



**Research Article**

**MATHEMATICS AND CITIZENSHIP: A PROPOSAL FROM PROBLEM SOLVING**

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**ABSTRACT**

A study in development is presented. This research is descriptive and exploratory and is still underway in Chile. Its objective is to develop a method of evaluation in the field of the promotion of values by solving types of mathematical problems. It is part of a more extensive investigation that aims to contribute to educational integration in Latin America, in particular to the development of proposals to link education for citizenship and mathematics lessons. This has been done by research teams from Chile, Costa Rica, Argentina, Mexico and Spain, at the Andrés Bello Chair of the Association of Latin American Universities. This research continues to be developed and implemented in higher education, using mixed methods. It includes a test of resolution of types of mathematical problems with ethical values that were administered in a pilot way to the students. The first results show the lack of integration between the teaching of values and mathematics

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**INTRODUCTION**

Education for citizenship is part of the curricula of every country. It is given in schools, mainly through one of the following approaches: as an independent subject, as part of another subject or subject field, or as a cross-curricular objective. This multidimensional nature of citizenship is reflected in the curricula of European countries. Schools set objectives related to it, not only in terms of knowledge of its theory, but also in terms of skills to be mastered, and attitudes and values to be fostered Eurydice (2012). The Eurydice report (2018) on citizenship education at school in Europe is the third Eurydice report on this subject area. Citizenship education is also taught as a compulsory separate subject at each of the three levels of the general education pathway in Estonia, Greece, France and Finland. This has also been the case in Belgium (French Community) since the 2017/2018. Furthermore, the compulsory separate subject of citizenship education is combined with the cross-curricular and integrated approaches at one education level in Cyprus, Lithuania, Poland, Slovenia, Montenegro and Turkey, this combination was also used in Ireland until the 2016/2017 school year.

For Olivo (2017) the development of citizenship in the Latin American region goes through the recognition of values such as social development, gender equity, respect for diversity, protection of the environment, etc.

According Valero *et al.* (2015) in the recent discourse of many international organizations, mathematics is positioned as a primordial tool for citizenship in the context of globalization and economic competitiveness. The OECD (2004) refers to the role of mathematics as part of what it means to be informed and reflective citizens. However, for Barwell (2013) the orientation given to the teaching of mathematics is to interpret information and solve problems, instead of being elements for the formulation of a social or political criticism.

Education for citizenship will be more effective if it is supported by a school environment where students are given the opportunity of experiencing values and principles. One way to do this is linking it to mathematics problem solving. According to Callejo (2000), mastering mathematics for citizenship does not only involve knowing the mathematics language, facts, concepts, and algorithms, but also knowing more complex processes like the mathematization of situations and problem solving in context.

Considering the large amount of research in the field of mathematics education, according to Domite (2004) there is little research on the analysis of the training of teachers as a social subject of their own actions and for Vanegaand Jimenez (2015) the role of mathematics education in the development of cross curricular competencies. Learning values as a cross-curricular competency is a topic discussed lately it is not hard to find people who state that: values have been lost depending on Orozco (2006). According to Andreoli (2008), even though knowledge is always useful, it does not guarantee job opportunities or success. This study is important and necessary

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because it is an answer to a topic that is crucial through the educational system. It is also important because it has made the integration of mathematics problem solving and fostering values possible. It is not only about doing mathematics, but also using mathematics in context. In other words, the student it is not studying statistics or functions in isolation, but in situations where solutions can be given by considering values such as solidarity, participation, education for health, etc.

### **Theoretical Framework**

The development of citizenship in education that includes ethical value education from a cross-curricular and interdisciplinary point of view D'Ambrosio (2011) is conceptualized. To understand values education through mathematics, we start from the affective domain perspective related to achievement in mathematics. DeBellis and Goldin (2006) recognize values/moral/ethics as a sub-domain, thus creating a tetrahedral model of interaction between values, emotions, attitudes, and beliefs. Values, including ethics and morals, that ultimately refer to personal truths or commitments of individuals, according to Reis and Amado (2012).

For Bishop (1999), values and beliefs are not the same, even though both constructs are related, and there is plenty of research about beliefs in mathematics education, but little about values. He distinguishes three kinds of basic values in mathematics education: Values of mathematics, that corresponds to those that have been developed according to the particular culture; General educational values, linked to the norms of the society and the particular school; Mathematic educational values, present in the curriculum, textbooks, the classroom, etc., as a result of the other groups of values.

Studies of Nucci and Narvaez (2008), Arthur and Wilson (2010) and Lovat *et al.* (2011) reinforce the conceptual proposal and an empirical verification of the inherent link between the pedagogy of values and the holistic student welfare, including academic improvement, have increased in number and scope. In some of these studies there is relevant information from the research and projects related to the Australian Values Education Programme. Based on evidence, it was concluded that the pedagogy of values has the potential of having a positive effect in the overall learning environment of a school, which causes strengthening the student-teacher relationship, environment, attitudes and values in the classroom, attitudes and behavior of the students, knowing and recognizing students, as well as their accomplishments.

Benninga *et al.* (2006) used the California Academic Performance Index as a guide to show a correlation between the fostering of higher order values (character) and the improvement of academic performance. Davidson *et al.* (2010) explain and demonstrate that there is a similar correlation between performance character and moral character, because both are closely linked to the development of personality. Osterman (2010) presents more evidence of these combined effects by showing that it is the teacher who teaches quality contents in the context of effective pedagogy. He/she builds positive relationships to create strong relationships among students and through the values that arise there, academic performance is improved.

### **Problem solving and ethical values**

For Puig (1996) at the beginning, the study of problem solving was centered in the outcomes of the activities of the ones who

solved the problems, and in how was it possible to teach more effective problem solving methods. Afterwards, the focus is shifted to the problem solving process and the subject that solves the problem. Isoda (2007) quotes some authors like Poincaré (1908), Dewey (1910), Polya (1945), Kilpatrick (1985), Schoenfeld (1985) that, both from Mathematics and from Psychology, have been considered the pioneers of the theories that seek to describe the problem solving process. Clearly, problem solving is an important aspect of the curriculum, and even more important if the problems are related to a context. For this reason, we opted for the classification of types of problems of the authors Diaz and Poblete (2015, 2016).

### **Types of problems**

*Routine Problems* are those for which students know a pre-established routine to solve them.

- A routine problem under its context is defined as Real Problem if it occurs in real life and triggers the students' action in that situation.
- A routine problem under its context is defined as Realist Problem if it is one that is likely to occur. It is a simulation of reality or a part of reality.
- A routine problem under its context is defined as Fantastical Problem if it is one that originates in imagination and has no relation with reality.
- A routine problem under its context is defined as Purely Mathematical if it is exclusively referred to mathematical objectives: numbers, relationships and arithmetic operations, geometric figures, etc.

*Non-Routine Problems*: occur if a student does not know a pre-established answer, procedure or routine to find the answer.

Although the resolution of problems has been recognized in different educational reforms in Latin America. however, is it possible to affirm that teachers are accustomed to teaching their discipline considering concepts, procedures and values? based on this research question, and considering the fundamental transversal objectives of the Chilean curriculum, we formulated the following objective: design an evaluation method in the field of ethical value development, considering the resolution of types of problems in mathematics.

## **METHODOLOGY**

Mixed-methods were used, including the administration of a test of mathematics problem solving that included ethics and values. This was administered to students of the first course of Didactics of Mathematics of the Mathematics and Information Technology Teaching Program at Universidad de Los Lagos. Interviews that included situations involving values and questions related to knowledge of Fundamental Cross-curricular Objectives of the Chilean curriculum were also carried out. These were literally transcribed in research notes of each research session.

The test was designed, validated by expert judgment, and administered in order to know student's competence in problem solving and the extent to which secondary teachers of mathematics managed to foster and integrate the teaching of values with the contents of the discipline. Once the test was submitted to the pilot application, it was left with 12 mathematics problem solving situations that involved specific values like solidarity, comradeship, education for

health, environmental education, tolerance and respect, justice, teamwork, responsibility, respect towards nature, kindness, and self-care. These should be recognized by them through the problems that are presented. Only a first pilot application has been achieved el 2019, whose reliability measured by Alpha de Cronbach was 0.89. The first results show the lack of integration between the teaching of ethical values and problem solving in mathematics.

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