International Journal of Current Advanced Research
ISSN: O: 2319-6475, ISSN: P: 2319-6505, Impact Factor: 6.614
Available Online at www.journalijcar.org
Volume 7; Issue 10(F); October 2018; Page No.16087-16090
DOI: http://dx.doi.org/10.24327/ijcar.2018.16090.2955

Research Article

AN EXTENDED STOCHASTIC MULTIVARIATE TIME SERIES MODEL FOR MODELLING METEOROLOGICAL VARIABLES

Ibrahim Lawal Kane[1] and Umar Saminu[2]

[1]Department of Mathematics and Computer Science, Umaru Musa Yar'adua University, 2218, Katsina – Nigeria
[2]Department of Geography, Umaru Musa Yar'adua University, 2218, Katsina – Nigeria

A R T I C L E I N F O

Article History:

Received 4th July, 2018
Received in revised form 25th August, 2018
Accepted 23rd September, 2018
Published online 28th October, 2018

Key words:

Climate Change, Fourier based VAR Modelling, Fourier Based VAR-GARCH, Seasonality, and Volatility.

A B S T R A C T

Climate change phenomena have been a global topic in environmental and water resources management. Nowadays weather is not as it used to be. High variability and persistence are common features of climatological time series data. Seasonality is one of the most key factors influencing variations in a time series. In several time series analysis, variations prompted by seasonal factor sometimes dictate the variations of the original series. This issue often occurs due to the environmental influence, such as periodic trend and variance clustering. Fourier based approach to seasonality modelling on vector autoregressive framework of rainfall and temperature series of Katsina was developed. An FtVAR(2) model for the series was able to capture the temporal linear properties of the two climate variables. However, it is shown that the bi-climate history is affected by nonlinear characteristics of the variance often referred to as variance clustering or volatility; this was captured by incorporating GARCH family model to the fitted FtVAR which resulted to FtVAR-mGARCH model. Comparing the efficiency of the two developed models, the FtVAR-mGARCH model performs better than FtVAR model arbitrating based onRMSE, MAE and MAPE of both models and therefore it's best captured the dynamics of the climate variables. This result may give insight on the temporal dynamics of tropical continental weather variable for proper planning environmental and water resources management.

INTRODUCTION

Climate change has been an overall global issue and one of the most vital topics in environmental and water resources. Global climatic conditions influence the tropical continental region. These conditions produce a high variability and persistence that are common features of climatological time series data. Stochastic models are often used to analyse these climatological time series. A Multivariate time-series approach is an indispensable tool to study the attitude of time dependent variables based on the history of the data variation (for example, Shahin, 2014;Norrulashikin *et al*. 2015; Norrulashikin *et al*. 2018). Weather parameters such as Precipitation and Temperature, have convincing effects on hydrological cycle and the environments. These variables not only have relationships with each other, but also are dependent.

Seasonal influence is one of the most key factors impelling variations in time series analysis. In several time series, variations prompted by seasonal factor sometimes dictate the variations of the original series. This issue often occurs due to the environmental influence, such as periodic trend.

Corresponding author:* **Ibrahim Lawal Kane
Department of Mathematics and Computer Science, Umaru Musa Yar'adua University, 2218, Katsina-Nigeria

A seasonal time series is considered as a non-stationary time series that follow some kind of seasonal periodic trend (Ming-Hung Shu *et al*., 2014). In general models for time series data can have many forms and represent different stochastic processes. Seasonal Autoregressive Integrated Moving Average (SARIMA) models have been proposed in many literatures, for example, (Fadhilah and Ibrahim, 2012; Bako *et al*., 2013; Jamaludin *et al*., 2014; etc). Another approach to modeling seasonal time series is proposed in this work using Fourier analysis. Fourier analysis is a fundamental tool used in all areas of science and engineering; it is originally concerned with representing and analysing periodic phenomena.Fourier transformation has been popular for reconstruction and classification purposes (Kauppinen*et al*., 1992), power distribution system (Shenbaga, 2017), wireless, signal processing (Bucci and Migliore, 2016), and industrial application (Zheng *et al*., 2016).

Another important characteristics of climate variables is the climate volatility which is one of the challenging issue nowadays, it effect long-run economic growth of a country (Abebayehu, 2015). Multivariate GARCH models specify equations for how the variances move over time (Tse, 2000). The crucial stage in MGARCH modelling is to provide a realistic but parsimonious specification of the variance matrix ensuring its positivity. Applications of mGARCH can be found in (Shohreh, 2004; Goeij, 2004; etc).

A better understanding of the temporal variability of historic climate data is essential to develop better perspectives of long-term variations of the climate variables. This paper tends to provide extended Vector Autoregressive (VAR) methodology in modelling framework for the quantitative analysis of the relative dynamics of different meteorological variable on one another. The joint quantitative analysis of these stochastic meteorological variables and the understanding of their fluctuations on different scales, i.e. their general variability, should be of fundamental importance in researches on climate change. Nigeria been one of the country effected by climate change, however, receives less attention in such type of study. Therefore, a new framework for modelling climate dynamics is presented.

METHODOLOGY

Data Used

This work observed 30 years data of rainfall and temperature from Katsina which belong to steppe climates under Köppen Climate Classification system from 1946 to 2014. The time series plots of all the variables considered are plotted against time these plots are shown in Figure.1.

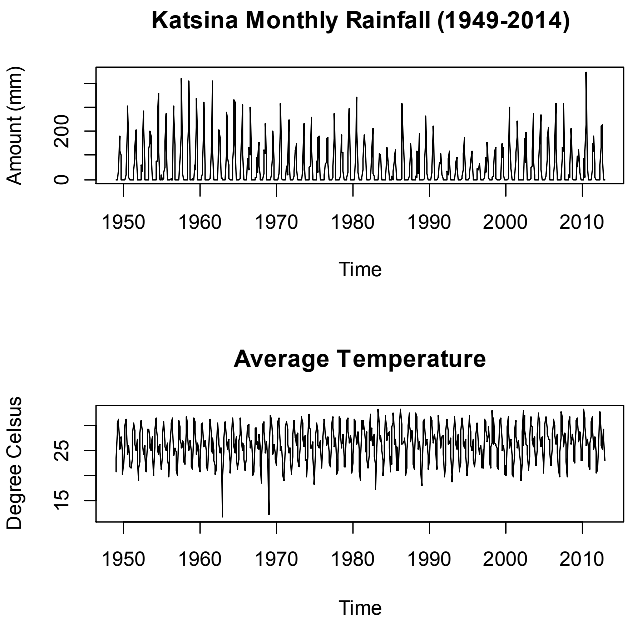

Figure 1 Rainfall and temperature time series of Katsina

Fourier analysis

Fourier analysis was originally concerned with representing and analysing periodic phenomena and extending these periodic intuitions to nonperiodic phenomena, via the Fourier transform. Katsina Rainfall and Temperature time series are periodic in time (12 months). The basic idea behind the proposed modelling is to re express the original seasonal time series of temperature x(t) and rainfall y(t) as a new sequence in term of frequency domain x(f) and y(f) given as: $x(f)=\sum_{t=1}^{n}x(t)e^{-i2\pi ft}, y(f)=\sum_{t=1}^{n}y(t)e^{-i2\pi ft}$ respectively, where f represents the frequency at which x or y is evaluated and $e^{-i2\pi ft}=\cos(2\pi ft)-i\sin(2\pi ft)$.

In general, the temperature and rainfall curves from the original time series are given as:

$$x(t)=\tfrac{1}{2}a_0+a_1\cos(\omega_1 t)+b\sin(\omega_1)+a_2 2\cos(\omega_2 t)+b_2\sin(\omega_2 t) \quad \ldots 1$$

and

$$y(t)=\tfrac{1}{2}a_0+a_1\cos(\omega_1 t)+b\sin(\omega_1)+a_2 2\cos(\omega_2 t)+b_2\sin(\omega_2 t) \quad \ldots 2$$

1 & 2 are incorporated into VAR modelling strategy.

Vector Autoregressive VAR Modelling Strategy

The description of the Vector Autoregressive (VAR) Modelling methodology which serves as the basic framework of this work is presented in Figure 1.

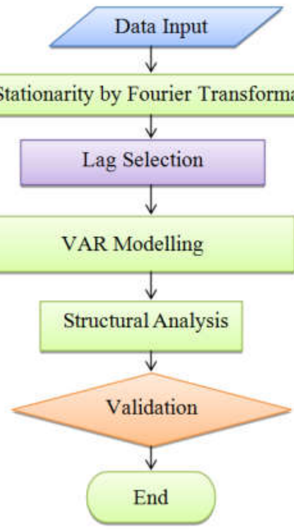

Figure 1 VAR methodology framework

Fourier Based VAR Model

The basic idea behind the proposed model is to re express the original time series of temperature x(t) and rainfall y(t) as a new sequence in term of frequency domain x(f) and y(f) given as: $x(f)=\sum_{t=1}^{n}x(t)e^{-i2\pi ft}, y(f)=\sum_{t=1}^{n}y(t)e^{-i2\pi ft}$ respectively, where f represents the frequency at which x or y is evaluated and $e^{-i2\pi ft}=\cos(2\pi ft)-i\sin(2\pi ft)$ For a set of k variables let ${}_{Ft}Y_t=(y_{1t}\ldots y_{kt})$ denote an (n×1) vector of Fourier transformed time series variables. The proposed Fourier based VAR (FtVAR (*p*)) model can be expressed as:

${}_{Ft}Y_t=c+\phi_1 {}_{Ft}Y_{t-1}+\ldots+\phi_p {}_{Ft}Y_{t-p}+{}_{Ft}\varepsilon_t$, for ($t$ = 1, …,N),

which can be rewritten as: $\phi(L){}_{Ft}Y_t=c+{}_{Ft}\varepsilon_t$ where c is the constant term, ${}_{Ft}\varepsilon_t=({}_{Ft}\varepsilon_{1t}+{}_{Ft}\varepsilon_{2t}+\ldots+{}_{Ft}\varepsilon_{kt})$ is an unobserved zero means independent white noise process with time invariant positive definite covariance matrix $E({}_{Ft}\varepsilon_t {}_{Ft}\varepsilon_t')=\Sigma_k$ and $\phi(L)=1-\phi_1 L-\phi_2 L^2-\ldots-\phi_p L^p$ is a matrix of a lag polynomial with k x k coefficient matrices ϕ_j, for j = 1,…, p.

Fourier Based VAR-m GARCH Model

Consider the FtVAR model express as;

$$\phi(L){}_{Ft}Y_t=c+{}_{Ft}\varepsilon_t$$

the error term ${}_{Ft}\varepsilon_t$ can be expressed as;

$${}_{Ft}\varepsilon_t=D_t R_t D_t$$

where $D_t = diag(h_{1t}^{\frac{1}{2}}, h_{2t}^{\frac{1}{2}}, \ldots, h_{kt}^{\frac{1}{2}})$ and R_t is the conditional correlation matrix. Following Carnero & Eratalay (2014) R_t is positive definite and it is clear that as long as conditional variances, h_{it} are positive for any i= 1, 2, …, k, the conditional variance-covariance matrix, H_t will be also positive definite. The conditional variances h_{it} are assumed to follow a GARCH(1,1) model. Then,

$$h_t = \omega + A\varepsilon_{t-1}^{(2)} + Gh_{t-1}$$

where $h_t = [h_{1t} h_{2t} \cdots h_{kt}]'$ and $\varepsilon_t^{(2)} = [\varepsilon_{1t}^{(2)} \varepsilon_{2t}^{(2)} \cdots \varepsilon_{kt}^{(2)}]'$ are $k \times 1$ vectors of conditional variances and squared errors respectively and ω is a $k \times 1$ and A and G are $k \times k$ matrices of coefficients.

FtVAR Model Estimation

The Fourier based VAR model of rainfall and temperature series is estimated using number of lags order based on information criteria that is values of AIC, HQC, FPE and BIC. The AIC and FPE suggested lag length of 2 while HQC and SC suggested lag length of 1. We decided to model the VAR process using both lag order of p=1 and p=2 and by comparing the mean square error and the mean absolute error of the models FtVAR(2) model seems to give better result based on low values of MAE and RMSE and the equation of the model is presented as;

$$\begin{bmatrix} FtR_{(t)} \\ FtT_{(t)} \end{bmatrix} = \begin{bmatrix} -0.1112 \\ -0.0658 \end{bmatrix} t + \begin{bmatrix} -0.0735 & 0.0460 \\ 0.0416 & 0.0079 \end{bmatrix} \begin{bmatrix} FtR_{(t-1)} \\ FtT_{(t-1)} \end{bmatrix} + \begin{bmatrix} 0.0005 & 0.0476 \\ 0.0459 & -0.0819 \end{bmatrix} \begin{bmatrix} FtR_{(t-2)} \\ FtT_{(t-2)} \end{bmatrix} + \begin{bmatrix} Ft\varepsilon_{(1t)} \\ Ft\varepsilon_{(2t)} \end{bmatrix}$$

Tests based on Breusch-Godfrey (BG) for residual autocorrelation and Breusch-Pagan for heteroskedasticity applied to the residuals from the developed FtVAR model to confirm the model adequacy. The results of these tests show a clear evidence of model adequacy at level and reject the null hypothesis of no autoregressive conditional heteroskedasticity (ARCH) effect see table 2. Hence, it indicates that generalized ARCH (GARCH) modelling is necessary.

FtVAR-mGARCH Model Estimation

Using the two residual series from the fitted FtVAR model, we estimate FtVAR-CCC-GARCH model and the results of the estimated parameters are given in Table 1.

Table 1 Parameter estimates of FtVAR-mGARCH Model

Parameter	μ_1	μ_2	β_{11}	β_{22}	ω_1	ω_2	α_{11}	α_{22}	γ_{11}	γ_{22}	ρ_{12}
FtVAR-mGARCH	0.09	0.08	0.007	-0.05	0.016	0.017	0.07	0.07	0.57	0.71	0.59

The residuals of the FtVAR-CCC-GARCH model were tested for model adequacy. Results based on Breusch-Godfrey Lagrange multiplier test for residuals auto correlation, Breusch-Pagan for heteroskedasticity and the Anderson Darling test for normality of the residuals are displayed in Table 2. From Table 2, it could be observed that the model FtVAR-CCC-GARCH model passed all the tests demonstrating that it captures the dynamics of the climate data considered. The efficiency of the model was further measured via simulation in comparison with FtVAR model.

Table 2 Models Diagnostic Check Results

	FtVAR Model	FtVAR-mGARCH
Breusch-Godfrey LM test	0.0971	0.1428
Breusch-Pagan test	0.0426	0.0764
Anderson Darling test	0.2172	0.4321

Simulation Studies

Simulation is used for model evaluation. In this section, the efficiency of VAR, FtVAR and FtVAR-mGARCH models were tested via simulation. Ten (10) replicates of the meteorological variables were randomly generatedbased on the fitted models and the average of the forecast errors are presented in Table 3.

Table 3 Models Comparison

Model	FtVAR	FtVAR-mGARCH
RMSE	1.010	0.901
MAE	0.904	0.637
MAPE	14.024	10.337

It can be observed from Table 3 that the FtVAR-mGARCH model performs better than FtVAR and VAR models arbitrating from the RMSE, MAE and MAPE of both models that shows lower values of the error components.

SUMMARY AND CONCLUSION

The study of joint modelling of climate variablesiscrucial for understanding the future changes in climate and executing important strategies. Seasonality is one of the most key factors influencing variations of a time series. In several time series, variations prompted by seasonal factor sometimes dictate the variations of the original series. This issue often occurs due to the environmental influence, such as periodic trend. Fourier based approach to seasonality modelling on vector autoregressive framework of rainfall and temperature series of Katsina was developed. An FtVAR (2) model for the series was able to capture the temporal linear properties of the two climate variables. However, it is shown that rainfall and temperature data are affected by nonlinear characteristics of the variance often referred to as variance clustering or volatility, this was captured by incorporating GARCH family model to the FtVAR which resulted to FtVAR-mGARCH model. Comparing the efficiency of the two developed models, the FtVAR-mGARCH model performs better than FtVAR model arbitrating based on RMSE, MAE and MAPE of both models and therefore it's best captured the dynamics of the climate variables.

Acknowledgement

The authors would like to thank Umaru Musa Yar'adua University, Katsina and TETFund grant vote number UMYU/UBR/BC/002 for funding the research.

References

1. Shahin M.A., Ali M.A., Ali A.B.M.S. (2014) Vector Autoregression (VAR) Modeling and Forecasting of Temperature, Humidity, and Cloud Coverage. In: Islam T., Srivastava P., Gupta M., Zhu X., Mukherjee S. (eds) Computational Intelligence Techniques in Earth and Environmental Sciences. Springer, Dordrecht.
2. Norrulashikin, S.M, Yusof, F & Kane, I.L. (2015). Modern Applied Science; Vol. 9, No. 11; pp89-100. ISSN 1913-1844 E-ISSN 1913-1852.
3. Norrulashikin, S.M, Yusof, F & Kane, I.L. (2018). Meteorological Multivariable Approximation and Prediction with Classical VAR-DCC Approach. Sains Malaysiana 47(2): 409-417. http://dx.doi.org/10.17576/jsm-2018-4702-24

4. S. Ming-Hung, H. Wei-Ju, H. Bi-Min, N. Thanh-Lam, L. Chunwei, “Forecasting with Fourier Residual Modified ARIMA Model- An Empirical Case of Inbound Tourism Demand in New Zealand,” WSEAS Transactions on Mathematics, Vol. 13, E-ISSN: 2224-2880, PP 12-21, 2014
5. F. Yusof and I. L. Kane, “Modeling Monthly Rainfall Time Series Using ETS State Space and SARIMA Models,” *International Journal of Current Research*, vol. 4:9, pp. 195-200, September, 2012.
6. H. Y. Bako, M. S. Rusiman, I. L. Kane, H. M Matias-Peralta, “Predictive Modeling of Pelagic Fish Catch using Seasonal ARIMA Models,” *Agriculture, Forestry and Fisheries*, 2(3), 136-140, 2013.
7. A. R Jamaludin Bt, F. Yusof, I. L. Kane, “Temporal Dynamics of Trend in Relative Humidity with RH-SARIMA Model,” Modern Applied Science; Vol. 9, No. 3, 2015.
8. H. Kauppinen, T. Seppanen, M. Pietikainen, “An Experimental Comparison of Autoregressive and Fourier-based Descriptors in 2-D Shape Classification,” IEEE Transaction on Pattern Analysis and Machine Intelligence, vol. 17, pp. 201-207, 1995.
9. S. E. Shenbaga, “Real Time Application of Fourier Transfprms,” *Indonesian Journal of Electrical Engineering and Computer Science*, Vol. 8, No. 2, pp. 574 -577, 2017.
10. O. M. Bucci, M. D. Migliore, M.D, “A Novel Non Uniform Fast Fourier Transform Algorithm and its Application to Aperiodic Arrays,” *IEEE Antennas and Wireless Propagation Letters*, 2016
11. S. Zheng, H. Cai, Y. Gu, L. K. Chin, A. Q. Liu, “On-chip Fourier transform spectrometer for chemical sensing applications," In *CLEO, Applications and Technology*, Optical Society of America, pp. AM1J-6, 2016.
12. G. G.Abebayehu (2015) Impacts of Climate Volatility on Long-run Economic Growth: Cross-Country Growth Regressions, Journal of Economics and Sustainable Development, Vol.6, No.15, 2015.
13. Tse, Y. K. 2000. “A test for constant correlations in a multivariate GARCH model”. *Journal of Econometrics* 98: 107-127
14. ShohrehValiani (2004) adopted MGARCH specification that has been applied to estimate the time varying correlations of underlying assets and related currency forwards in order to hedge the currency exposure risk in an international portfolio context.
15. De Goeij, Peter, Marquering Wessel. 2004. “Modeling the Conditional Covariance Between Stock and Bond Returns: A Multivariate GARCH Approach”. *Journal of Financial Econometrics*. Vol. 2, No. 4: 531-564
16. M. A. Carnero and M. H. Eratalay. (2014) Estimating VAR-MGARCH models in multiple steps. Stud. Nonlinear Dyn. E. 2014; 18(3): 339-365. DOI 10.1515/snde-2012-0065

How to cite this article:

Ibrahim Lawal Kane and Umar Saminu (2018) 'An Extended Stochastic Multivariate Time Series Model for Modelling Meteorological Variables', *International Journal of Current Advanced Research,* 07(10), pp. 16087-16090.
DOI: http://dx.doi.org/10.24327/ijcar.2018.16090.2955

****