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Research Article

DOMINATION NUMBERS, CHROMATIC NUMBERS AND TOTAL DOMINATION NUMBERS OF ALL POWERS OF PATHS USING AN INTERVAL GRAPH G

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ABSTRACT

Interval graphs are rich in combinatorial structures and have drawn the attention of research for over number of years. They are extensively studied and revealed their practical relevance for modeling problems arising in the real world. A dominating set is used as a backbone for communication. Among various applications of the theory of domination, chromatic number and total domination the most often discussed is a communication network. The aim of this paper is to find the domination numbers, Chromatic numbers and Total domination numbers of all powers of paths using an interval graph G corresponding to an interval family I using an algorithm.

INTRODUCTION

Graphs are useful in enhancing the understanding of the organization and behavioral characteristics of computer system its rigor and mathematical elegance appear in problem solving. Problems in almost every conceivable discipline can be solved using graph models. Graph theory is the study of graphs, which are mathematical structures used to demonstrate pair wise relations between objects. Graph theory is too much old subject; even it is too much young due to its progressive applications in various fields like, Operations Research, Computer Science, Decision Theory, Game Theory, etc. Now-a-days the graph theory is used in communication system, internet, mobile, Computer design, Social networks, etc. And it has extensive applications in computer science, engineering science, mathematical science, physical science etc. So almost every real world problem can be composed by using graph theory. One of the beauties of Graph Theory is that it depends very little on the other branches of Mathematics. There are several reasons for the acceleration of interest in Graph Theory. One of the attractive features of Graph Theory is its inherent pictorial character.

Here we have taken an interval graph G corresponding to an interval family I. interval graphs have drawn the attention of many researchers for over 25 years.

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They have extensively been studied and revealed their particle relevance for modeling problems arising in the real world. Interval graphs found applications in Archaeology, Genetics, Ecology, Psychology, Traffic control, Computer scheduling, storage information, retrieval and electronic circuit design and a wide variety of algorithms have been developed. Interval graph have studied from both point of view theoretical as well as algorithmic.

The main focus of the paper is structured on the theory of domination and coloring. For finding these various types of dominating sets in different intervals we introduced the different types of algorithms. The theory of domination in graphs by Ore [1] and Berge [2] is an emerging area of research in graph theory. A vertex v in a graph G is said to dominate both itself and its neighbors, that is v dominates every vertex in its closed neighborhood N[v]. A dominating set is used as a backbone for communication. Among various applications of the theory of domination, chromatic number and total domination the most often discussed is a communication network. And also we have introduced a new graph coloring concept more exactly; we study the chromatic number on several classes of graphs as well as finding general parameters like domination number, chromatic number, and total domination.

Graph coloring problems are bottomed by frequency assignment in broadcast communication, traffic planning, register allocation problem, task assignment, fleet maintenance and much more. In the fundamental graph coloring problem two adjoining vertices are colored by various colors. This coloring problem is known as vertex coloring problem or optimal coloring problem. By imposing several conditions different graph coloring problems are raised by different researchers.

We need to make explicit our assumptions about the kinds of computer we expect the algorithm to be executed on. The assumptions we make can have important consequences with respect to how fast a problem can be solved. This has given scope to consider faster computers and the need for faster computers has increased in recent years. As a consequence there has been considerable interest in demising parallel algorithms for solving various computational problems.

Preliminaries

Let I = $\{I_{1,}\ I_{2,}\ I_{3,}\ldots\ldots I_n\}$ be an **interval family**, where each I_i is an interval on the real line and represented by $I_i = [a_i\ , b_i]$ for i = 1,2,3,….,n.

Here a_i is called the left end point and b_i is called the right end point of I_i without loss of generality we assume that all end points of the interval in I are distinct numbers between 1 and 2n. The intervals are labeled in the increasing order of their right end points. Two intervals I and j are said to intersect each other if they have nonempty intersection. Two intervals are said to overlap if they have nonempty intersection and neither one of them contains the other.

Let G= (V, E) be a graph G is called an **interval graph** if there is a one-to-one correspondence between V and I such that two vertices of G are joined by an edge in E if and only if their corresponding intervals in I intersect. The interval representation of G is denoted by I and the graph G is referred to as the IG for I. Also, it is to be noted that an interval I_k of I and a vertex v_k of V are one and the same. With no loss of generality, it is assumed that no two intervals share a same endpoint.

Let G be a graph and the **neighborhood** [6] of a vertex v in G is defined as the set of vertices adjacent with v (including v) and is denoted by nbd[v]. A set S of vertices in G is called a neighborhood set of G if G=$\bigcup_{v \in S} G[nbd[v]]$, where G[nbd[v]] is the vertex induced subgraph of G.

Let v_i be a vertex in a graph G then the degree $d_G(v)$ of the vertex v_i in G is the number of edges of G that incident with v_i. We denote the degree of a vertex v in G by $d_G(v)$= = $|N_G(v)|$. The minimum and **maximum degree** among all vertices of G are denoted by δ(G) and Δ(G), respectively.

Let G be a graph. A subset D of V is said to be a **dominating set** of G if every vertex in V\D is adjacent to a vertex in D [4][5]. The **domination number** of the graph G is the minimum cardinality of the dominating set in G denoted by γ(G).

A proper coloring of a graph G= (V, E) is a function from the vertices of the graph to a set of colors such that any two adjacent vertices have different colors [9]. Another way of saying is for every graph G, a vertex coloring is a mapping f: V (G) → {0, 1, 2, …..,} so that no two adjacent vertices get the same color and every vertex get one color. A p-coloring of a graph consist of p distinct colors and then the graph G is called p-colorable. For any graph G the lease number p which subsists a p-coloring of G is called the **Chromatic number** of the graph G and it is denoted by χ(G). If χ(G)=p then the graph is said to be p-chromatic [8].

Let G be a graph. A set 'S' of vertices in G is said to be a **total dominating set** of G if every vertex of G is adjacent to some vertex in 'S' and the subgraph induced by 'S' has no isolated vertices. The smallest cardinality of a total dominating set is called **total domination number** which is denoted by γ_t(G) [7].

In this paper we considered all graphs here are without loops and multiple edges, connected, finite and undirected graphs Let G= (V, E) be a graph the **order of graph** G is number of vertices in G that is |V|=n.

The **power of graph** [3] G is donated as G^k and defined as 'u' and 'v' are two vertices in G^k if the distance $d(u,v) \leq k$ then 'u', 'v' are adjacent vertices in G^k.

An open walk in which no vertex appears more than once is called a path. The number of edges present in a path is called its length. A graph G with 'n' vertices is said to be a **path** if v_i and v_{i+1} are adjacent for $1 \leq i \leq n-1$ denoted as P_n.

The k^{th} **power of path** of order 'n' is denoted as P_n^k and defined as v_i is adjacent to v_{i+1}, v_{i+2}, . . v_{i+k} for $1 \leq i \leq n-k$.

In this paper we are going to find Domination number, Chromatic number and Total Domination number of all powers of paths using order of path 'n' and power of path 'k' for all powers of paths of order 'n'.

Main theorems

Theorem 1: If I= {I_1, I_2, ….. I_n} be an interval family and G=P_n^k is an interval graph corresponding to an interval family 'I'. We consider first 2k+1 consecutive intervals as S_1 and another 2k+1 consecutive intervals as S_2 and so on then the domination number of G is γ(G)=$\left\lceil \frac{n}{2k+1} \right\rceil$ where 'n' is order of the path and 'k' is power of the path.

Proof: Let G=P_n^k be an interval graph corresponding to an interval family 'I' where I={I_1,I_2,….I_n}. Our aim to find the domination number of G=P_n^k using power of path 'k' and order of path 'n'. The k^{th} power of path of order n is corresponding to an interval family, intervals in such a way that I_i interval in I is adjacent to I_{i+1}, I_{i+2}, . . . I_{i+k} intervals for $1 \leq i \leq n-1$. Vertex v_i in G is adjacent to v_{i+1}, v_{i+2}, . . .v_{i+k} vertices for $1 \leq i \leq n-1$ in G.

We divide 'n' (finite) intervals into few disjoint sets. We consider first 2k+1 consecutive intervals as S_1 and another 2k+1 consecutive intervals as S_2 and so on. Each set contains 2k+1 consecutive intervals where 'k' is power of path. $(k+1)^{th}$ interval in first set I_{k+1} dominates I_1, I_2, . . I_k, I_{k+2}, . . . I_{2k+1} intervals. Corresponding to this v_{k+1} vertex in G dominates v_1, v_2, . . .v_k, v_{k+2}, . . . v_{2k+1} vertices in G. Middle interval in first set dominates all other intervals in that set. Similarly I_{3k+2}, dominates in second set. Likewise in all other sets.

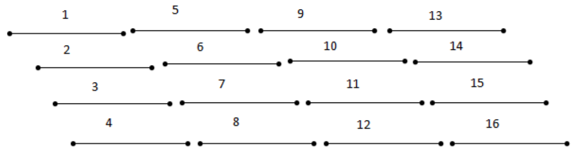
V_{k+1}, v_{3k+2}, . . .∈D

In each set middle interval ∈D we are choosing the elements for D such a way that those elements are not connected. Since sets are disjoint. From each set we are taking one interval namely middle interval. So the domination number of the graph G is equal to the number of sets that is $\left\lceil\frac{n}{2k+1}\right\rceil$ which is equal to $\left\lceil\frac{n}{\Delta+1}\right\rceil$.

Algorithm

Step 1 start
Step 2 D=ϕ
Step 3 max{nbd[v_1]}=v_{k+1}∈D
Step 4 max{nbd[v_{k+1}]}=v_{2k+1}
Step 5 max{nbd[v_{2k+2}]}=v_{3k+2}∈D
Step 6 repeat steps 4 and 5
Step 7 in repeating if v_n in 4^{th} step, stop
If v_n in 5^{th} step v_n∈D and stop

Experimental problem 1: Find Domination number of G=P_{16}^3



Interval Family I

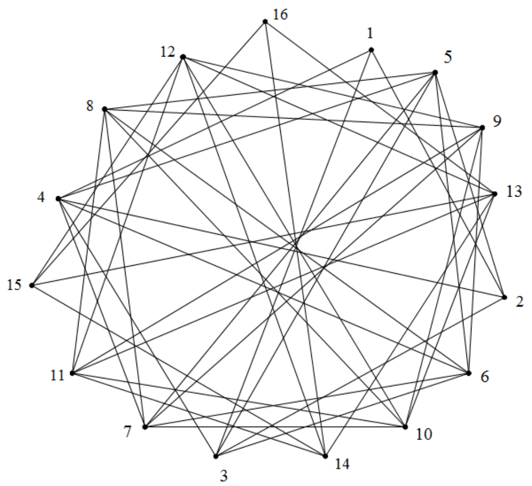


Fig 1 Interval graph G=P_{16}^3

Domination number of G=P_{16}^3

Here order of the path n=16
Power of the path k=3
Maximum degree of the path Δ=6

Domination number of G is γ(G) =$\left\lceil\frac{n}{2k+1}\right\rceil$ which is equal to $\left\lceil\frac{n}{\Delta+1}\right\rceil$

$\left\lceil\frac{16}{2(3)+1}\right\rceil=\left\lceil\frac{16}{6+1}\right\rceil = \left\lceil\frac{16}{7}\right\rceil=3$

Domination number of G is γ(G) =3

Find the closed neighborhood of v_i nbd[v_i] and Maximum of nbd[v_i] for $1 \le i \le 16$ to the graph G=P_{16}^3

nbd[v_1]={v_1,v_2,v_3,v_4 Max{nbd[v_1]}=v_4
nbd[v_2]={v_1,v_2,v_3,v_4,v_5} Max{nbd[v_2]}=v_5
nbd[v_3]={v_1,v_2,v_3,v_4,v_5,v_6} Max{nbd[v_3]}=v_6
nbd[v_4]={$v_1,v_2,v_3,v_4,v_5,v_6,v_7$} Max{nbd[$v_4$]}=$v_7$
nbd[v_5]={$v_2,v_3,v_4,v_5,v_6,v_7,v_8$} Max{nbd[$v_5$]}=$v_8$
nbd[v_6]={$v_3,v_4,v_5,v_6,v_7,v_8,v_9$} Max{nbd[$v_6$]}=$v_9$
nbd[v_7]={$v_4,v_5,v_6,v_7,v_8,v_9,v_{10}$} Max{nbd[$v_7$]}=$v_{10}$
nbd[v_8]={$v_5,v_6,v_7,v_8,v_9,v_{10},v_{11}$} Max{nbd[$v_8$]}=$v_{11}$
nbd[v_9]={$v_6,v_7,v_8,v_9,v_{10},v_{11},v_{12}$} Max{nbd[$v_9$]}=$v_{12}$
nbd[v_{10}]={$v_7,v_8,v_9,v_{10},v_{11},v_{12},v_{13}$} Max{nbd[$v_{10}$]}=$v_{13}$
nbd[v_{11}]={$v_8,v_9,v_{10},v_{11},v_{12},v_{13},v_{14}$} Max{nbd[$v_{11}$]}=$v_{14}$
nbd[v_{12}]={$v_9,v_{10},v_{11},v_{12},v_{13},v_{14},v_{15}$} Max{nbd[$v_{12}$]}=$v_{15}$
nbd[v_{13}]={$v_{10},v_{11},v_{12},v_{13},v_{14},v_{15},v_{16}$} Max{nbd[$v_{13}$]}=$v_{16}$
nbd[v_{14}]={$v_{11},v_{12},v_{13},v_{14},v_{15},v_{16}$} Max{nbd[$v_{14}$]}=$v_{16}$
nbd[v_{15}]={$v_{12},v_{13},v_{14},v_{15},v_{16}$} Max{nbd[$v_{15}$]}=$v_{16}$
nbd[v_{16}]={$v_{13},v_{14},v_{15},v_{16}$} Max{nbd[$v_{16}$]}=$v_{16}$

Algorithm verification for Domination number of G=P_{16}^3

Step 1 start
Step 2 D=ϕ
Step 3 max{nbd[v_1]}=v_4∈D
Step 4 max{nbd[v_4]}=v_7
Step 5 max{nbd[v_8]}=v_{11}∈D
Step 6 max{nbd[v_{11}]}=v_{14}
Step 7 max{nbd[v_{15}]}=v_{16}∈D
Step 8 D= {v_4, v_{11}, v_{16}}
Step 9 stop

Dominating set of Graph G is D= {v_4, v_{11}, v_{16}}
Domination number of G is γ(G) =3

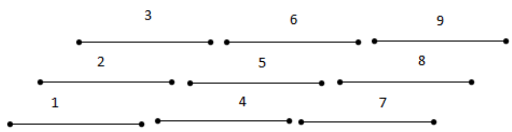
Our algorithm is verified true for Domination number of G=P_{16}^3

Theorem 2: If I= {I_1, I_2, …. I_n} be an interval family and G=P_n^k is an interval graph corresponding to an interval family 'I' then the Chromatic number of G is χ(G)=k+1 where 'K' is power of the path.

Proof: Let G=P_n^k be an interval graph corresponding to an interval family 'I' where I={$i_1,i_2,\ldots.i_n$}. Our aim to find the Chromatic number of G=P_n^k using power of path 'k'. k^{th} power of path of order n is corresponding to an interval family I, intervals in such a way that i_i interval in I is adjacent to i_{i+1}, i_{i+2}, . . . i_{i+k} intervals. Vertex v_i in G is adjacent to v_{i+1}, v_{i+2}, . . .v_{i+k} vertices in G.

We divide 'n' (finite) intervals into few disjoint sets. Consider each set contains k+1 consecutive interval where 'k' is power of path. Give colors 1, 2, …, k+1 to v_1, v_2. . . .v_{k+1} respectively in first set. Repeat the colors in same order in all other sets. By definition of k^{th} power of path v_1 is adjacent to v_2, v_3, . . .v_{k+1} only for this reason we should give k+1 different colors and v_1 is not adjacent to v_{k+2} so we can repeat the color 1 similarly other colors in the same order. Here we need minimum colors to color all the vertices of G is k+1 so the chromatic number of G=P_n^k is χ(G)=k+1.

Experimental problem 2: Find Chromatic number of G=P_9^2



Interval Family I

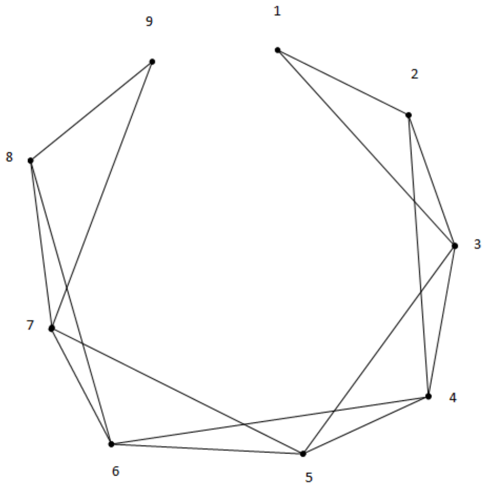


Fig 2 Interval graph G=P_9^2

Chromatic number of G=P_9^2

Vertex v_1 is adjacent to vertices v_2 and v_3 not adjacent to any other vertex in the graph. So we need at least 3 different colors to color the graph such a way that no two adjacent vertices get the same color. We can repeat the 3 colors in the same order so that we will get different colors for adjacent vertices in the graph.

By the definition of Chromatic number of a graph for every graph G, a vertex coloring is a mapping f: V(G) → {0, 1, 2, …..,} so that no two adjacent vertices get the same color and every vertex get one color. A p-coloring of a graph consist of p distinct colors and then the graph G is called p-colorable. For any graph G the least number p which subsists a p-coloring of G is called the Chromatic number of the graph G and it is denoted by χ(G). If χ(G)=p then the graph is said to be p-chromatic.

So the chromatic number of G=P_9^2 is χ(G)=3

Verification of our formula for the Chromatic number of G=P_9^2

Here order of the path n=9
Power of the path k=2
Chromatic number of G is χ(G)=k+1 that is χ(G)=2+1=3

Our formula is verified true for the Chromatic number of G=P_9^2

Theorem 3: If I= {I_1, I_2, …..I_n} be an interval family and G=P_n^k is an interval graph corresponding to an interval family 'I' We consider first k+1 consecutive intervals as S_1 and another k+1 consecutive intervals as S_2 and so on with a condition that last interval of S_i is first interval of S_{i+1} then the total domination number of G is γ_t(G)=P-1 for n≤PK+1 where P∈N, 'n' is order of the path and 'K' is power of the path.

Proof: Let G=P_n^k be an interval graph corresponding to an interval family 'I' where I={i_1,i_2,….i_n}. Our aim to find the total domination number of G=P_n^k using power of path 'k' and order of path 'n'. k^{th} power of path of order n is corresponding to an interval family, intervals in such a way that i_i interval in I is adjacent to i_{i+1}, i_{i+2}, . . . i_{i+k} intervals. Vertex v_i in G is adjacent to v_{i+1}, v_{i+2}, . . .v_{i+k} vertices in G.

We divide 'n' (finite) intervals into few sets. We consider first k+1 consecutive intervals as S_1 and another k+1 consecutive intervals as S_2 and so on Such a way that last interval of one set is first interval in next set. Each set contains k+1 consecutive intervals where 'k' is power of path. By the definition of k^{th} power of path and our consideration of sets the intervals in intersection of two sets are adjacent and dominate all other intervals. Corresponding vertices in G=P_n^k are adjacent and dominates all other vertices in G. v_{k+1}, v_{2k+1}, . . . which satisfies total domination. Choose P∈N such that n≤PK+1 where 'n' is order of the path and 'K' is power of the path. Then the total domination number of G=P_n^k is γ_t(G)=P-1.

Algorithm

Step 1 start
Step 2 D=ϕ
Step 3 max{nbd[v_1]}=v_{k+1}∈D
Step 4 max{nbd[v_{k+1}]}=v_{2k+1}∈D
Step 5 repeat step 4
Step 6 in repeating if v_n is adjacent to v_i where v_i∈D stop.

Experimental problem 3: Find Total domination number ofG=P_{15}^2

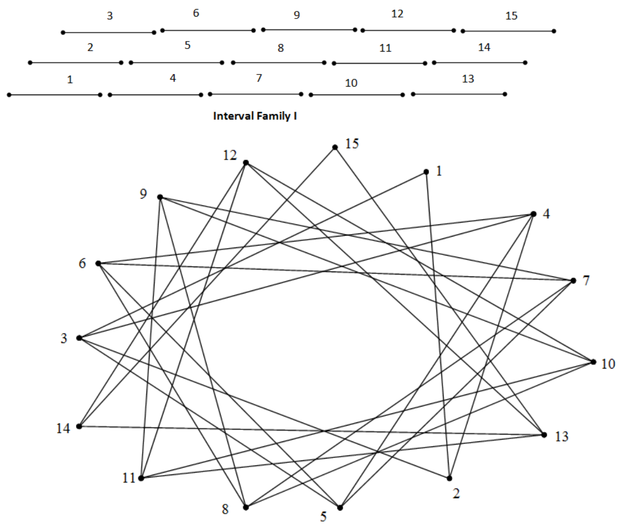


Fig 3 Interval graph G=P_{15}^2

Total Domination number of G=P_{15}^2

Here order of the path n=15
Power of the path k=2

$\frac{n-1}{k} \leq P$ Where P∈N

$\frac{15-1}{2} = \frac{14}{2} = 7,$ P=7

Total domination number of G=P_{15}^2 is γ_t(G)=P-1 that is 7-1=6
γ_t(G)=6

Find the closed neighborhood of v_i nbd[v_i] and Maximum of nbd[v_i] for $1 \leq i \leq 15$ to the graph G=P_{15}^2

nbd[v_1]={v_1,v_2,v_3,} Max{nbd[v_1]}=v_3
nbd[v_2]={v_1,v_2,v_3,v_4} Max{nbd[v_2]}=v_4
nbd[v_3]={v_1,v_2,v_3,v_4,v_5} Max{nbd[v_3]}=v_5

$nbd[v_4]=\{v_2, v_3, v_4, v_5, v_6\}$ Max$\{nbd[v_4]\}=v_6$
$nbd[v_5]=\{v_3, v_4, v_5, v_6, v_7\}$ Max$\{nbd[v_5]\}=v_7$
$nbd[v_6]=\{v_4, v_5, v_6, v_7, v_8\}$ Max$\{nbd[v_6]\}=v_8$
$nbd[v_7]=\{v_5, v_6, v_7, v_8, v_9\}$ Max$\{nbd[v_7]\}=v_9$
$nbd[v_8]=\{v_6, v_7, v_8, v_9, v_{10}\}$ Max$\{nbd[v_8]\}=v_{10}$
$nbd[v_9]=\{v_7, v_8, v_9, v_{10}, v_{11}\}$ Max$\{nbd[v_9]\}=v_{11}$
$nbd[v_{10}]=\{v_8,v_9, v_{10}, v_{11}, v_{12}\}$ Max$\{nbd[v_{10}]\}=v_{12}$
$nbd[v_{11}]=\{v_9,v_{10},v_{11}, v_{12}, v_{13}\}$ Max$\{nbd[v_{11}]\}=v_{13}$
$nbd[v_{12}]=\{v_{10},v_{11},v_{12}, v_{13}, v_{14}\}$ Max$\{nbd[v_{12}]\}=v_{14}$
$nbd[v_{13}]=\{v_{11}, v_{12}, v_{13}, v_{14}, v_{15}\}$ Max$\{nbd[v_{13}]\}=v_{15}$
$nbd[v_{14}]=\{v_{12}, v_{13}, v_{14}, v_{15}\}$ Max$\{nbd[v_{14}]\}=v_{15}$
$nbd[v_{15}]=\{v_{13},v_{14}, v_{15}\}$ Max$\{nbd[v_{15}]\}=v_{15}$

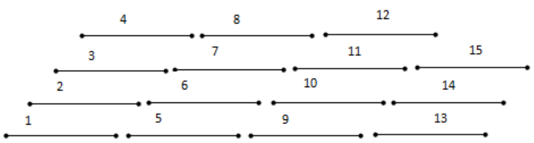
Algorithm verification for Total Domination number of G=P_{15}^2

Step 1 start
Step 2 D=ϕ
Step 3 max$\{nbd[v_1]\}=v_3 \in D$
Step 4 max$\{nbd[v_3]\}=v_5 \in D$
Step 5 max$\{nbd[v_5]\}=v_7 \in D$
Step 6 max$\{nbd[v_7]\}=v_9 \in D$
Step 7 max$\{nbd[v_9]\}=v_{11} \in D$
Step 8 max$\{nbd[v_{11}]\}=v_{13} \in D$
Step 9 max$\{nbd[v_{13}]\}=v_{15}$
Step 10 D= $\{v_3, v_5, v_7, v_9, v_{11}, v_{13}\}$
Step 11 stop.

Total Dominating set of Graph G is D= $\{v_3, v_5, v_7, v_9, v_{11}, v_{13}\}$
Total Domination number of G is γ_t(G)=6

Our algorithm is verified true the Total domination number of G=P_{15}^2

Experimental problem 4: Find Domination number, Chromatic number and Total domination number of G=P_{15}^3



Interval Family I

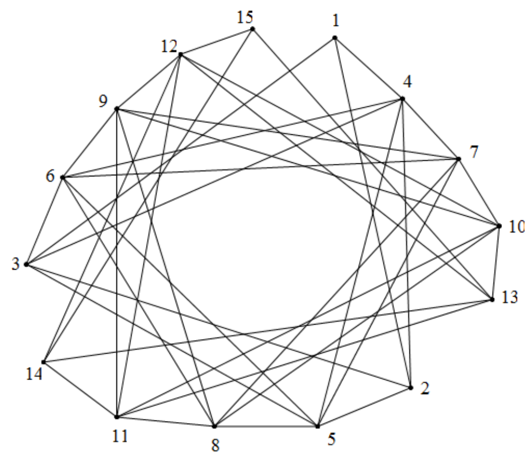


Fig 4 Interval graph G=P_{15}^3

Domination number of G=P_{15}^3

Here order of the path n=15
Power of the path k=3
Maximum degree of the path Δ=6

Domination number of G is γ(G) $=\left\lceil\frac{n}{2k+1}\right\rceil$ which is equal to$\left\lceil\frac{n}{\Delta+1}\right\rceil$

$\left\lceil\frac{15}{2(3)+1}\right\rceil=\left\lceil\frac{15}{6+1}\right\rceil = \left\lceil\frac{15}{7}\right\rceil=3$

Domination number of G is γ(G) =3

Find the closed neighborhood of v_i nbd[v_i] and Maximum of nbd[v_i] for $1 \le i \le 16$ to the graph G=P_{15}^3

$nbd[v_1]=\{v_1,v_2,v_3,v_4\}$ Max$\{nbd[v_1]\}=v_4$
$nbd[v_2]=\{v_1,v_2,v_3,v_4,v_5\}$ Max$\{nbd[v_2]\}=v_5$
$nbd[v_3]=\{v_1,v_2,v_3,v_4,v_5,v_6\}$ Max$\{nbd[v_3]\}=v_6$
$nbd[v_4]=\{v_1,v_2,v_3,v_4,v_5,v_6,v_7\}$ Max$\{nbd[v_4]\}=v_7$
$nbd[v_5]=\{v_2,v_3,v_4,v_5,v_6,v_7,v_8\}$ Max$\{nbd[v_5]\}=v_8$
$nbd[v_6]=\{v_3,v_4,v_5,v_6,v_7,v_8,v_9\}$ Max$\{nbd[v_6]\}=v_9$
$nbd[v_7]=\{v_4,v_5,v_6,v_7,v_8,v_9,v_{10}\}$ Max$\{nbd[v_7]\}=v_{10}$
$nbd[v_8]=\{v_5,v_6,v_7,v_8,v_9,v_{10},v_{11}\}$ Max$\{nbd[v_8]\}=v_{11}$
$nbd[v_9]=\{v_6,v_7,v_8,v_9,v_{10},v_{11},v_{12}\}$ Max$\{nbd[v_9]\}=v_{12}$
$nbd[v_{10}]=\{v_7,v_8,v_9,v_{10},v_{11},v_{12},v_{13}\}$ Max$\{nbd[v_{10}]\}=v_{13}$
$nbd[v_{11}]=\{v_8,v_9,v_{10},v_{11},v_{12},v_{13},v_{14}\}$ Max$\{nbd[v_{11}]\}=v_{14}$
$nbd[v_{12}]=\{v_9,v_{10},v_{11},v_{12},v_{13},v_{14},v_{15}\}$ Max$\{nbd[v_{12}]\}=v_{15}$
$nbd[v_{13}]=\{v_{10},v_{11},v_{12},v_{13},v_{14},v_{15}\}$ Max$\{nbd[v_{13}]\}=v_{15}$
$nbd[v_{14}]=\{v_{11},v_{12},v_{13},v_{14},v_{15}\}$ Max$\{nbd[v_{14}]\}=v_{15}$
$nbd[v_{15}]=\{v_{12},v_{13},v_{14},v_{15}\}$ Max$\{nbd[v_{15}]\}=v_{15}$

Algorithm verification for Domination number of G=P_{15}^3

Step 1 start
Step 2 D=ϕ
Step 3 max$\{nbd[v_1]\}=v_4 \in D$
Step 4 max$\{nbd[v_4]\}=v_7$
Step 5 max$\{nbd[v_8]\}=v_{11} \in D$
Step 6 max$\{nbd[v_{11}]\}=v_{14}$
Step 7 max$\{nbd[v_{15}]\}=v_{15} \in D$
Step 8 D= $\{v_4, v_{11}, v_{15}\}$
Step 9 stop

Dominating set of Graph G is D= $\{v_4, v_{11}, v_{15}\}$
Domination number of G is γ(G) =3

Our algorithm is verified true for Domination number of G=P_{15}^3

Chromatic number of G=P_{15}^3

Vertex v_1 is adjacent to vertices v_2, $v_3$and $v_4$not adjacent to any other vertex in the graph. So we need at least 4 different colors to color the graph such a way that no two adjacent vertices get the same color. We can repeat the 4 colors in the same order so that we will get different colors for adjacent vertices in the graph.

By the definition of Chromatic number of a graph for every graph G, a vertex coloring is a mapping f:V(G)→{0,1,2,…..,} so that no two adjacent vertices get the same color and every vertex get one color. A p-coloring of a graph consist of p distinct colors and then the graph G is called p-colorable. For any graph G the least number p which subsists a p-coloring of G is called the Chromatic number of the graph G and it is denoted by χ(G). If χ(G)=p then the graph is said to be p-chromatic.

So the chromatic number of G=P_{15}^3 is χ(G)=4

Verification of our formula for the Chromatic number of G=P_{15}^{3}

Here order of the path n=15
Power of the path k=3
Chromatic number of G is χ(G)=k+1 that is χ(G)=3+1=4

Our formula is verified true for the Chromatic number of G=P_{15}^{3}

Total Domination number of G=P_{15}^{3}

Here order of the path n=15
Power of the path k=3

$\frac{n-1}{k} \leq P$ Where P∈N

$$\frac{15-1}{3}=\frac{14}{3}<5$$

P=5

Total domination number of G=P_{15}^{3} is γ_t(G)=P-1 that is 5-1=4

γ_t(G)=4

Algorithm verification for Total Domination number of G=P_{15}^{3}

Step 1 start
Step 2 D=ϕ
Step 3 max{nbd[v_1]}=v_4∈D
Step 4 max{nbd[v_4]}=v_7∈D
Step 5 max{nbd[v_7]}=v_{10}∈D
Step 6 max{nbd[v_{10}]}=v_{13}∈D
Step 7 max{nbd[v_{13}]}=v_{15}
Step 8 D= {v_4, v_7, v_{10}, v_{13}}
Step 9 stop.

Total Dominating set of Graph G is D= {v_4, v_7, v_{10}, v_{13}}
Total Domination number of G is γ_t(G)=4

Our algorithm is verified true the Total domination number of G=P_{15}^{3}

CONCLUSION

In this papers we find the domination number, chromatic number and total domination number of all powers of paths using an interval graph G with an algorithm and formulated as γ(G)=$\left\lceil\frac{n}{2k+1}\right\rceil$ which is equal to $\left\lceil\frac{n}{\Delta+1}\right\rceil$, χ(G)=k+1 and γ_t(G)=P-1 for n≤PK+1 respectively. Where P∈N, 'n' is order of the path and 'K' is power of the path. Further we will try to work for all powers of cycles using circular-arc graph G.

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