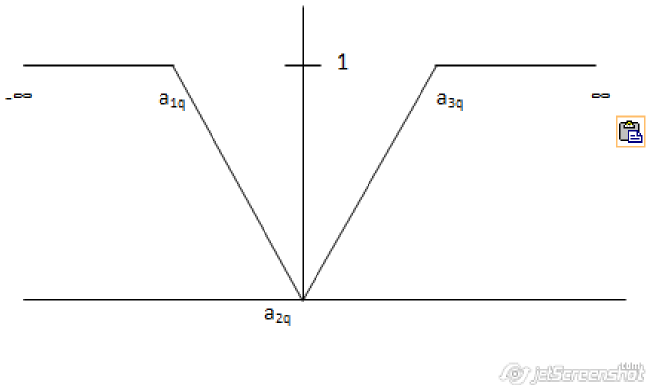
A fuzzy set is a fuzzy number if it satisfies the following four conditions

1. It is a convex set
2. It is normalised
3. It is defined on the real number R
4. It is piecewise continuous

Definition (Triangular trident Fuzzy Number)

A fuzzy number A =(a_{1q}, a_{2q}, a_{3q}) is called triangular trident fuzzy number if its membership function is given by

$$\mu_{A_q}(x)=\begin{cases} 1^{\frac{1}{3}}, & x<a_{1q} \\ \left(\dfrac{a_{2q}-x}{a_{2q}-a_{1q}}\right)^{\frac{1}{3}}, & a_{1q}\le x\le a_{2q} \\ 0, & x=a_{2q} \\ \left(\dfrac{x-a_{2q}}{a_{3q}-a_{2q}}\right)^{\frac{1}{3}}, & a_{2q}\le x\le a_{3q} \\ 1^{\frac{1}{3}}, & x>a_{3q} \end{cases}$$



Definition 2.4.(α-cut for triangular trident Fuzzy number)

The α-cut of triangular trident fuzzy number A_q is the closed interval A_{q_α} = $\left[a_{2_q}-\alpha^3\left(a_{2_q}-a_{1_q}\right),\ a_{2_q}+\alpha^3\left(a_{3_q}-a_{2_q}\right)\right]$, $\alpha\in[0,1]$.

Definition 2.5 (Signed distance method)

The formula for defuzzifying triangular trident fuzzy number using signed distance method is

$$R=\frac{1}{2}\int_0^1 a_2-\alpha^3(a_2-a_1)+a_2+\alpha^3(a_3-a_2)\,d\alpha$$

$$=\frac{1}{2}\left[a_2\alpha-\frac{\alpha^4(a_2-a_1)}{4}+a_2\alpha+\frac{\alpha^4(a_3-a_2)}{4}\right]_0^1$$

$$=\frac{a_1+2a_2+a_3}{8}$$

Definition (Graded Mean Integration Representation method)

The formula for defuzzifying using gradient mean integration representation method is

$$R=\frac{1}{2}\int_0^1 (a_2-\alpha^3(a_2-a_1)+a_2+\alpha^3(a_3-a_2))\;\alpha\,d\alpha$$

$$=\frac{1}{2}\left[a_2\frac{\alpha^2}{2}-\frac{\alpha^5}{5}(a_2-a_1)+a_2\frac{\alpha^2}{2}+\frac{\alpha^5}{5}(a_3-a_2)\right]_0^1$$

$$=\frac{a_1+3a_2+a_3}{10}$$

Inventory model in crisp and fuzzy sense

Assumptions

1. The inventory system involves production of single item.
2. Lead time is zero and shortages are not allowed.
3. Demand is time dependent.
4. Replenishment is instantaneous.

Notations

A - set up cost per cycle
$\tilde{A}$ - fuzzy set up cost
θ - deterioration rate independent of time
$\tilde{\theta}$ - fuzzy deterioration rate independent of time
T - cycle length
P - production rate
$\tilde{P}$ - fuzzy production rate
h - holding cost per unit per unit time
$\tilde{h}$ - fuzzy holding cost per unit per unit time
d - ` deterioration cost per unit per unit time
$\tilde{d}$ - fuzzy deterioration cost per unit per unit time
D - demand rate which depends exponentialy over time
$\tilde{D}$ - fuzzy demand rate
t_1 - duration of production
$I_1(t)$ - inventory level at time t, $0\le t\le t_1$
$I_2(t)$ - inventory level at time t, $t_1\le t\le T$
C - total cost for the period [0,T]
$\tilde{C}$ - fuzzy total cost for the period [0,T]
$d_F\tilde{C}$ - defuzzified value of $\tilde{C}$

Description of Inventory model in crisp sense

At t = 0, the inventory level is zero and it increases in $[0,t_1]$ due to the production at the constant rate P.

At t = T again it reaches the inventory level zero. This is due to demand and deterioration of the item. This can be represented by the following figure 1

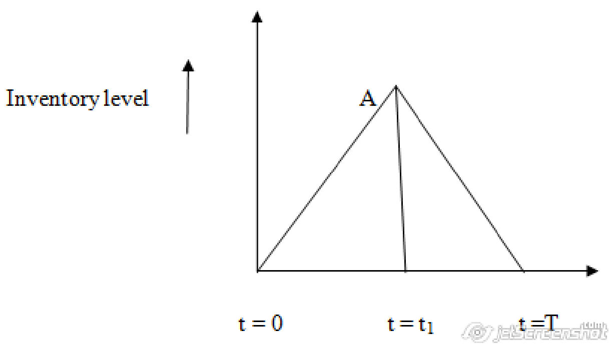


Figure 1

For $0 \le t \le t_1$

The differential equation governing the situation is

$$\frac{d}{dt} I_1(t) = P - D - \theta I_1(t)$$

$\frac{d}{dt} I_1(t) + \theta I_1(t) = P - Ke^{-\lambda t}$, where K is the initial demand and λ is the decreasing rate of demand. K > 0 and 0 < λ < θ

$$\frac{d}{dt} I_1(t) + \theta I_1(t) = P - Ke^{-\lambda t}$$

Now apply the initial condition $I_1(t) = 0$ when t = 0 we get

$$I_1(t) = \frac{K}{\theta - \lambda}\left(e^{-\theta t} - e^{-\lambda t}\right) + \frac{P}{\theta}(1 - e^{-\theta t})$$

For $t_1 \le t \le T$

The differential equation governing the above condition is

$\frac{d}{dt} I_2(t) + \theta I_2(t) = - Ke^{-\lambda t}$, where K is the initial demand and λ is the decreasing rate of demand. K > 0 and 0 < λ < θ.

The solution of the linear equation after applying the condition $I_2(t) = 0$ at t = T is

$$I_2(t) = \frac{K}{\theta - \lambda}\left(e^{\theta T - \lambda T - \theta t} - e^{-\lambda t}\right)$$

Holding cost can be calculated by using the formula

$$H.C = h\left[\int_0^{t_1} I_1(t)dt + \int_0^{t_2} I_2(t)dt\right]$$ we get

$$h\left[\frac{K}{\theta - \lambda}\left(\frac{1}{\theta} - \frac{1}{\lambda}\right)\left(1 - e^{-\lambda T}\right) + \frac{P}{\theta} t_1\right]$$

The deteriorating cost can be found out by the formula

$$DC = d\left[\int_0^{t_1} \theta I_1(t)dt + \int_{t_1}^{T} \theta I_2(t)dt\right]$$

$$= d\left[\frac{K\theta}{\theta - \lambda}\left(\frac{1}{\theta} - \frac{1}{\lambda}\right)\left(1 - e^{-\lambda T}\right) + Pt_1\right]$$

Total cost can be defined as

$$C = \frac{1}{T}\left(A + HC + DC\right)$$

$$Total\cos t = C = \frac{1}{T}\left(A + \frac{h + d\theta}{2} KT^2 - \frac{1}{2}(h + d\theta)\frac{K^2T^2}{P}\right) \text{------(1)}$$

[9]

The optimum value of T can be found out by differentiating with respect to T

$$\frac{\partial C}{\partial T} = \frac{-A}{T^2} + \frac{(h + d\theta)K}{2}\left(1 - \frac{K}{P}\right) \quad and \quad \frac{\partial^2 C}{\partial T^2} = \frac{2A}{T^3} > 0 .$$

Now equate the first derivative to zero to obtain optimum time period T^*

$$\frac{-A}{T^2} + \frac{(h + d\theta)K}{2}\left(1 - \frac{K}{P}\right) = 0$$

$$T^* = \sqrt{\frac{A}{\frac{(h + d\theta)K}{2}\left(1 - \frac{K}{P}\right)}} \text{------(2)}$$

Description of Inventory model in fuzzy sense

Triangular trident fuzzy number

It is not always possible to define certain parameters with certainty for which we fuzzify some parameters A , h, d , θ , P , D , K.

We consider triangular trident fuzzy numbers for the above parameters as

$\widetilde{A} = (a_1, a_2, a_3)$, $\widetilde{h} = (h_1, h_2, h_3)$ $\widetilde{d} = (d_1, d_2, d_3)$, $\widetilde{\theta} = (\theta_1, \theta_2, \theta_3)$, $\widetilde{P} = (p_1, p_2, p_3)$, $\widetilde{D} = (d_1, d_2, d_3)$, $\widetilde{K} = (k_1, k_2, k_3)$

Therefore, (1) becomes

$$C_i = \frac{1}{T}\left(A_i + \frac{h_i + d_i\theta_i}{2} K_i T^2 - \frac{1}{2}(h_i + d_i\theta_i)\frac{K_i^2 T^2}{P_{4-i}}\right), \ i = 1, 2, 3$$

To find the optimum value , we have to differentiate the above equation with respect to T

$$\frac{d}{dt}(C_i) = \frac{-A_i}{T^2} + \frac{1}{2}(h_i + d_i\theta_i)K_i - \frac{1}{2}(h_i + d_i\theta_i)\frac{K_i^2}{P_{4-i}}, \ i = 1,2,3$$ and $$\frac{d^2}{dt^2}(C_i) = \frac{2A_i}{T^3}, \ i = 1,2,3$$

Now defuzzifying using signed distance method

$$d_F\widetilde{C} = \frac{1}{8}\left(c_1 + 2c_2 + c_3\right)$$

$$\frac{d}{dt}\left(d_F\widetilde{C}\right) = \frac{1}{8}\left(\frac{dc_1}{dt} + 2\frac{dc_2}{dt} + \frac{dc_3}{dt}\right)$$

$$\frac{d^2}{dt}\left(d_F\widetilde{C}\right) = \frac{1}{8}\left[\frac{d^2c_1}{dt^2} + 2\frac{d^2c_2}{dt^2} + \frac{d^2c_3}{dt^2}\right]$$

$$= \frac{1}{8}\left[\frac{2a_1}{T^3} + \frac{2a_2}{T^3}2 + \frac{2a_3}{T^3}\right]$$

$$= \frac{a_1 + 2a_2 + a_3}{4T^3}$$ which is greater than zero .

Therefore we get the minimum total cost.

Now let us find optimum solution of total cost by putting

$$\frac{d}{dt}\left(d_F\widetilde{C}\right) = 0$$

$$\frac{1}{2}(h_1 + d_1\theta_1)k_1 + \frac{2}{2}(h_2 + d_2\theta_2)k_2 + \frac{1}{2}(h_3 + d_3\theta_3)k_3$$
$$- \frac{1}{2}(h_1 + d_1\theta_1)\frac{k_1^2}{p_3} - \frac{2}{2}(h_2 + d_2\theta_2)\frac{k_2^2}{p_2} - \frac{1}{2}(h_3 + d_3\theta_3)\frac{k_3^2}{p_1} = \frac{a_1 + 2a_2 + a_3}{T^2}$$

$$T = \frac{\sqrt{a_1 + 2a_2 + a_3}}{\sqrt{\frac{(h_1 + d_1\theta_1)k_1}{2}\left(1 - \frac{k_1}{p_3}\right) + 2(h_2 + d_2\theta_2)k_2\left(1 - \frac{k_2}{p_2}\right) + \frac{(h_3 + d_3\theta_3)k_3}{2}\left(1 - \frac{k_3}{p_1}\right)}} \quad (3)$$

we get the minimum total cost as

$$\text{Total cost} = \frac{1}{8}\left[\frac{a_1 + 2a_2 + a_3}{T} + T\frac{(h_1 + d_1\theta_1)k_1}{2}\left(1 - \frac{k_1}{p_3}\right) + T\frac{(h_2 + d_2\theta_2)k_2}{1}\left(1 - \frac{k_2}{p_2}\right) + T\frac{(h_3 + d_3\theta_3)k_3}{2}\left(1 - \frac{k_3}{p_1}\right)\right] \text{------------------(4)}$$

Similarly defuzzifying using graded mean integration method, we get the optimum time period and minimum total cost as

$$T = \frac{\sqrt{a_1 + 3a_2 + a_3}}{\sqrt{\frac{(h_1 + d_1\theta_1)k_1}{2}\left(1 - \frac{k_1}{p_3}\right) + \frac{3}{2}(h_2 + d_2\theta_2)k_2\left(1 - \frac{k_2}{p_2}\right) + \frac{(h_3 + d_3\theta_3)k_3}{2}\left(1 - \frac{k_3}{p_1}\right)}} \quad (5)$$

$$\text{Total cost} = \frac{1}{10}\left[\frac{a_1 + 2a_2 + a_3}{T} + T\frac{(h_1 + d_1\theta_1)k_1}{2}\left(1 - \frac{k_1}{p_3}\right) + T\frac{3(h_2 + d_2\theta_2)k_2}{2}\left(1 - \frac{k_2}{p_2}\right) + T\frac{(h_3 + d_3\theta_3)k_3}{2}\left(1 - \frac{k_3}{p_1}\right)\right] \text{--------------(6)}$$

Numerical Example

Crisp Model

Suppose A = 54, h = 8, θ = 0.010, P = 550, d = 1.5, K = 500

By using the formula (1) and (2), we get $T^* = 0.54$

And total cost C = 198.37

Fuzzy Model

Signed Distance method

A = (20,26,32), h = (4,6,8), θ=(0.002,0.006,0.010), P =(500,550,600), d =(1,1.3,1.6) K=(450,500,550)
Using (3) and (4) we get
T^*= 0.6 and total cost = 42.495

Graded mean Integration method

A = (20,26,32), h = (4,6,8), θ=(0.002,0.006,0.010), P =(500,550,600), d =(1,1.3,1.6) K=(450,500,550)
Using (5) and (6) we get the optimum solution as
T^*= 0.56 and total cost = 46.41

Sensitivity Analysis

The following table shows the effect of change of each parameter in (3),(4), (5) & (6)

Table 1

$\tilde{A}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(20,26,32)	0.56	46.41	0.6	42.50
(30,34,38)	0.6406	53.08	0.6997	48.59
(50,52,54)	0.7922	65.64	0.8654	60.09
(60,70,80)	0.9191	70.25	1.004	69.72

Table 2

$\tilde{d}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(1,1.6,2.2)	0.6406	53.08	0.6999	48.58
(1,1.3,1.6)	0.6406	53.08	0.6997	48.59
(1.3,1.7,2.1)	0.6405	53.08	0.6997	48.59
(2,2.5,3)	0.6404	53.09	0.6997	48.59

Table 3

$\tilde{\theta}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(0.002,0.006,0.010)	0.6406	53.08	0.6997	48.59
(0.003,0.006,0.009)	0.6405	53.09	0.6996	48.6
(0.005,0,010,0.015)	0.6403	53.10	0.6995	48.61
(0.008,0.015,0.022)	0.6401	53.12	0.6993	48.62

Table 4

$\tilde{K}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(450,500,550)	0.6406	53.08	0.6997	48.59
(400,425,450)	0.3592	94.66	0.3638	93.45
(350,410,470)	0.3554	95.65	0.3630	93.65
(200,230,260)	0.2935	115.83	0.2942	115.56

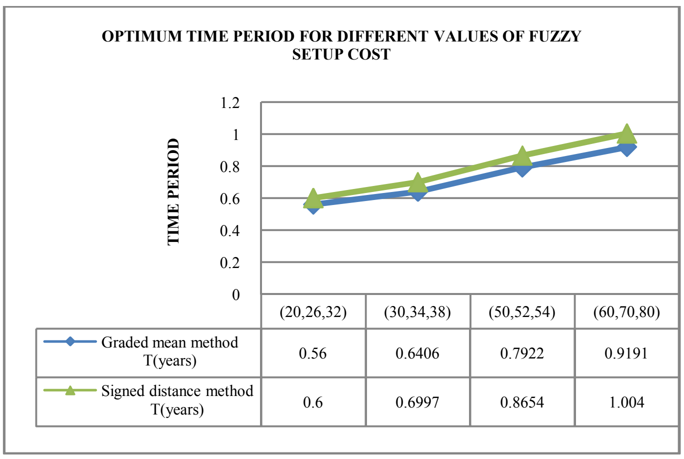
Table 5

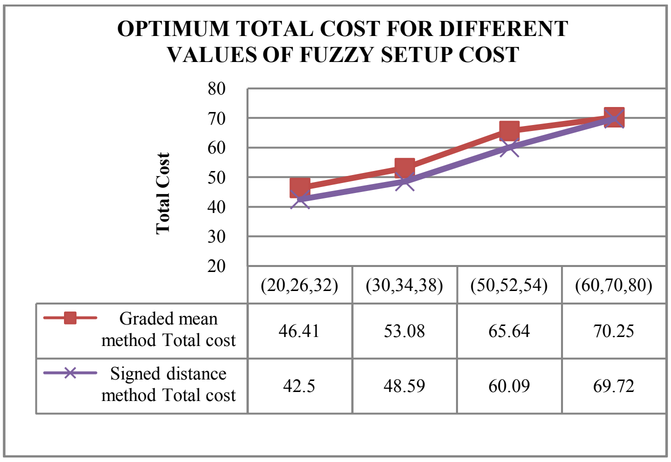
$\tilde{h}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(2,7,12)	0.8086	42.05	1.16	29.26
(4,6,8)	0.6406	53.08	0.6997	48.59
(3,9,15)	0.6777	50.17	0.9069	37.49
(10,14,18)	0.4077	83.38	0.4396	77.35

Table 6

$\tilde{P}$	Graded mean method		Signed distance method	
	T(years)	Total cost	T(years)	Total cost
(500,550,600)	0.6406	53.08	0.6997	48.59
(550,575,600)	0.4573	74.35	0.4695	72.41
(600,700,800)	0.3019	112.62	0.3078	110.48
(625,705,785)	0.2946	115.42	0.2989	113.75

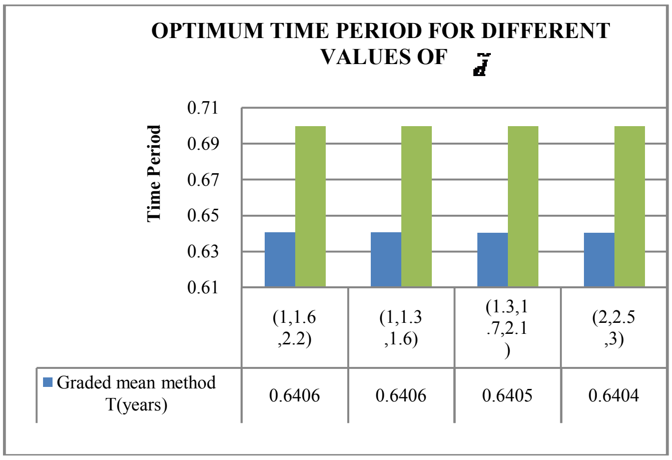
From the Table 1, we observe that increase in setup cost increases the time period and the total cost. Table 2 shows that if the deterioration cost increases then there is no rapid change in the time period and total cost.

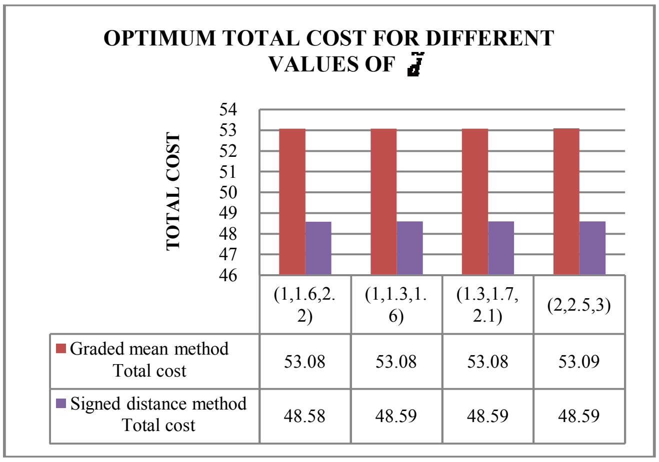




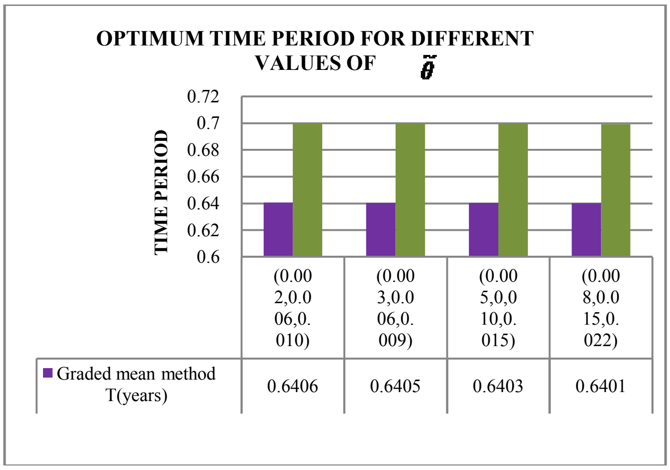
Graphical representation of **Table 1**

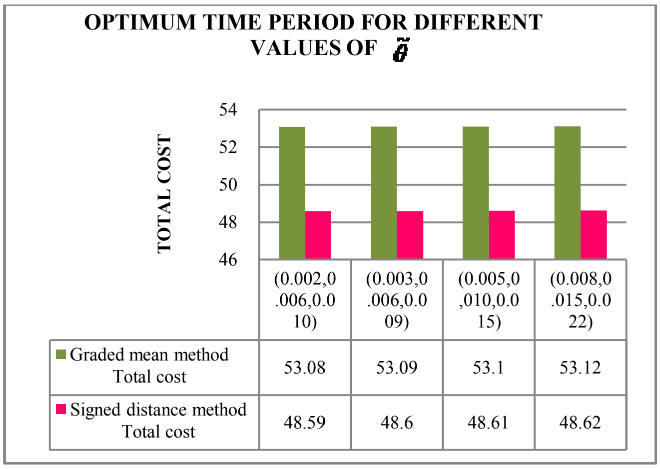
Since the deterioration rate is very minimum in our example problem, there is not much change in the time period and total cost if the deterioration rate of material increases which is depicted in Table 3. If the production is large while the demand is less then there is huge rise in total cost and the time period of one cycle is very minimum which is shown in Table 4 and Table 6. Table 5 shows that if the holding cost of material for placing in the inventory increases then the cost incurred for the cycle also increases.



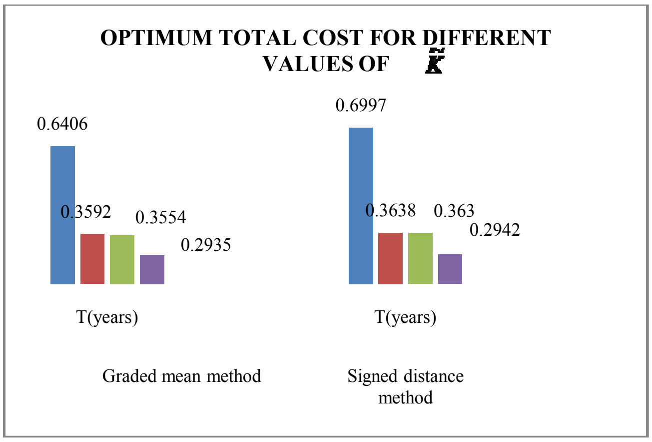


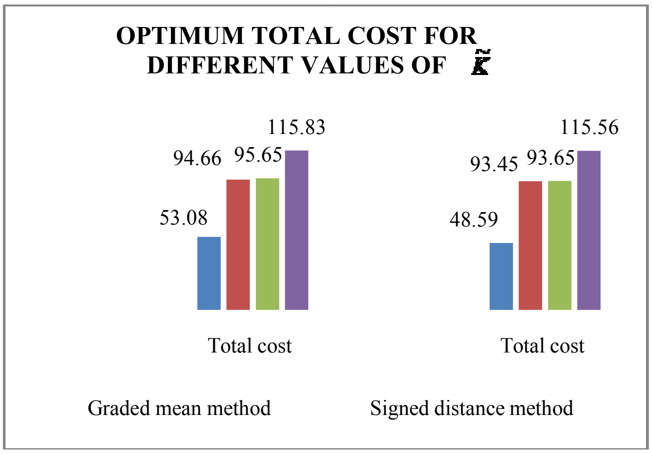
Graphical representation of **Table 2**



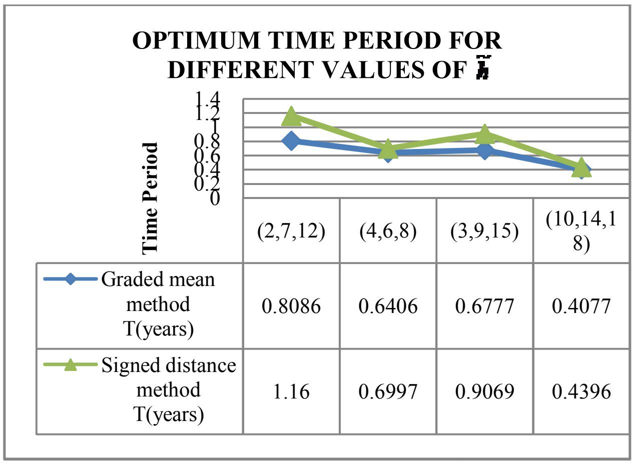


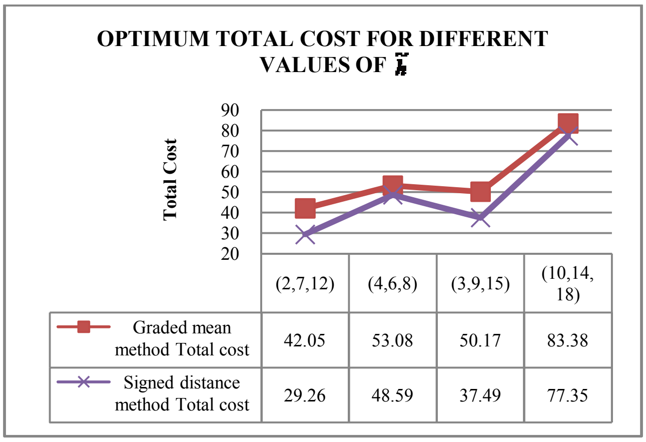
Graphical representation of **Table 3**



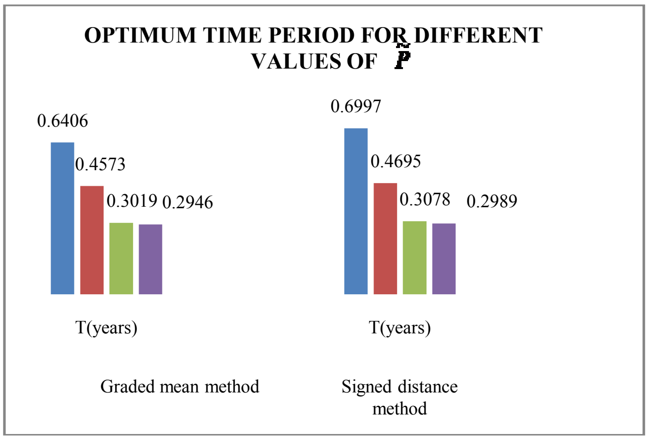


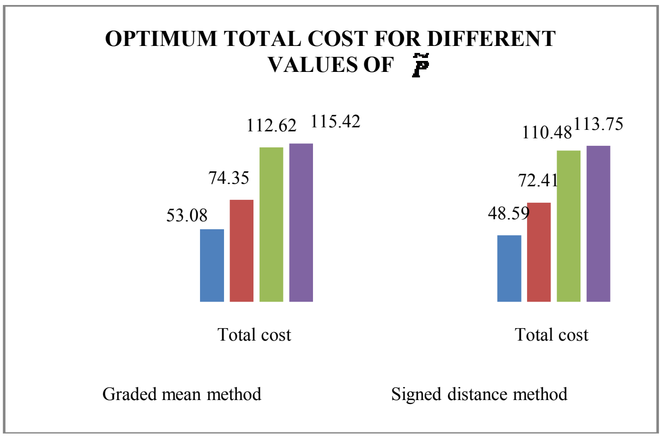
Graphical representation of **Table 4**





Graphical representation of **Table 5**





Graphical representation of **Table 6**

CONCLUSION

In this paper an inventory model is considered. The description for the model is given in crisp and fuzzy environment. The crisp model of the problem was already discussed in [9] . In Paper [9], the fuzzy numbers we used are triangular and trapezoidal fuzzy numbers. In our present paper, we used triangular trident fuzzy number. Signed distance method and graded mean integration method are used for defuzzification. We observe that, we obtain minimum total cost and optimum time period while using Triangular trident fuzzy number.

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