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Research Article

A CLOSE EXAMINATION OF AL-JABR WAL MUQABALA BY AL-KHWARIZMI

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ABSTRACT

This article focuses on reviewing al-Khwarizmi's mathematical work, in particular his text al-Jabr Wal Muqabala. It explores an answer to the question: can this book be considered a treatise in algebra? Or is it a practical manual? Also, sheds some light about different possible influences, reinforcing the discussion by explaining particular examples from the text.

INTRODUCTION

The House of Wisdom was a library and learning institution in Baghdad in the Ninth Century CE, early in the Islamic Golden Age.[1] Its most famous scholar was Muhammad Ibn Musa Al-Khwarizmi, often credited with developing the new field that we now call algebra.[2] I will closely analyze al-Khwarizmi's most celebrated text.

Objectives

The goal of this review study is to shed some light and draw some conclusions about al-Khwarizmi's scholarly life, his mathematics and the interplay between them.

I will concentrate on al-Khwarizmi's mathematical work, in particular his text al-Jabr Wal Muqabala. I will investigate different possible influences, reinforcing the discussion by exploring particular examples from the text.

Al-Khwarizmi's Algebra

Survey of Al-Jabr Wal Muqabala

Let us begin with an excerpt from al-Khwarizmi's preface, which contains a kind of mission statement about the text.“That fondness for science, by God has distinguished the Imam Al-Ma'mun, the commander of the faithful-that affability which he shows to the learned, that promptitude with which he protects and supports them in the elucidation of obscurities and in the removal of difficulties, has encouraged me to compose a short work on calculating by Al-Jabr and Al-Muqabala conning it to what is easiest and most useful in arithmetic, such as men constantly require in cases of inheritance, legacies, partition, law suits and trades, and in all their dealings with one another, or where the measuring of lands, the digging of canals, geometrical computation and other objects of various sorts and kinds are concerned”.

From this excerpt, one would say that this does not sound like a description of a book on algebra at all but rather a practical handbook containing useful calculations. We will explore each chapter of the book and try to decide whether Al-Khwarzimi meant for it to be an illustration of pure algebra or a practical manual. He begins by explaining the natural numbers. This might seem trivial to us now that we are familiar with numbers but it is important to note the new depth of abstraction he showed here. The first part of the book contains all types of quadratic equations. Al-Khwarizmi said that there are three types of terms involved in quadratics: roots (jothoor), squares (amwal) and a single number (al-adad al-mufrad). Where a root is the unknown x, the square is x^2 and the single number is what we identify as the unit or constant. He then noted that there are three types of quadratics that only contain two of these terms. Those are: squares equal to roots ($ax^2 = bx$), squares equal to numbers ($ax^2 = c$) and roots equal to numbers ($bx = c$).He explained the solution of each type by giving several numerical examples and mentioning to the reader that whenever he is faced with one of these types to follow the solutions of the examples. Then he noted that there are three types of quadratics that involve all three of the terms. Those are: squares and roots equal to numbers ($ax^2 +bx = c$), squares and numbers equal to roots ($ax^2 +c = bx$) and finally roots and numbers equal to squares ($bx + c = ax^2$). Again, he gave several examples and justified his solutions geometrically. This is why al-Khwarizmi's work is considered to be systematic and comprehensive. He gave algorithms that the

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reader can follow for any similar situation. In total, al-Khwarizmi classified quadratics into six types. This is because the coefficients had to be positive and he did not accept negative solutions. It is important to mention that al-Khwarizmi does deal with negatives in other chapters of the book; like in the chapter on number operations. He explains sign rules there. Hence, we can certainly say that al-Khwarizmi identifies negative numbers and knows how to deal with them but it does not make any sense to him that they are solutions to equations. This is because he calls the square *mal* which means capital or money and in that sense, all his quadratics deal with money and practical situations. Moreover, the solutions had to make sense even with regards to the wording of the problem. Let us look at an example: “I have divided ten into two parts; I have multiplied the one by ten and the other by itself and the products are the same”. This can be written as $10x = (10-x)^2$. The solution al-Khwarizmi gives is $x = 15-\sqrt{125}$. He ignores the other root $x = 15+\sqrt{125}$because it did not meet the conditions of the problem, i.e., it could not be a part of ten. In this first part of al-Khwarizmi's text the problems consisted of just the equations themselves; that is, it starts by stating a quadratic and trying to solve it. For example, “What must be the square which, when increased by ten of its own roots, amounts to thirty-nine*?*” which can be written in modern notation as $x^2 + 10x = 39$.

The second part of the text is about number operations: multiplication, addition, subtraction and division of terms. Al-Khwarizmi deals here with radicals, negatives and irrationals. He illustrates the sign rules in arithmetic and gives several examples. He solves them without any justification except two examples where he gave a geometrical reasoning. The third part consists of word problems that would lead to one of the types of quadratic equations previously explained in part one. For example: “If a person puts such a question to you as: I have divided ten into two parts, and multiplying one of these by the other, the result was twenty-one" (Rosen 41). This can be written as: $(10-x)\, x = 21$ which is reduced to $x^2 + 21 = 10x$. There are a few examples that would include the word *dirhams* (currency used in Islamic lands) instead of just numbers. Only one problem deals with grain and wheat (things that a common trader of that time will deal with). The fourth part is about mensuration. Al-Khwarizmi gives formulas of areas and volumes of different surfaces and solids and even gives a proof of the Pythagorean Theorem. Those formulas are given in a systematic way just like in a math book studied in schools in modern times. That is, they are not just used to solve a certain problem. These can be considered as useful in the measuring of lands and digging of canals as al-Khwarizmi mentioned in his preface. Finally, almost the final third of the book consists of complicated inheritance and legacies problems; one needs to be familiar with Islamic inheritance laws to understand them. The equations they are reduced to are linear equations.

In summary, the first part of the book was an elaboration on what we would today recognize as algebra. Then comes the applications which inheritance problems take a huge bulk of, and the ones with quadratic equations are not very practical. The part on inheritance might as well be declared as a handbook for the judges who practice these laws in court. The theory of quadratic polynomials is rarely used in a practical sense despite the fact that al-Khwarizmi spends much time explaining it and even proving his methods. Can this book be considered as both a treatise on a new (at the time) theoretical subject, namely algebra, and as a practical manual for any one (even non-mathematicians) to read? I believe that al-Khwarizmi was mainly asked by the caliph to give a manual on real life applications and so he tried, and the book was intended to be highly practical but he could not resist the temptation to explore the theoretical side. Al-Khwarizmi's choice to use the word *mal* (capital, in the sense of money) for x^2 throughout the text seems to supports this belief.[3,4,5]

Debates about Inheritance

Gandz gives in an article he wrote about al-Khwarizmi different quotes of other historians and mathematicians. Those quotes reflect their opinion on al-Khwarizmi and show that the consensus was that he was one of the greatest mathematicians of all time.In 1831, however, when Frederic Rosen translated al-Khwarizmi's book Al-Kitab al-Mukhtasar fe al-Jabr Wal Muqabala and edited it, he added some notes that criticized al-Khwarizmi. In page 133 of the English translation, Rosen remarks in his note:

"The solutions which the author has given of the remaining problems of his treatise are mathematically considered, for the most part incorrect. It is not that the problems when once reduced into equations, are incorrectly worked out; but that in reducing them to equations, arbitrary assumptions are made, which are foreign or contradictory to the data first enounced..." This is harsh comment and certainly does not support the idea that al-Khwarizmi is one of the greatest mathematicians of all time. To the contrary, it makes al-Khwarizmi seem as a very poor under qualified mathematician. The phrase “arbitrary assumptions” alone is enough to show Rosen's ignorance of Islamic inheritance rules and laws. The assumptions are in no way arbitrary; they were made according to established laws of inheritance. For example, suppose a muslim dies and leaves three sons and one daughter. The ideal solution which is simply arithmetical and independent of any laws would say: the estate should be divided into four parts since there are four heirs. Each heir receives one part as his share. But the Islamic law provides that a son gets twice the share of a daughter. So, al-Khwarizmi says to divide the estate into seven parts each son gets 2/7 of the estate while the daughter gets 1/7. Now this would seem arbitrary to a critic of Rosen's type. That al-Khwarizmi arbitrarily divided the estate into 7 parts. Gandz proposes to defend al-Khwarizmi against the attacks of this kind on him in his study *The Algebra of Inheritance: A Rehabilitation of Al-Khuwarizmi*. He first tries to explain some important laws of inheritance. Then he dissects each problem stated in the part on legacies and inheritance of al-Khwarizmi's treatise. He does that by solving the problems after clarifying why certain assumptions were made and then explains where Rosen's errors and misunderstandings occur and corrects them. Gandz mentions that the law of inheritance was taught in school books and that every educated Muslim was familiar with it, not only judges and lawyers. Thus “Al-Khuwarizmi took it for granted that every reader of his book will be well acquainted with this elementary school-subject, hence he did not trouble to cite the provisions of the law". Therefore, one must first learn the law before taking up the algebra of it

(Gandz, Inheritance 328). Other than the rule given above which is: the son gets twice the share of the daughter, there are many rules and conditions that may cause confusions in the understanding of the problems. For example, Rosen points out on page 135 that a husband sometimes inherits 1/4 (as in page 92-93) and sometimes 1/2 of his wife's estate (as in page 135). This is not a contradiction. There is a very simple explanation: if they had children he inherits 1/4 if not he inherits 1/2. This law is stated in the Quran. Another important fact to keep in mind regarding legacies and is used many times in the treatise is that a testator is allowed to bequeath at most 1/3 of his estate to a stranger. If he bequeathed more than that, the amount exceeding 1/3 has no validity and the legatee receives only 1/3 (Gandz, Inheritance332).

The part that was greatly misunderstood by Rosen, as Gandz mentions, is the section devoted to problems in the Computation of Returns, where all cases involve "damages" to be paid for the illegal cohabitation with a woman slave. What adds to the confusion is that most of these problems deal with last illness, i.e., illness resulting in death, and in Islamic law donations made in this period are considered to be legacy and the same restrictions on legacies apply (like not being able to bequeath more than 1/3 to a stranger).[3,4,5,6,7]

Example: Legacies

I will now take a closer look at a specific example explained on page 379 of Gandz's article about inheritance and on page 164 of Rosen's English translation, that deals with legacies, slavery, and terminal illness. Al-Khwarizmi says the following:

A man, in his last illness, gives to somebody his woman slave as a present, and dies without leaving any other property. The slave is worth 300 dirhems, and her dowry (price of cohabitation) is 100 dirhems. The man to whom she has been given to cohabits with her.

Gandz gives explanations to this case and others on pages 37-376 of his article on al-Khwarizmi's algebra of inheritance. Since in this case the donator died after this illness, the gift turns out to be a legacy. As mentioned before if the legacy exceeds 1/3 of the estate it becomes invalid.

Let a = the price of the girl, so a = 300. Let x = valid legacy. There are two owners: the original master has a-x and the legatee has x. The price of cohabitation in this case is 1/3 her price. Since the legatee in this case cohabited with her, he has to pay 1/3(300-x) to the heirs of the master. Then M = (300 - x) +1/3 (300 - x) = (300 - x) + 100 – 1/3x = 2x.

Where M is what the donor's heirs obtain. Note that $M = 2x$ because if y is the whole estate then the legacy x is equal to 1/3y (since this is the Islamic law) and therefore what the heirs get which is M is 2/3y so $M = 2(1/3y) = 2x$.

$400 – 4/3x = 2x$

$400 = 10/3x$

$x = 120$

Al-Khwarizmi's rhetorical solution is stated as follows.

Call the legacy of the person to whom the girl is presented, thing. Subtract this from the donation: there remain three hundred less thing. One-third of this difference returns to donor on account of dowry (since the dowry is one-third of the price): this is one hundred dirhems less one-third of thing. The donor's heirs obtain, therefore, four hundred less one and one-third of thing, which is equal to twice the legacy, which is thing, or to two things. Transpose the one and one-third thing from the four hundred, and add it to the two things; then you have four hundred, equal to three things and one-third. One thing is, therefore, equal to three-tenths of it, or to one hundred and twenty dirhems, and this is the legacy (Rosen 164)

The importance in presenting this example is reminding the reader that this is genuine example of applicable algebra in al-Khwarizmi's text.[3,4,5,6,7]

CONCLUSION

Al-Khwarizmi writes in his preface that he is composing a book for practical use. He does have a big part on inheritance problems and legacies, a part on mensuration which is helpful for land measuring and a few practical problems about grains and trades. In the first part of the book, however, he explains a systematic way of solving quadratics. Al-Khwarizmi was indeed set out to compose a practical manual most probably because of external patronage pressures.[4] He knew quadratics may arise in practical situations including the cases he mentions in his preface. He did not prefer to just solve specific problems but rather show the reader all kinds of quadratics he might be faced with. Al-Khwarizmi then, most importantly, pioneered the new sub-field of mathematics: algebra.

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