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Research Article

STUDY OF POLYNOMIAL AND NON POLYNOMIAL SPLINE BASED APPROXIMATION

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A R T I C L E I N F O

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A B S T R A C T

The purpose of this paper is to discuss numerical solutions of differential equations including the evolution, progress and types of differential equations, special attention is given to the solution of differential equations by application of spline functions. Here we are interested in differential equation based problems and their solutions using polynomial and non polynomial splines of different orders.

INTRODUCTION

Differential equations are mathematically studied from several different perspectives, mostly concerned with their solutions – the set of functions that satisfy the equation. Only the simplest differential equations admit solutions given by explicit formulae; however, some properties of solutions of a given differential equation may be determined without finding their exact form. If a self contained formula for the solution is not available, the solution may be numerically approximated using computers. In general, it is not possible to obtain the analytical solution of a system of differential equations, obtained from obstacle, unilateral, moving and free boundary-value problems and problems of the defection of plates and in a number of other scientific applications, while many numerical methods have been developed to determine solutions with a given degree of accuracy. In the present paper we discuss the history, classification and numerical solution of differential equations. Here we are merely concerned about the solution of these boundary-value problems by application of spline functions.

Let $x_n, x_{n+1}, \dots\dots x_{n+k}$ be asset of k+1equispaced tabular points with spacing h and $u_n, u_{n+1}, \dots\dots u_{n+k}$ be the corresponding values of a function $u(x)$ at these points, that is,$x_{n+i} = x_n + ih, u_{n+i} = u(x_{n+i}), i = 0,1, \dots\dots k,$ for some integer k. A relationship between u_n and the differences $\Delta u_n, \Delta^2 u_n, \dots\dots \Delta^k u_n$ is called difference equation and hence it can be regarded as a relation among $u_n, u_{n+1}, \dots\dots, u_{n+k}$.

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We include here one paper based on application of various spline functions to solve different systems of differential equations. The paper is organized as follows: in section 2, we will discuss a brief history of differential equations. In section 3, we consider the general introduction of differential equations. In section 4, we discuss about types of differential equations. In section 5, subdivision of differential equations, in section 6, initial & boundary value problems, in section 7 types of boundary value problems, in section 8 differential equations associated with physical problems arising in engineering is discussed. In section 9, numerical solution of differential equations, in section 10, general introduction to spline, in section 11 spline solution of differential equations and finally in section 12 the conclusion and further development is given.

General Introduction of Differential Equations

There is difference between differential equations and ordinary equations of mathematics. The differential equations, in addition to variables and constants, also contain derivatives of one or more of the variables involved. In general, a differential equation is an equation which involves the derivatives of an unknown function represented by a dependent variable. It expresses the relationship involving the rates of change of continuously changing quantities modeled by functions and are used whenever a rate of change (derivative) is known. A solution to a differential equation is a function whose derivatives satisfy the equation.

Types of Differential Equations

There are five types of differential equations:

1. ***Ordinary differential equation:*** An ordinary differential equation (ODE) is a differential equation in which the unknown function is a function of a single independent variable.
2. ***Partial differential equation:*** A partial differential equation (PDE) is a differential equation in which the unknown function is a function of multiple independent variables and their partial derivatives.
3. ***Delay differential equation:*** A delay differential equation (DDE) is a differential equation in which the derivative of the unknown function at a certain time is given in terms of the values of the function at previous times.
4. ***Stochastic differential equation:*** A stochastic differential equation (SDE) is a differential equation in which one or more of the terms are a stochastic process, thus resulting in a solution which is itself astochastic process.
5. ***Differential algebraic equation:*** A differential algebraic equation (DAE) is a differential equation comprising differential and algebraic terms, given in implicit form.

Subdivision of Differential Equations

Each of types of differential equations mentioned above is divided into two subcategories - linear and nonlinear. A differential equation is linear if it involves the unknown function and its derivatives only to the first power; otherwise the differential equation is nonlinear. Thus if y' denotes the first derivative of y, then the equation $y'=y$ is linear, while the equation $y'=y^2$ is nonlinear. Solutions of a linear equation in which the unknown function or its derivative or derivatives appear in each term (linear homogeneous equations) may be added together or multiplied by an arbitrary constant in order to obtain additional solutions of that equation, but there is no general way to obtain families of solutions of nonlinear equations, except when they exhibit symmetries. Linear equations frequently appear as approximations to nonlinear equations, and these approximations are only valid under restricted conditions.

Numerical techniques to solve the boundary value problems include some of the following methods:

Shooting Methods: These are initial value problem methods. In this method, we convert the given boundary value problem to an initial value problem by adding sufficient number of conditions at one end and adjust these conditions until the given conditions are satisfied at the other end.

Finite Difference Methods: In finite difference method (FDM), functions are represented by their values at certain grid points and derivatives are approximated through differences in these values. For the finite difference method, the domain under consideration is represented by a finite subset of points. These points are called "nodal points" of the grid. This grid is almost always arranged in (uniform or non-uniform) rectangular manner.

The differential equation is replaced by a set of difference equations which are solved by direct or iterative methods.

Finite Element Methods: In finite element method (FEM), functions are represented in terms of basic functions and the ODE is solved in its integral (weak) form. In this method the domain under consideration is partitioned in a finite set of elements. In this the differential equation is discretized by using approximate methods with the piecewise polynomial solution.

Spline Based Methods: In spline based methods, the differential equation is discretized by using approximate methods based on spline. The end conditions are derived for the definition of spline. The algorithm developed not only approximates the solutions, but their higher order derivatives as well.

General Introduction to Spline

When computers were not available, the draftsman used a device to draw a smooth curve through a given set of points such that the slope and curvature are also continuous along the curve, that is, $f(x), f'(x)$ and $f''(x)$ are continuous on the curve. Such a device was called a spline and plotting of the curve was called spline fitting.

The given interval [a, b] is subdivided $[x_0, x_1]$, $[x_1, x_2]$, ……,$[x_{n-1}, x_n]$ where $a = x_0<x_1<x_2<\ldots<x_n = b$. The nodes(knots) $x_1, x_2, \ldots, x_{n-1}$ are called internal nodes.
Example: Given the data

x	0	1	2	3
f(x)	1	2	33	244

we write the spline approximation as

$p_1(x) = a_1x^2 + b_1x + c_1$,
$p_2(x) = a_2x^2 + b_2x + c_2$,
$p_3(x) = a_3x^2 + b_3x + c_3$,

after putting the value and solving we get the splines

$p_1(x) = x + 1$
$p_2(x) = 30x^2 – 59x + 31$
$p_3(x) = 150x^2 – 539x + 511$

Cubic Spline Techniques to Solve Boundary Value Problems

A cubic spline function $S\Delta(x)$ of class $C^2[a,b]$ interpolating to a function u(x) defined on [a,b] is such that

(a) In each interval $[x_{j-1}, x_j]$=1,2,….,N, $S\Delta(x)$ is a Polynomial of degree at most three,
(b) The first and second derivatives of $S\Delta(x)$ are continuous on [a,b]. Considering the article by E. A. Al-Said having the system of second-order boundary value problem of the type

$$u'' = \begin{cases} f(x), & a\le x \le c \\ g(x)u(x) + f(x) + r, & c\le x \le d \\ f(x), & d\le x \le b \end{cases} \qquad (7)$$

with the boundary conditions

$$u(a) = \alpha_1 \text{ and } u(b) = \alpha_2 \qquad (8)$$

and assuming the continuity conditions of u and u' at c and d. Here, f and g are continuous functions on [a,b] and [c,d], respectively. The parameters r, α_1 , α_2 are real finite constants. The main purpose of this article is to use uniform cubic spline functions to develop some consistency relations which are then used to develop a numerical method for computing smooth approximations to the solution and its derivatives for a system of second-order boundary-value problems of the type (7). In this paper the author has shown that the present method gives approximations which are better than those produced by other collocation, finite-difference, and spline methods.

Quartic Spline Techniques to Solve Boundary Value Problems

A quartic spline function $S\Delta(x)$, interpolating to a function u(x) defined on [a, b] is such that

1. In each subinterval $[x_{j-1}, x_j]$=1,2,….,N,, $S\Delta(x)$ is a polynomial of degree at most four.
2. The first, second and third derivatives of $S\Delta(x)$ are continuous on [a,b].

Considering the paper by E. A. Al-Said [8] having the system of fourth order boundary value problem of the type

$$u^{(iv)} = \begin{cases} f(x), & a \le x \le c \\ g(x)u(x) + f(x) + r, & c \le x \le d \\ f(x), & d \le x \le b \end{cases} \tag{9}$$

with the boundary conditions

$$u(a) = u(b) = \alpha_1 \text{ and } u''(a) = u''(b) = \alpha_2$$
$$u(c) = u(d) = \beta_1 \text{ and } u''(c) = u''(d) = \beta_2 \tag{10}$$

where f and g are continuous functions on [a,b] and [c,d] respectively. The parameters r, α_i and β_i, i=1,2 are real constants.

Quintic Spline Techniques to Solve Boundary Value Problems

A quintic spline function $S\Delta(x)$, interpolating to a function u(x) on [a, b] is defined as:

1. In each subinterval $[x_{j-1}, x_j]$=1,2,….,N, $S\Delta(x)$ is a polynomial of degree at most five.
2. The first, second, third and fourth derivatives of $S\Delta(x)$ are continuous on [a ,b].

To be able to deal effectively with such problems we introduce 'spline functions' containing a parameter ω. These are 'non-polynomial splines' defined through the solution of a differential equation in each subinterval. The arbitrary constants are being chosen to satisfy certain smoothness conditions at the joints. These 'splines' belong to the class C^4[a,b] and reduce into polynomial splines as parameter $\omega \to 0$. A paper based on quintic spline is as follows –

Considering the paper by Arshad Khan and Tariq Aziz [9] having a third-order linear and non-linear boundary value problem of the type

$$y'''(x) = f(x, y),\ a \le x \le b, \tag{11}$$

Subject to

$$y(a)=k_1,\ y'(a) = k_2,\ y(b) = k_3 \tag{12}$$

In this paper, the methods discussed are tested on two problems from the literature [10], and absolute errors in the analytical solutions are calculated.

Sextic Spline Techniques to Solve Boundary Value Problems

A sextic spline function $S\Delta(x)$, interpolating to a function u(x) on [a, b] is defined as:

1. In each interval $[x_{j-1}, x_j]$=1,2,….,N, $S\Delta(x)$ is a polynomial of degree at most six.
2. The first fifth derivatives of $S\Delta(x)$ are continuous on [a,b].
3. $S\Delta(x_i)=u(x_i)$, i=0(1)N+1.

Consider the paper [16] having a system of second-order boundary-value problem of the type (7) and (8). Here we have considered the obstacle boundary-value problem of finding y such that, on $\Omega = [0, \pi]$,

$$\begin{aligned} &-y'' \ge f(x), \\ &y(x) \ge \Psi(x), \\ &[y''-f(x)][y(x)-\Psi(x)]=0, \\ &y(0) = y(\pi) = 0 \end{aligned} \tag{13}$$

Where f(x) is a given force acting on the string and Ψ(x) is the elastic obstacle.

Septic Spline Technique to Solve Boundary Value Problems

A septic spline function $S\Delta(x)$, interpolating to a function u(x) on [a,b] is defined as:

1. In each interval $[x_{j-1}, x_j]$, $S\Delta(x)$ is a polynomial of degree at most seven.
2. The first six derivatives of $S\Delta(x)$ are continuous on [a,b].
3. $S\Delta(x_i)=u(x_i)$, i=0(1)N+1.

In a non polynomial septic spline we introduce a parameter k. The arbitrary constants are being chosen to satisfy certain smoothness conditions at the joints. This 'spline' belongs to the class C^6[a,b] and reduces into polynomial splines as parameter $k \to 0$.

Considering the system of sixth-order boundary value problem of the type

$$\begin{aligned} &y^{(6)}(x) + f(x)y(x) = g(x), \\ &y(a) = \alpha_0,\ y(b) = \alpha_1, \\ &y^{(1)}(a) = \gamma_0,\ y^{(1)}(b) = \gamma_1, \\ &y^{(2)}(a) = \delta_0,\ y^{(2)}(b) = \delta_1, \end{aligned} \tag{14}$$

where α_i, γ_i and δ_i i = 0, 1 are finite real constants and the functions f(x) and g(x) are continuous on [a, b].

Octic Spline Technique to Solve Boundary Value Problems

An octic spline function $S\Delta(x)$, interpolating to a function u(x) on [a,b] defined as:

1. In each interval $[x_{j-1}, x_j]$=1,2,….,N, $S\Delta(x)$ is a polynomial of degree at most eight.
2. The first seventh derivatives of $S\Delta(x)$ are continuous on [a,b].
3. $S\Delta(x_i)=u(x_i)$, i=0(1)N+1.

Considering the system of eighth-order boundary value problem of the type

$$y^{(viii)} + \Phi(x)y + \varphi(x)$$

where y=y(x) and Φ(x) and φ(x) are continuous function defined in the interval $x \in [a,b]$. Ai and Bi, i=0,2,4,6 are finite real constants.

In this paper, the authors have used octic spline to solve the problem of the type (15). The spline function values at the mid knots of the interpolation interval and the corresponding values of the even order derivatives are related through consistency relations. The algorithm developed approximates the solutions, and their higher-order derivatives, of differential equations. Four numerical illustrations are given to show the practical usefulness of the algorithm developed. It is observed that this algorithm is second-order convergent.

Nonic Spline Technique to Solve Boundary Value Problems

A nonic spline function $S\Delta(x)$, interpolating to a function u(x) on [a,b] defined as:

1. In each interval $[x_{j-1}, x_j]$=1,2,….,N, $S\Delta(x)$ is a polynomial of degree at most nine.
2. The first eighth derivatives of $S\Delta(x)$ are continuous on [a,b].
3. $S\Delta(x_i)=u(x_i)$, i=0(1)N+1.

Considering the system of eighth-order boundary value problem of the type

$$\begin{aligned}&y^{(8)}(x) + f(x)y(x) = g(x),\\ &y(a) = \alpha_0,\ y(b) = \alpha_1,\\ &y^{(1)}(a) = \gamma_0,\ y^{(1)}(b) = \gamma_1,\\ &y^{(2)}(a) = \delta_0,\ y^{(2)}(b) = \delta_1,\\ &y^{(3)}(a) = v_0,\ y^{(2)}(b) = v_1,\end{aligned} \tag{16}$$

where $\alpha_i, \gamma_i\ \delta_i$ and v_i, i = 0, 1 are finite real constants and the functions f(x) and g(x) are continuous on [a, b].

Tenth Degree Spline Technique to Solve Boundary Value Problems

A Tenth degree spline function $S\Delta(x)$, interpolating to a function u(x) on [a,b] defined as:

1. In each interval $[x_{j-1}, x_j]$=1,2,….,N, $S\Delta(x)$ is a polynomial of degree at most ten.
2. The first ninth derivatives of $S\Delta(x)$ are continuous on [a,b].
3. $S\Delta(x_i)$=u(xi), i=0(1)N+1.

Considering the system of tenth-order boundary value problem of the type

$$\begin{aligned}&y^{(x)}(x) + \Phi(x)y = \Psi(x),\\ &y^{(a)} = A_0,\ y^{(ii)}(a) = A_2,\\ &y^{(iv)}(a) = A_4,\ y^{(iv)}(a) = A_6,\\ &y^{(viii)}(a) = A_8,\ y(b) = B_0,\ y^{(ii)}(b) = B_2,\\ &y^{(iv)}(b) = B_4,\ y^{(vi)}(b) = B_6,\\ &y^{(viii)}(b) = B_8,\end{aligned} \tag{17}$$

where y = y(x), $\Phi(x)$ and $\Psi(x)$ are continuous function defined in the interval $x\in$ [a,b] and A_i and B_i, i=0,2,4,6,8 are finite real constants.

In the present paper, linear, tenth-order boundary value problems (special case) are solved, using polynomial splines of degree ten. The spline function values at mid knots of the interpolation interval and the corresponding values of the even order derivatives are related through consistency relations.

Eleventh Degree Spline Technique to Solve Boundary Value Problems

An Eleventh degree spline function $S\Delta(x)$, interpolating to a function u(x) on [a,b] defined as:

1. In each interval $[x_{j-1}, x_j]$=1,2,….,N, $S\Delta(x)$ is a polynomial of degree at most eleven.
2. The first tenth derivatives of $S\Delta(x)$ are continuous on [a,b].
3. $S\Delta(x_i)=u(x_i)$, i=0(1)N+1.

In a non polynomial Eleventh degree spline we introduce a parameter k. The arbitrary constants are being chosen to satisfy certain smoothness conditions at the joints. This 'spline' belongs to the class C^{10}[a,b] and reduces into polynomial splines as $k\rightarrow 0$.

Considering the system of tenth-order boundary value problem of the type

$$\begin{aligned}&y^{(10)}(x) + f(x)y(x) = g(x),\\ &y(a) = \alpha_0,\ y(b) = \alpha_1,\\ &y^{(1)}(a) = \gamma_0,\ y^{(1)}(b) = \gamma_1,\\ &y^{(2)}(a) = \delta_0,\ y^{(2)}(b) = \delta_1,\\ &y^{(3)}(a) = v_0,\ y^{(2)}(b) = v_1,\\ &y^{(4)}(a) = \xi_0,\ y^{(4)}(b) = \xi_1,\end{aligned} \tag{18}$$

where $\alpha_i, \gamma_i\ \delta_i\ v_i$ and ξ, i = 0, 1 = are finite real constants and the functions f(x) and g(x) are continuous on [a,b].

Twelfth Degree Spline Technique to Solve Boundary Value Problems

A twelfth degree spline function $S\Delta(x)$, interpolating to a function u(x) on [a,b] defined as:

1. In each interval $[x_{j-1}, x_j]$=1,2,….,N, $S\Delta(x)$ is a polynomial of degree at most twelve.
2. The first eleven derivatives of $S\Delta(x)$ are continuous on [a,b].
3. $S\Delta(x_i)=u(x_i)$, i=0(1)N+1.

Considering the paper having the system of twelfth-order boundary value problem of the type

$$y^{(xii)} + \Phi(x)y = \Psi(x), \qquad -\infty < a \le x \le b < \infty$$

$$y^{(2k)}(a) = A_{2k,} y^{(2k)}(b) = B_{2k,}\ k = 0, 1. 2,….,5 \tag{19}$$

where y=y(x), $\Phi(x)$ and $\Psi(x)$ are continuous function defined in the interval $x\in$ [a,b] and A_i and B_i, i=0,2,4,6,8,10 are finite real constants. The spline function values at mid knots of the interpolation interval and the corresponding values of the even-order derivatives are related through consistency relations. The algorithm developed approximates the solutions and their higher-order derivatives, of differential equations. Two numerical illustrations are given to show the practical usefulness of the algorithm developed. It is observed that this algorithm is second-order convergent.

Thirteenth Degree Spline Technique to Solve Boundary Value Problems

A thirteen degree spline function $S\Delta(x)$, interpolating to a function u(x) on [a,b] defined as:

1. In each interval $[x_{j-1}, x_j]$=1,2,….,N, $S\Delta(x)$ is a polynomial of degree at most thirteen.
2. The first twelve derivatives of $S\Delta(x)$ are continuous on [a,b].
3. $S\Delta(x)$=u(x), i=0(1)N+1.

In a non polynomial thirteenth degree spline we introduce a parameter k. The arbitrary constants are being chosen to satisfy certain smoothness conditions at the joints. This 'spline' belongs to the class C^{12}[a,b] and reduces into polynomial splines as $k\rightarrow 0$.

Considering the system of twelfth-order boundary value problem of the type

$$\begin{aligned}&y^{(12)}(x) + f(x)y(x) = g(x),\\ &y(a) = \alpha_0,\ y(b) = \alpha_1,\\ &y^{(1)}(a) = \gamma_0,\ y^{(1)}(b) = \gamma_1,\\ &y^{(2)}(a) = \delta_0,\ y^{(2)}(b) = \delta_1,\\ &y^{(3)}(a) = v_0,\ y^{(2)}(b) = v_1,\\ &y^{(4)}(a) = \xi_0,\ y^{(4)}(b) = \xi_1,\\ &y^{(5)}(a) = \omega_0,\ y^{(5)}(b) = \omega_1,\end{aligned} \tag{20}$$

where $\alpha_i, \gamma_i \ \delta_i \ v_i \ \xi$ and ω_i, i = 0, 1 = are finite real constants and the functions f(x) and g(x) are continuous on [a,b].
In this paper numerical solutions of the twelfth order linear special case boundary value problems are obtained using thirteen degree spline. The algorithm developed, approximates not only the solution but its higher order derivatives as well. Numerical illustrations are tabulated to demonstrate the practical usefulness of the method. This paper is organized in three sections.

CONCLUSION

This paper is devoted to the evolution, progress, types and spline solutions of differential equations. There is now considerable evidence that in many circumstances a spline function is a more adaptable approximating function than a polynomial involving a comparable number of parameters. Recent trends in computational mathematics, mathematical physics and mechanics are toward the wide use of spline functions to solve such problems. The main advantages of application of spline function are its stability (the local behaviour of a spline at a point does not affect its overall behaviour) and calculation simplicity In solving problems arising in astrophysics, problem of heating of infinite horizontal layer of fluid, eigen value problems arising in thermal instability, obstacle, unilateral, moving and free boundary value problems, problems of the deflection of plates and in a number of other problems of scientific applications, spline functions are not only more accurate but also we have a variety of choices to use quadratic, cubic, quartic, quintic, sextic, septic, octic, nonic or higher splines to solve them.

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