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Research Article

A CLASS OF TESTS FOR TESTING PARAMETRIC REGRESSION AGAINST NONPARAMETRIC ALTERNATIVES

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A B S T R A C T

A class of tests based on adaptive varying kernel regression estimator is proposed for testing parametric regression against nonparametric regression. The tests are integrated squared error functions of adaptive Demir-Toktamiş estimator and conditional expectation of regression function obtained from adaptive varying kernel density estimator. Properties of the proposed tests and their asymptotic distributions are derived. Their performances in terms of empirical power are obtained and are compared with existing tests in the literature. Application of the tests is provided through a real data.

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INTRODUCTION

Nonparametric regression is a widely used data analytic tool since it does not assume a predetermined form for the function of the predictor. Suppose a random sample of size n from a bivariate population having density $f(x,y)$ is taken whose relation is modeled and given by

$$Y_i = m(X_i) + \varepsilon_i, i = 1,2,\dots,n \tag{1}$$

where $m(.)$ is unknown regression function, $m(x) = E(Y|X = x)$ and ε_i are independent and identically distributed (iid) random errors with mean zero and finite variance σ^2.

Nonparametric regression employs smoothing techniques to estimate the regression function $m(x)$. One of such techniques based on varying badwidth is due to Demir and Toktamiş (2010) which is adaptive Nadaraya - Watson (NW) estimator studied by Nadaraya (1964) and Watson (1964). Further, Abramson (1982), Silverman (1986), Aljuhani and Al Turk (2014, AA estimator), Joshi and Deshpande (2016), Deshpande and Bhat (2019) and Bhat and Deshpande (2019b) carry out study on adaptive varying kernel regression estimators. Tests due to Härdle and Mammen (1993), Stute (1997), Koul and Ni (2004) and Bhat and Deshpande (2019a) based on kernel regression estimator are developed for testing parametric regression against nonparametric regression. González-Manteiga and Crujeiras (2013) contains discussion on various tests for nonparametric regression.

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Test due to Härdle and Mammen (1993) based on Nadaraya-Watson (NW) estimator is given by

$$T = nh^{\frac{1}{2}} \int \left(\widehat{m}_h - \hat{l}_h\right)^2 \pi(x) f(x) dx, \tag{2}$$

where $\hat{l}_h(x) = \frac{\sum_{i=1}^{n} K_h(x-X_i) m_{\hat{\theta}}(x)}{\sum_{i=1}^{n} K_h(x-X_i)}$, is conditional expectation of $\widehat{m}_h$, estimated by pilot density,

$$\tilde{f}(x) = \frac{1}{n}\sum_{i=1}^{n} K_h(x - X_i), \tag{3}$$

$\pi(x)$ is a weight function, $f(x)$ is true density at $x, \widehat{m}_h(x)$ is NW kernel regression estimator with fixed bandwidth h given by

$$\widehat{m}_h(x) = \frac{\sum_{i=1}^{n} K_h(x-X_i) Y_i}{\sum_{i=1}^{n} K_h(x-X_i)}, \tag{4}$$

$K_h(x - X_i) = \frac{1}{h} K(\frac{x-X_i}{h})$ is a kernel function with fixed bandwidth h and $m_{\hat{\theta}}(x)$ is parametric estimate at x, $\hat{\theta}$ being least square estimate.

The alternative form of T is given by

$$T = h^{1/2} \sum_{i=1}^{n} \left(\widehat{m}_h(x_i) - \hat{l}_h(x_i)\right)^2 \pi(x). \tag{5}$$

For mathematical simplicity, $\pi(x)$ is taken to be unity by Härdle and Mammen (1993) while obtaining properties of T. Bhat and Deshpande (2019a) modified T-test by replacing $\widehat{m}_h$ by $\widehat{m}_{h_*}$ where h_* is varying bandwidth based on functions of range. In the present work, we propose a class of tests based on ISE of $\widehat{m}_{h_*}$ and conditional expectation of $\widehat{m}_{h_*}$ estimated by the density estimator based on varying kernel regression estimator obtained from the same function using which $\widehat{m}_{h_*}$ is proposed. We take various statistical functions to define $\widehat{m}_{h_*}$ which is also adaptive NW and adaptive AA estimators and modify the tests due to Bhat and Deshpande (2019a) by considering $(\widehat{m}_{h_*} - \hat{l}_{h_*})^2$ instead of $(\widehat{m}_{h_*} - \hat{l}_h)^2$.

we propose a class of tests for testing parametric regression against nonparametric regression and we derive its distribution in. Performance of the class of tests in terms of empirical power is studied through simulation Application of the tests is illustrated and conclusions are recorded.

A Class of Tests

Suppose we are testing

$$H_0: m(x) = m_{\theta_0}(x) \quad \text{against} \quad H_1: m(x) = m_\theta(x) > m_{\theta_0}(x), \tag{6}$$

where $m_{\theta_0}(x)$ is a parametric regression function and $m_\theta(x)$ is nonparametric regression function. We propose a test

$$A_j = n(h_{*j})^{\frac{1}{2}} \int \left(\widehat{m}_{h_{*j}} - \hat{l}_{h_{*j}}\right)^2 \pi(x) f(x) dx, \quad j = 1,2,\ldots,6 \tag{7}$$

where

$$\widehat{m}_{h_{*j}}(x) = \frac{\sum_{i=1}^n K_{h_{*j}}(x - X_i) Y_i}{\sum_{i=1}^n K_{h_{*j}}(x - X_i)}, \quad \hat{l}_{h_{*j}}(x) = \frac{\sum_{i=1}^n K_{h_{*j}}(x - X_i) m_{\hat{\theta}}(x)}{\sum_{i=1}^n K_{h_{*j}}(x - X_i)} \tag{8}$$

and $\hat{l}_{h_{*j}}$ is $E(Y|X = x)$ obtained from $\hat{f}(x) = \frac{1}{n}\sum_{i=1}^n K_{h_{*j}}(x - X_i)$. We take $\pi(x) = 1$ and $K_{h_{*j}}(.) = \frac{1}{h_{*j}} K\left(\frac{x - X_i}{h_{*j}}\right)$ is kernel function with varying bandwidth h_{*j},

$$\text{where } h_{*j} = h\lambda_{*j}, \quad \lambda_{*j} = \left[\frac{\tilde{f}(x)}{g_j(\tilde{f}(x))}\right]^{-0.5}, \tag{9}$$

$\tilde{f}(x)$ is given by (3). Here $g_1\left(\tilde{f}(x)\right) = \frac{R}{2}$, $g_2\left(\tilde{f}(x)\right) = \frac{R}{n}$, $R = \tilde{f}(x_{(n)}) - \tilde{f}(x_{(1)})$,

$$g_3\left(\tilde{f}(x)\right) = \frac{MR}{2}, \quad g_4\left(\tilde{f}(x)\right) = \frac{MR}{n}, \quad MR = \frac{\{\tilde{f}(x_{(n)}) + \tilde{f}(x_{(1)})\}}{2},$$

$$g_5\left(\tilde{f}(x)\right) = \frac{\widetilde{M}D}{2},$$

$g_6(\tilde{f}(x)) = \frac{\widetilde{M}D}{n}$, $\widetilde{M}D = \frac{\sum_{i=1}^n |\tilde{f}(x_i) - \tilde{\tilde{f}}(x)|}{n}$, $\tilde{\tilde{f}}(x)$ is median of $\tilde{f}(x), \tilde{f}(x_{(i)})$ is i^{th} order statistic of $\tilde{f}(.)$ and $|.|$ is absolute value. A_j is obtained by substituting $g_j(.)$ in (9) for $j = 1,\ldots,6$.

An alternative expression for (7) is given by

$$A_j = (h_{*j})^{\frac{1}{2}} \sum_{i=1}^n \left(\widehat{m}_{h_{*j}}(x_i) - \hat{l}_{h_{*j}}(x_i)\right)^2 \pi(x). \tag{10}$$

The tests reject the null hypothesis for their large values.

Distribution of the Tests

In this section, the consistency and asymptotic distribution of the proposed tests for a parametric regression $H_0: m(x) = m_{\theta_0}(x)$ against an alternative nonparametric regression $H_1: m_\theta(x) = m_{\theta_0}(x) + c\Delta(x)$, where $\Delta(x) = (x - 1/4)(x - 1/2)(x - 3/4)$ is established.

$E_{H_0}(A_j) = 0$ since $h_{*j} = h$ implies $\widehat{m}_{h_{*j}} = \hat{l}_{h_{*j}}$ under $H_0, j = 1,\ldots,6$.

Under H_1, when $\widehat{m}_{h_{*j}} \neq \hat{l}_{h_{*j}}$, it is obvious that $(\widehat{m}_{h_{*j}} - \hat{l}_{h_{*j}})^2 > 0$.

That is, $E_{H_1}(A_j) = (h_{*j})^{1/2} \pi(x) \sum_{i=1}^n E(\widehat{m}_{h_{*j}} - \hat{l}_{h_{*j}})^2 > 0, \quad j = 1,\ldots,6$.

Therefore the proposed tests are consistent.

T follows asymptotic normal distribution due to Mammen (1992) and Härdle and Mammen (1993). Using similar assumptions and arguments, we derive the asymptotic distribution of the proposed tests.

We note that,

$$\left.\begin{aligned} \tilde{f}(x) &= f(x) + O_p(n^{-2/5}\sqrt{logn}), \\ \hat{f}(x) &= f(x) + O_p(n^{-2/5}\sqrt{logn}) \\ \text{and} \quad \widehat{m}_{h_{*j}} &= m(x) + O_p(n^{-2/5}\sqrt{logn}). \end{aligned}\right\} \tag{11}$$

Using (11) the proposed tests $A_j, j = 1,\ldots,6$ can be written as

$$A_j = nh_{*j}^{1/2} \int_0^1 \frac{\left\{\frac{1}{n}\sum_{i=1}^n K_{h_{*j}}(x - X_i)(y_i - m_{\hat{\theta}}(x))\right\}^2}{f^2(x)} dx + o_p(1). \tag{12}$$

We know that, $Y_i = m(X_i) + \varepsilon_i$. (13)

Taking, $m(X_i) - m_{\hat{\theta}}(X_i) = -\frac{1}{n}\sum_{i=1}^n G(X_i)\Sigma^{-1} W(X_j) G(X_j)$ (14)

where $\Sigma = E(W(X_1)G(X_1)G(X_1)')$, is positive definite matrix and $G(X_i)'$ is transpose of $G(X_i)$, we get

$$A_j = nh_{*j}^{1/2} \int_0^1 \frac{\left\{\frac{1}{n}\sum_{i=1}^n K_{h_{*j}}(x - X_i)(\varepsilon_i - \frac{1}{n}\sum_{i=1}^n G(X_i)'\Sigma^{-1} W(X_j) G(X_j))\right\}^2}{f^2(x)} dx + o_p(1), j = 1,\ldots,6. \tag{15}$$

Defining,

$$W_1 = \frac{\frac{1}{n}\sum_{i=1}^n K_{h_{*j}}(x - X_i)\varepsilon_i}{f(x)}, \tag{16}$$

$$W_2 = -\frac{\frac{1}{n}\sum_{i=1}^n K_{h_{*j}}(x - X_i)\frac{1}{n}\sum_{i=1}^n G(X_i)'\Sigma^{-1} W(X_j) G(X_j)}{f(x)}, \tag{17}$$

following de Jong (1987), on simplification, we get

$$\mu_j = E(A_j) = b_{h_{*j}} + o_p(1) \quad \text{where} \quad b_{h_{*j}} = h_{*j}^{1/2} K_{h_{*j}}^{(2)}(0) \int_0^1 \frac{\sigma^2(x)\pi(x)}{f(x)} dx, \tag{18}$$

$$\tau_j^2 = Var(A_j) = \frac{2h_{*j}}{n} K_{h_{*j}}^{(4)}(0) \int_0^1 \frac{\sigma^4(x)\pi^2(x)}{f^2(x)} dx, \tag{19}$$

$K_{h_{*j}}^{(2)}$ is two-fold convolution product of $K_{h_{*j}}, j = 1,\ldots,6$. The test statistics A_j asymptotically follows $N(\mu_j, \tau_j^2)$.

Table 1 Power of $A_j, j = 1,\ldots,6$ for different values of c, h and n at 5% level of significance.

n	c	h	A_1	A_2	A_3	A_4	A_5	A_6
25	1	0.05	0.1503	0.089	0.5186	0.3071	0.3819	0.1919
		0.1	0.1539	0.1073	0.3153	0.4024	0.5137	0.2434
		0.15	0.157	0.1412	0.2298	0.3775	0.3856	0.3201
		0.2	0.149	0.1217	0.2283	0.3194	0.2913	0.318
		0.25	0.1588	0.0957	0.3092	0.4282	0.3322	0.346
		0.3	0.1473	0.118	0.2288	0.3592	0.2642	0.4053
	2	0.05	0.3138	0.191	0.6855	0.4634	0.5461	0.3173
		0.1	0.3519	0.2346	0.535	0.575	0.6904	0.402
		0.15	0.3498	0.2848	0.4383	0.5451	0.5722	0.4669
		0.2	0.3363	0.2758	0.4434	0.5151	0.5091	0.4877
		0.25	0.3398	0.2386	0.5303	0.6279	0.5557	0.5292
		0.3	0.3248	0.2648	0.4235	0.5658	0.479	0.5753
	3	0.05	0.5643	0.3891	0.8486	0.6733	0.7447	0.5325
		0.1	0.5991	0.4534	0.7573	0.7572	0.8454	0.6064
		0.15	0.606	0.5232	0.6813	0.7534	0.7837	0.6915
		0.2	0.5902	0.5172	0.6753	0.7282	0.7313	0.6946
		0.25	0.5749	0.4741	0.7278	0.8115	0.764	0.7277
		0.3	0.5548	0.5207	0.6537	0.7753	0.7142	0.7719
	4	0.05	0.7769	0.6318	0.9389	0.8411	0.8841	0.7386
		0.1	0.8086	0.695	0.8996	0.9007	0.9432	0.8065
		0.15	0.7896	0.7449	0.8463	0.8853	0.9082	0.8396
		0.2	0.778	0.751	0.8423	0.8816	0.8815	0.8593
		0.25	0.7642	0.7165	0.8765	0.9276	0.9089	0.8802
		0.3	0.7519	0.7516	0.813	0.9033	0.8694	0.8962
	5	0.05	0.9119	0.8238	0.9775	0.9347	0.9542	0.8806
		0.1	0.9137	0.8512	0.9600	0.9557	0.9784	0.9138
		0.15	0.9016	0.8908	0.9323	0.9562	0.9668	0.9323
		0.2	0.8877	0.8876	0.9284	0.9519	0.9531	0.9414
		0.25	0.8784	0.8653	0.9398	0.972	0.9606	0.9495
		0.3	0.8633	0.8873	0.9056	0.9636	0.9475	0.9626
	6	0.05	0.9647	0.9211	0.9912	0.9723	0.9819	0.9479
		0.1	0.964	0.941	0.9841	0.986	0.9927	0.9671
		0.15	0.9528	0.9532	0.9693	0.9823	0.9865	0.9762
		0.2	0.9442	0.9523	0.9674	0.9821	0.9831	0.9768
		0.25	0.9417	0.9449	0.9742	0.9892	0.9842	0.9808
		0.3	0.9283	0.9527	0.9541	0.9865	0.9774	0.9857
n	c	h	A_1	A_2	A_3	A_4	A_5	A_6
50	1	0.05	0.3286	0.3673	0.0863	0.0282	0.1091	0.1277
		0.1	0.3470	0.2748	0.1290	0.0468	0.1196	0.1102
		0.15	0.2941	0.2045	0.1524	0.0776	0.2169	0.0951
		0.2	0.2801	0.1677	0.1506	0.0975	0.2648	0.1129
		0.25	0.2730	0.2062	0.1455	0.1223	0.2470	0.1149
		0.3	0.3059	0.2321	0.1425	0.0733	0.2247	0.1120
	2	0.05	0.6715	0.6034	0.3400	0.1109	0.3331	0.3063
		0.1	0.7072	0.5425	0.4435	0.1809	0.3941	0.3047
		0.15	0.6492	0.4777	0.5037	0.2747	0.5632	0.2801
		0.2	0.6399	0.4482	0.4982	0.3449	0.6279	0.3231
		0.25	0.6356	0.5072	0.4675	0.3996	0.6111	0.3293
		0.3	0.6502	0.5687	0.4397	0.3198	0.5859	0.3265
	3	0.05	0.9220	0.8648	0.7334	0.3824	0.6961	0.6308
		0.1	0.9280	0.8375	0.8231	0.5249	0.7645	0.6324
		0.15	0.9081	0.8121	0.8413	0.6427	0.8696	0.6070
		0.2	0.8919	0.7987	0.8284	0.7270	0.9079	0.6663
		0.25	0.8931	0.8427	0.8053	0.7811	0.8992	0.6794
		0.3	0.8963	0.8720	0.7792	0.7140	0.8892	0.6920
	4	0.05	0.9892	0.9726	0.9395	0.7320	0.9255	0.8829
		0.1	0.9890	0.9697	0.9650	0.8319	0.9528	0.8880
		0.15	0.9825	0.9611	0.9673	0.9060	0.9803	0.8809
		0.2	0.9801	0.9585	0.9608	0.9340	0.9875	0.9071
		0.25	0.9762	0.9732	0.9504	0.9524	0.9861	0.9172
		0.3	0.9810	0.9793	0.9389	0.9355	0.9811	0.9195
	5	0.05	0.9986	0.9951	0.9910	0.9260	0.9880	0.9768
		0.1	0.9984	0.9967	0.9954	0.9706	0.9955	0.9811
		0.15	0.9976	0.9948	0.9939	0.9863	0.9981	0.9771
		0.2	0.9956	0.9941	0.9917	0.9914	0.9984	0.9826
		0.25	0.9954	0.9978	0.9898	0.9938	0.9983	0.9861
		0.3	0.9960	0.9991	0.9854	0.9915	0.9980	0.9878
	6	0.05	0.9999	0.9996	0.9991	0.9877	0.9981	0.9967
		0.1	0.9997	0.9996	0.9993	0.9963	0.9991	0.9975
		0.15	0.9998	0.9995	0.9995	0.9978	0.9996	0.9957
		0.2	0.9998	0.9994	0.9993	0.9991	0.9997	0.9977
		0.25	0.9990	1.0000	0.9985	0.9994	0.9999	0.9977
		0.3	0.9994	0.9997	0.9974	0.9987	0.9997	0.9982

n	c	h	A_1	A_2	A_3	A_4	A_5	A_6
100	1	0.05	0.3635	0.1794	0.3321	0.4652	0.0297	0.0011
		0.1	0.4230	0.4295	0.3818	0.3519	0.0528	0.0025
		0.15	0.4598	0.3581	0.3659	0.2893	0.0788	0.0032
		0.2	0.4721	0.3797	0.3972	0.2325	0.1092	0.0045
		0.25	0.4586	0.3961	0.3766	0.2808	0.1358	0.0062
		0.3	0.4058	0.3969	0.3446	0.2740	0.1081	0.0080
	2	0.05	0.8613	0.5228	0.8343	0.7938	0.3003	0.0252
		0.1	0.8988	0.7981	0.8779	0.7414	0.4439	0.0411
		0.15	0.9091	0.7813	0.8866	0.7431	0.5596	0.0573
		0.2	0.9151	0.8146	0.8835	0.6939	0.6439	0.0748
		0.25	0.9036	0.8334	0.8684	0.7720	0.7011	0.1067
		0.3	0.8700	0.8447	0.8361	0.7744	0.6422	0.1341
	3	0.05	0.9945	0.8958	0.9914	0.9764	0.8208	0.2467
		0.1	0.9960	0.9807	0.9965	0.9743	0.9198	0.3358
		0.15	0.9963	0.9819	0.9954	0.9748	0.9537	0.3978
		0.2	0.9969	0.9873	0.9935	0.9709	0.9702	0.4781
		0.25	0.9953	0.9914	0.9924	0.9836	0.9786	0.5371
		0.3	0.9891	0.9910	0.9889	0.9882	0.9672	0.6040
	4	0.05	1.0000	0.9936	1.0000	0.9994	0.9906	0.7237
		0.1	0.9998	0.9996	0.9999	0.9993	0.9973	0.8095
		0.15	1.0000	0.9998	1.0000	0.9992	0.9988	0.8650
		0.2	1.0000	0.9998	1.0000	0.9995	0.9999	0.8999
		0.25	0.9998	0.9997	0.9999	0.9997	0.9997	0.9263
		0.3	0.9999	0.9998	0.9997	0.9998	0.9989	0.9559
150	1	0.05	0.3056	0.0308	0.2376	0.0315	0.0440	0.0088
		0.1	0.3379	0.0876	0.3596	0.1143	0.0750	0.0088
		0.15	0.2880	0.0819	0.3524	0.1697	0.1516	0.0058
		0.2	0.3094	0.0711	0.3396	0.1758	0.1549	0.0065
		0.25	0.2450	0.1064	0.3532	0.2224	0.2077	0.0182
		0.3	0.2427	0.0824	0.3468	0.2084	0.2499	0.0202
	2	0.05	0.9133	0.2690	0.8760	0.2835	0.4977	0.1156
		0.1	0.9297	0.5107	0.9433	0.5644	0.6757	0.1279
		0.15	0.9064	0.5493	0.9403	0.7132	0.8225	0.1255
		0.2	0.9117	0.5753	0.9350	0.7552	0.8266	0.1425
		0.25	0.8705	0.6686	0.9201	0.8205	0.8813	0.2447
		0.3	0.8522	0.6409	0.9101	0.8285	0.8905	0.3122
	3	0.05	0.9996	0.8188	0.9991	0.8199	0.9647	0.6074
		0.1	0.9994	0.9524	0.9994	0.9627	0.9913	0.6434
		0.15	0.9989	0.9662	0.9998	0.9885	0.9976	0.669
		0.2	0.9989	0.9736	0.9994	0.9927	0.9978	0.7161
		0.25	0.9969	0.9876	0.9991	0.9972	0.999	0.8349
		0.3	0.9956	0.9849	0.9983	0.9976	0.9993	0.8848
n	c	h	A_1	A_2	A_3	A_4	A_5	A_6
200	1	0.05	0.5565	0.4695	0.3918	0.1267	0.2473	0.0210
		0.1	0.6331	0.4931	0.5804	0.1724	0.3175	0.0664
		0.15	0.6428	0.6025	0.5948	0.2020	0.3716	0.1480
		0.2	0.5853	0.6474	0.6097	0.2629	0.3573	0.1722
		0.25	0.5941	0.5745	0.5822	0.3404	0.4122	0.1185
		0.3	0.5166	0.5357	0.4833	0.3665	0.4123	0.1434
	2	0.05	0.9868	0.8965	0.9715	0.6024	0.8912	0.2295
		0.1	0.9941	0.9339	0.9915	0.7537	0.9483	0.4348
		0.15	0.9952	0.9743	0.9920	0.8206	0.9717	0.6536
		0.2	0.9922	0.9816	0.9928	0.8977	0.9715	0.7167
		0.25	0.9914	0.9793	0.9888	0.9375	0.9794	0.6706
		0.3	0.9792	0.9760	0.9765	0.9522	0.9821	0.7355
	3	0.05	1.0000	0.9981	1.0000	0.9774	0.9998	0.8167
		0.1	1.0000	0.9998	1.0000	0.9942	1.0000	0.9361
		0.15	1.0000	0.9999	1.0000	0.9975	1.0000	0.9829
		0.2	1.0000	1.0000	1.0000	0.9997	1.0000	0.9917
		0.25	1.0000	1.0000	1.0000	0.9998	1.0000	0.9897
		0.3	1.0000	1.0000	1.0000	1.0000	1.0000	0.9927
250	1	0.05	0.5232	0.0813	0.3909	0.0124	0.5875	0.2035
		0.1	0.6455	0.2016	0.4820	0.0077	0.6231	0.2996
		0.15	0.6495	0.3209	0.4589	0.0189	0.6631	0.3670
		0.2	0.6418	0.3728	0.5036	0.0962	0.5873	0.2022
		0.25	0.5886	0.3123	0.4352	0.1224	0.6433	0.2108
		0.3	0.5439	0.3360	0.4704	0.1922	0.6777	0.2480
	2	0.05	0.9936	0.5701	0.9876	0.2739	0.9891	0.7034
		0.1	0.9972	0.8388	0.9949	0.3481	0.9954	0.8251
		0.15	0.9974	0.9234	0.9912	0.5505	0.9977	0.8913
		0.2	0.9974	0.9568	0.9916	0.8234	0.9960	0.7993
		0.25	0.9959	0.9552	0.9863	0.8703	0.9971	0.8381
		0.3	0.9919	0.9665	0.9871	0.9327	0.9984	0.8853
	3	0.05	1.0000	0.9844	1.0000	0.9166	1.0000	0.9883
		0.1	1.0000	0.9993	1.0000	0.9599	1.0000	0.9974
		0.15	1.0000	0.9998	1.0000	0.9896	1.0000	0.9995
		0.2	1.0000	1.0000	1.0000	0.9991	1.0000	0.9982
		0.25	1.0000	1.0000	1.0000	0.9997	1.0000	0.9991
		0.3	1.0000	0.9999	1.0000	1.0000	1.0000	0.9990

Table 2 Power of $B_j, j = 1, \dots, 6$ at 5% level of significance for different values of c, h and n

n	c	h	B_1	B_2	B_3	B_4	B_5	B_6
25	1	0.05	0.1189	0.0326	0.4408	0.2675	0.4653	0.0728
		0.10	0.0781	0.1223	0.2833	0.4429	0.4759	0.2434
		0.15	0.1570	0.4052	0.3395	0.6893	0.6023	0.5969
		0.20	0.1490	0.4593	0.3095	0.7327	0.5950	0.7243
		0.25	0.1588	0.5002	0.3421	0.8143	0.6698	0.8557
		0.30	0.1821	0.5984	0.2863	0.7753	0.6106	0.8589
	2	0.05	0.2696	0.0872	0.6191	0.4218	0.6244	0.1532
		0.10	0.2295	0.2601	0.5036	0.6145	0.6572	0.4020
		0.15	0.3498	0.5871	0.5523	0.8137	0.7582	0.7289
		0.20	0.3363	0.6405	0.5322	0.8560	0.7729	0.8424
		0.25	0.3398	0.6941	0.5630	0.9015	0.8193	0.9231
		0.30	0.3729	0.7522	0.4875	0.8734	0.7740	0.9231
	3	0.05	0.5124	0.2324	0.8097	0.6345	0.7991	0.3323
		0.10	0.4748	0.4808	0.7352	0.7867	0.8238	0.6064
		0.15	0.6060	0.7756	0.7702	0.9197	0.8966	0.8717
		0.20	0.5902	0.8208	0.7432	0.9374	0.8941	0.9215
		0.25	0.5749	0.8529	0.7547	0.9565	0.9166	0.9628
		0.30	0.6023	0.8955	0.7083	0.9486	0.9044	0.9699
	4	0.05	0.7440	0.4651	0.9200	0.8119	0.9128	0.5670
		0.10	0.7164	0.7168	0.8883	0.9155	0.9345	0.8065
		0.15	0.7896	0.9031	0.8937	0.9677	0.9592	0.9425
		0.20	0.7780	0.9277	0.8799	0.9775	0.9620	0.9733
		0.25	0.7642	0.9420	0.8910	0.9868	0.9714	0.9899
		0.30	0.7859	0.9601	0.8460	0.9850	0.9635	0.9900
	5	0.05	0.8935	0.7075	0.9702	0.9210	0.9680	0.7725
		0.10	0.8606	0.8654	0.9547	0.9646	0.9748	0.9138
		0.15	0.9016	0.9619	0.9561	0.9895	0.9863	0.9808
		0.20	0.8877	0.9713	0.9485	0.9915	0.9854	0.9890
		0.25	0.8784	0.9782	0.9470	0.9948	0.9883	0.9965
		0.30	0.8858	0.9886	0.9259	0.9945	0.9866	0.9963
	6	0.05	0.9552	0.8604	0.9878	0.9672	0.9873	0.8940
		0.10	0.9423	0.9463	0.9818	0.9876	0.9914	0.9671
		0.15	0.9528	0.9845	0.9821	0.9950	0.9942	0.9920
		0.20	0.9442	0.9908	0.9785	0.9976	0.9960	0.9974
		0.25	0.9417	0.9936	0.9771	0.9982	0.9960	0.9984
		0.30	0.9410	0.9953	0.9659	0.9976	0.9948	0.9992
n	c	h	B1	B2	B3	B4	B5	B6
50	1	0.05	0.3046	0.4974	0.0863	0.0508	0.3050	0.0106
		0.10	0.4391	0.9164	0.1715	0.5130	0.5144	0.4739
		0.15	0.3738	0.9200	0.2017	0.7208	0.6376	0.8632
		0.20	0.3610	0.9260	0.2384	0.8242	0.6755	0.9585
		0.25	0.3548	0.9344	0.2098	0.8002	0.6342	0.9668
		0.30	0.3604	0.9460	0.2348	0.8386	0.5765	0.9666
	2	0.05	0.6463	0.7216	0.3400	0.1678	0.5829	0.0642
		0.10	0.7758	0.9768	0.5068	0.7533	0.7887	0.7074
		0.15	0.7196	0.9765	0.5693	0.8950	0.8700	0.9515
		0.20	0.7117	0.9811	0.6089	0.9464	0.8988	0.9890
		0.25	0.7078	0.9846	0.5588	0.9364	0.8860	0.9928
		0.30	0.7016	0.9873	0.5607	0.9554	0.8575	0.9918
	3	0.05	0.9113	0.9197	0.7334	0.4780	0.8647	0.2566
		0.10	0.9514	0.9965	0.8619	0.9328	0.9545	0.9036
		0.15	0.9346	0.9970	0.8747	0.9797	0.9797	0.9919
		0.20	0.9217	0.9981	0.8868	0.9939	0.9848	0.9980
		0.25	0.9235	0.9988	0.8593	0.9924	0.9824	0.9988
		0.30	0.9177	0.9989	0.8557	0.9943	0.9781	0.9991
	4	0.05	0.9873	0.9853	0.9395	0.8002	0.9757	0.6116
		0.10	0.9923	0.9997	0.9737	0.9913	0.9957	0.9839
		0.15	0.9889	1.0000	0.9760	0.9980	0.9978	0.9994
		0.20	0.9873	0.9998	0.9777	0.9990	0.9987	0.9999
		0.25	0.9841	0.9999	0.9668	0.9996	0.9985	0.9996
		0.30	0.9858	1.0000	0.9634	0.9997	0.9965	0.9999
100	1	0.05	0.3895	0.9365	0.4764	0.9414	0.2330	0.0229
		0.10	0.5088	0.9967	0.5521	0.9981	0.4335	0.7613
		0.15	0.5822	0.9992	0.4761	0.9955	0.4481	0.9437
		0.20	0.5951	0.9991	0.4526	0.9951	0.5428	0.9917
		0.25	0.5199	0.9965	0.4857	0.9954	0.5569	0.9969
		0.30	0.4620	0.9938	0.3939	0.9871	0.5187	0.9966
	2	0.05	0.8775	0.9899	0.8997	0.9914	0.6886	0.1440
		0.10	0.9289	0.9998	0.9420	0.9999	0.8743	0.9381
		0.15	0.9450	0.9999	0.9265	1.0000	0.9024	0.9938
		0.20	0.9511	1.0000	0.9043	1.0000	0.9383	0.9996
		0.25	0.9240	0.9999	0.9197	1.0000	0.9449	0.9997
		0.30	0.8987	0.9998	0.8686	0.9992	0.9315	0.9999
	3	0.05	0.9956	0.9997	0.9957	0.9997	0.9672	0.5582
		0.10	0.9974	1.0000	0.9993	1.0000	0.9945	0.9960
		0.15	0.9982	1.0000	0.9970	1.0000	0.9970	0.9998
		0.20	0.9985	1.0000	0.9950	1.0000	0.9982	1.0000
		0.25	0.9966	1.0000	0.9968	1.0000	0.9987	1.0000
		0.30	0.9932	1.0000	0.9921	1.0000	0.9984	1.0000

n	c	h	B1	B2	B3	B4	B5	B6
150	1	0.05	0.3056	0.9704	0.3718	0.9811	0.6094	0.6221
		0.10	0.4042	0.9972	0.4798	0.9992	0.6697	0.9965
		0.15	0.4107	0.9972	0.4702	0.9988	0.6395	0.9995
		0.20	0.4136	0.9960	0.4049	0.9940	0.6616	0.9999
		0.25	0.3562	0.9892	0.4195	0.9944	0.6662	1.0000
		0.30	0.3487	0.9837	0.4172	0.9941	0.6495	0.9999
	2	0.05	0.9133	0.9983	0.9333	0.9991	0.9639	0.9076
		0.10	0.9481	1.0000	0.9685	1.0000	0.9843	0.9999
		0.15	0.9506	1.0000	0.9666	1.0000	0.9827	0.9999
		0.20	0.9467	1.0000	0.9516	0.9999	0.9883	1.0000
		0.25	0.9251	0.9997	0.9402	1.0000	0.9891	1.0000
		0.30	0.9106	0.9996	0.9346	1.0000	0.9856	1.0000
200	1	0.05	0.6091	0.9999	0.5761	0.9999	0.8661	0.9934
		0.10	0.7083	1.0000	0.6840	1.0000	0.8895	1.0000
		0.15	0.7360	1.0000	0.6683	1.0000	0.8994	1.0000
		0.20	0.6822	1.0000	0.6843	1.0000	0.8608	1.0000
		0.25	0.6722	1.0000	0.6553	1.0000	0.8424	1.0000
		0.30	0.5874	0.9997	0.5821	0.9996	0.8146	1.0000
	2	0.05	0.9916	1.0000	0.9896	1.0000	0.9982	0.9999
		0.10	0.9966	1.0000	0.9949	1.0000	0.9995	1.0000
		0.15	0.9974	1.0000	0.9955	1.0000	0.9995	1.0000
		0.20	0.9956	1.0000	0.9959	1.0000	0.9997	1.0000
		0.25	0.9953	1.0000	0.9939	1.0000	0.9991	1.0000
		0.30	0.9864	1.0000	0.9879	1.0000	0.9989	1.0000
250	1	0.05	0.5717	1.0000	0.5102	0.9999	0.9808	1.0000
		0.10	0.7339	1.0000	0.6135	1.0000	0.9766	1.0000
		0.15	0.6930	1.0000	0.5623	1.0000	0.9674	1.0000
		0.20	0.7305	1.0000	0.5678	1.0000	0.9537	1.0000
		0.25	0.6324	1.0000	0.5189	0.9989	0.9307	1.0000
		0.30	0.6112	0.9998	0.5402	0.9992	0.9048	1.0000
	2	0.05	0.9954	1.0000	0.9934	1.0000	1.0000	1.0000
		0.10	0.9990	1.0000	0.9983	1.0000	1.0000	1.0000
		0.15	0.9982	1.0000	0.9959	1.0000	1.0000	1.0000
		0.20	0.9985	1.0000	0.9946	1.0000	1.0000	1.0000
		0.25	0.9967	1.0000	0.9924	1.0000	0.9999	1.0000
		0.30	0.9950	1.0000	0.9921	1.0000	0.9998	1.0000

Table 3 Power of T at 5% level of significance for different values of c, h and n

n	c	h					
		0.05	**0.10**	**0.15**	**0.20**	**0.25**	**0.30**
25	**1**	0.1173	0.0794	0.1337	0.1033	0.1031	0.1099
	2	0.2794	0.2409	0.3324	0.2863	0.2544	0.2506
	3	0.5261	0.4988	0.6023	0.5442	0.4765	0.4511
	4	0.7531	0.7366	0.7930	0.7451	0.6830	0.6538
	5	0.9006	0.8813	0.9075	0.8725	0.8213	0.7883
	6	0.9579	0.9524	0.9581	0.9361	0.9086	0.8771
50	**1**	0.3276	0.3911	0.2947	0.2582	0.2308	0.2224
	2	0.6770	0.7627	0.6771	0.6279	0.5716	0.5200
	3	0.9254	0.9539	0.9244	0.8909	0.8602	0.8092
	4	0.9907	0.9951	0.9900	0.9807	0.9662	0.9475
	5	0.9985	0.9997	0.9987	0.9967	0.9940	0.9891
	6	0.9998	1.0000	0.9998	0.9997	0.9992	0.9982
100	**1**	0.3742	0.4291	0.4705	0.4493	0.3506	0.2781
	2	0.8799	0.9173	0.9226	0.9071	0.8336	0.7412
	3	0.9970	0.9976	0.9976	0.9961	0.9885	0.9652
	4	1.0000	0.9999	1.0000	1.0000	0.9996	0.9982
150	**1**	0.2730	0.3487	0.3271	0.2950	0.2159	0.1878
	2	0.9148	0.9417	0.9287	0.8959	0.8189	0.7280
	3	0.9998	0.9997	0.9993	0.9984	0.9931	0.9825
200	**1**	0.5468	0.6228	0.6323	0.5421	0.4803	0.3586
	2	0.9903	0.9954	0.9950	0.9889	0.9756	0.9200
	3	1.0000	1.0000	1.0000	0.9999	0.9999	0.9991
250	**1**	0.5160	0.6578	0.5912	0.5893	0.4308	0.3669
	2	0.9945	0.9987	0.9959	0.9952	0.9811	0.9501
	3	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999

Performance of the Class of Tests

In this section, using simulation, we study the performance of the proposed class of tests in terms of its empirical power. We apply wild bootstrap method due to Wu (1986) since it yields $E(Y_i|(X_i,Y_i)_{i=1,\dots,n}) = m_{\hat{\theta}}(X_i)$.

We generate n random numbers X_i from Uniform (0,1) distribution and obtain $m_{\hat{\theta}}(X_i)$ under H_0. We compute $\tilde{\varepsilon}_i = Y_i - m_{\hat{\theta}}(X_i), i = 1,\dots,n$. By wild bootstrapping $\tilde{\varepsilon}_i$, we obtain ε_i^* taking $\varepsilon_i^* \sim \hat{F}_i$ with $E_{\hat{F}_i}(Z) = 0,\ \ E_{\hat{F}_i}(Z^k) = \tilde{\varepsilon}_i^{\,k}, k = 2$ and 3 which is known as two-point distribution due to Härdle et. al. (2004). We compute $Y_i^* = m_{\hat{\theta}}(X_i) + \varepsilon_i^*$. This procedure is repeated 1000 times to approximate distribution of A_j empirically which is used to obtain critical values of the tests under the null hypothesis. We obtain the empirical power of A_j using 10000 replications. To study the performance of the test, we obtain the empirical power for various values of $c, h,$ and n, under quartic kernel, considering $y = 2x - x^2 + c(x - 1/4)(x - 1/2)(x - 3/4) + \varepsilon$ with $\varepsilon \sim N(0,0.01)$ for testing $H_0: c = 0$ versus $H_1: c > 0$.

For comparative study on performance of $A_j, j = 1, \dots, 6$, we consider T test due to Härdle and Mammen (1993) and $B_j = (h_{*j})^{1/2} \sum_{i=1}^{n} \left(\hat{m}_{h_{*j}}(x_i) - \hat{l}_h(x_i)\right)^2 \pi(x), \quad j = 1,2$ due to Bhat and Deshpande (2019a) which are modified T tests based on adaptive NW and adaptive AA estimators. We observe that, $\hat{m}_{h_{*j}}, j = 1,2$ is obtained by (8). We also consider B_3, B_4, B_5 and B_6 obtained respectively using $MR/2$, MR/n, $\widetilde{M}D/2$ and $\widetilde{M}D/n$ in $\hat{m}_{h_{*j}}$ for $j = 3, 4,$ 5 and 6. The empirical powers of $A_j, B_j, j = 1, \dots, 6$ and T at 5% level of significance are furnished respectively in tables 1, 2 and 3. The empirical power of $A_j, B_j, j = 1,3,5$ and $A_j, B_j, j = 2,4,6$ with T respectively are presented in figures 1 and 2 for $c = 1,2$ and $h = 0.05,0.3$.

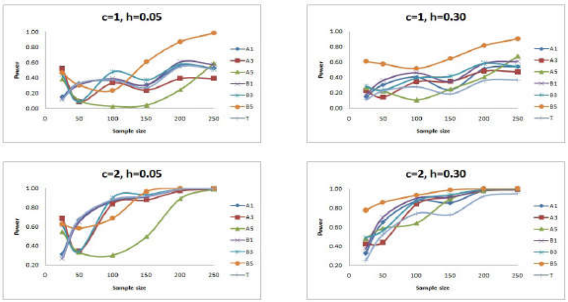

Figure 1: Empirical power of $A_j, B_j, j = 1, 3, 5$ and T.

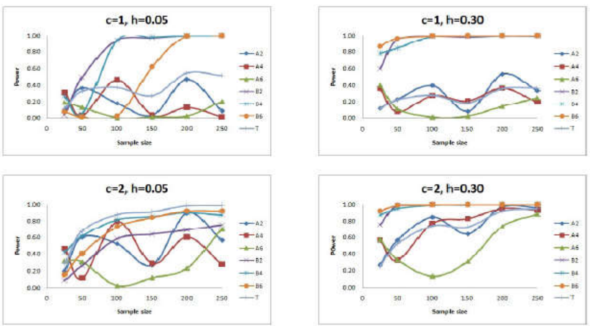

Figure 2: Empirical power of $A_j, B_j, j = 2, 4, 6$ and T.

From tables and figures, we observe that, the empirical power of $A_j, j = 1, \dots, 6$ increase as c increases and have higher power than T for all the values of n, h and c. A_j has higher power than $B_j, j = 1, \dots, 6$ for lower values of bandwidth and sample size. For all values of n, h and c, A_1, A_3, A_5 respectively exhibit higher power than A_2, A_4 and A_6. For $n = 25{,}150, h > 0.05$, A_3 has higher empirical power than that of A_1, A_5 and A_4 has higher empirical power than that of A_2, A_6. Also, A_3 possesses higher power than all other tests of the proposed class.

Application of the tests

As the tests A_3 and A_4 perform better than the other tests for small sample sizes, we illustrate the application of these tests on a real data in this section. We make use of salinity data set of R library comprising of 28 observations, considering discharge as predictor variable X and salinity as dependent variable Y.

The scatter plot of the original data along with parametric (quadratic) fit and nonparametric fits, $\hat{m}_{h_{*3}}$ and $\hat{m}_{h_{*4}}$ are given in figure 3.

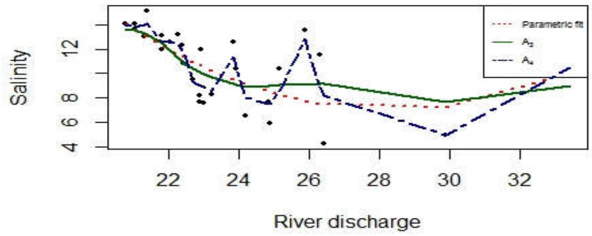

Figure 3 Regression fits with $m_{\hat{\theta}}$, $\hat{m}_{h_{*3}}$ and $\hat{m}_{h_{*4}}$ for salinity data.

From the figure, we observe that, $\hat{m}_{h_{*4}}$ is a suitable fit for the salinity data. The null hypothesis is rejected by A_3 and A_4 when $h = 2.7759$, $A_3 = 17.9249$ with critical value 15 and $A_4 = 47.4079$ with critical value 46.

CONCLUSIONS

A class of tests based on ISE of adaptive NW and adaptive AA estimators and regression function obtained from varying density estimates outperforms Härdle and Mammen test based on ISE of NW estimator and regression function obtained from pilot density estimates. Also, the members of the class have higher power than the members of the class of tests based on ISE of adaptive NW and adaptive AA estimators and regression function obtained from pilot density estimates due to Bhat and Deshpande (2019a) for smaller sample sizes and smaller bandwidths. The proposed class of tests follow asymptotic normal distribution. Among numerous members of the class, the test based on $(MR/2)$ possesses higher empirical power for $h > 0.05$ and $n = 25$ and 150.

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