



Research Article

ON THE ROLE OF QUANTUM COMPUTING IN GROUNDING MORPHOLOGICAL COMPLEXITY

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ABSTRACT

In this Short Communication we will discuss the role played by quantum computing within the emerging morphological paradigm in the unconventional natural computing. We intend merely introduce the main reasons why a coherent representation of Universality in morphological natural computing needs to be grounded on a version of Quantum Field Theory independent, in many senses, from the Quantum Mechanics formalism in fundamental physics, namely formulated as a thermal field theory. This theory describes the “emergence” of natural information and computation’s complexity at a quantum level, thus overcoming the trivializing argument against the so-called “pancomputationalism” in morphological natural computing.

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INTRODUCTION

Morphological computation is an emerging paradigm in the contemporary unconventional natural computing. An updated survey of the theoretical and engineering issues involved in morphological computation is given in (Müller and Hoffmann, 2017). Within this paradigm the notion of “form” (*morphé*), with its (often) unaware ontological background, has an operational scientific counterpart in the notion of information exchanges, embodied in the matter-energy exchanges of the physical body and/or the organism and/or of the brain, inside/outside them.

In (Dodig-Crnkovic, 2017) the author shows how morphological information and computation can interpret the information-theoretic approach to contemporary fundamental physics, connoted as the physics “beyond the Standard Model” because of the current crisis of the Standard Model, that is the theory describing three of the four known fundamental forces (electromagnetic, weak, and strong interactions) in the universe, as well as classifying all known elementary particles. The theoretical core of Dodig-Crnkovic’s argument is to suggest a morphological extension of Hewitt’s Actor Model for modelling, outside quantum Turing Universality, the evolution of Schrödinger’s statistical wave function with the connected amount of information it “contains”, made available via the mechanism of the decoherence, strictly depending on the measurement operation performed by the actor-observer.

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At a quantum level, natural information and computation are thus of a morphological type, embodied in observer-dependent quantum correlation effects (entanglements).

Commenting Wolfram’s digital understanding of the information-theoretic approach to fundamental (quantum) physics presented in *A New Kind of Science*, Dodig-Crnkovic argues that a Universal Cellular Automata cannot compute the Computable Universe in natural computing, because of the complexity of nature. This means, essentially, as Davies made clear referring to Benioff’s work (Davies, 2010), that natural laws, i.e., the Universe computing rules, unfold with natural evolution, so that the number of computations grows necessary for the growth in complexity of the nature itself. We are faced here with the notion of physical information (Goyal, 2012), intended, however, as a measure of the algorithmic complexity.

In (2017) Müller and Hoffmann generalize the above critical remark, turning it against the so-called pancomputationalism in morphological natural computing, which does not imply that universal computation is digital: “cognitive and morphological processes are trivially computational in the sense that everything else is” (p. 11). Dodig-Crnkovic’s suggestion does not respond to this criticism definitively. Indeed, the Computable Universe it describes grounds on Quantum Mechanics (QM) correlation effects that occur only at the microscopic level, so that universal computation implies reduction of mesoscopic, macroscopic and even cosmic morphological structures and processes to microscopic ones. As Feynman put it, in a QM’s Computable Universe “laws will turn out to be simple, like the chequer board with all its apparent complexity” (1985, p. 57).

In what follows we merely introduce the main reasons why a coherent representation of Universality in morphological natural computing needs to be grounded on a version of Quantum Field Theory (QFT), independent, in many senses, from the QM formalism in fundamental physics, i.e., formulated as a thermal field theory and not as a “second quantization” with respect to QM. The abstract model of thermal QFT computing satisfies a principle of Coalgebraic Universality in computation. This gives a higher degree of generality also to QM computations, since any Quantum Turing Machine (QTM) can be interpreted in terms of it, but not vice versa. On the contrary, the Coalgebraic Universality in computation is essential for QFT computing because Turing Universality does not hold in it.

DISCUSSION

Till now QFT was conceived as an extension of the QM, i.e., it was conceived according to the so-called “second quantization” as to QM, starting from P. Dirac’s and R. Feynman’s statistical and not dynamic interpretation of fields in QFT. In this interpretation, specifically in the study of the fundamental electromagnetic interactions of the Quantum Electrodynamics (QED) and of the color charge interactions of the strong force in the sub-nuclear physics of Quantum Chromodynamics (QCD), it continues to work the mechanistic scheme of the particles isolated from forces in the mechanical vacuum. According to this scheme, it is the same the study of the properties of a physical system abstractly considered as an object in mind, i.e., isolated from the dynamic forces (causes) acting on it, and the study of it as interacting with other physical systems, as it is in reality. Indeed, using the so-called asymptotic condition to study the particle behavior in the perturbative theory of many body dynamic systems, we can represent the system separating it at infinite spatio-temporal distances, so as to isolate the particles from interactions, “cutting-off” them and re-creating artificially the condition of the particle isolation in the mechanical vacuum of the Newtonian Mechanics (Bogolyubov 2011). The supposition is that such a mathematical modeling does not falsify the reality of the observed phenomena. The consequence is the absolute distinction between particles and interaction force fields. Now, it is precisely this distinction that constitutes the theoretical core of the Standard Model in QM.

To understand intuitively the relevance of such a perspective for QED and QCD mathematical formalism, think at the powerful calculus means of the so-called “Feynman diagrams” in QED and in QCD. In them the particles (fermions) are represented by straight lines, the quanta of the force fields mediating the interactions (gauge bosons) are represented by wavy lines, and the coupling strengths of interactions by the different angles of the diagrams. The intuitive mechanistic model in the background is that the particles (fermions) interact mechanically by exchanging reciprocally force quanta (bosons) in the vacuum. In other terms, in the Standard Model the mechanistic distinction particle-force is introduced by interpreting ontologically the distinction, in itself only statistical, between fermions and bosons like the mechanistic difference between particles and forces.

The dynamic substrate of the Quantum Vacuum (QV) is the core of the passage from the mechanistic paradigm in the ontology of fundamental (quantum) physics, proper of the QM, to the dynamic one of the QFT, interpreted as a thermal field

theory (Blasone *et al.*, 2011). QV becomes necessary when one tries to explain how the Higgs field works.

The Higgs field with the relative quantum, the Higgs boson, with a mass of about 126 GeV (126 protons) is needed to keep the law of conservation of charge, when the fields of massless particles interact, so as to give rise to the fields of massive particles. Indeed, the gauge bosons, per se, according to the fundamental Goldstone Theorem (Goldstone *et al.*, 1962), must not have masses like, effectively, it is the case of photons and gluons. Therefore, for giving rise, in the case of the electroweak force, to the tremendous mass of the Z boson (practically, it “weighs” like an iron atom), and of the W^\pm bosons, and, finally, for giving rise to the same mass of quarks and leptons, of which all the ordinary matter is constituted, the Higgs field becomes necessary. In other words, the masses of particles constituting the matter of all bodies are generated when the QV, namely the dynamic whole of all the quantum fields connecting dynamically everything, “produces” the Higgs field, a kind of viscous medium that “brakes” the propagation of other force fields. In this way, the fields “running over” the underlying Higgs field manifest a sort of inertia, and therefore they manifest the dynamic generation of a mass by the coupling with the Higgs field of the relative force field quanta. On the contrary, other fields like the electromagnetic one, whose quanta are the massless photons, by which quark and electrons interact, or the strong field, whose quanta are the massless gluons by which mainly quarks interact, do not couple with the Higgs field. For this reason, photons and gluons remain massless. In short, the mass of all the elementary particles is proportional to the “degree of coupling” of all their fields with the Higgs field, with a key-role of Z- W^\pm bosons exchange, and hence of the neutrino physics, given that neutrinos interact with other fermions, mainly quarks, only through the weak force. From the universal dynamic substratum of the QV all the particles and all their systems emerge thus dynamically like as many Spontaneous Symmetry Breakdowns (SSBs) of the QV at its ground state, without any input from the outside—because there is no “outside” of the QV, since everything is “inside” it. The association of whichever mole of matter with a force field, and therefore the existence of the QV as the totality of the quantum force fields, is an immediate consequence of the Third Law of Thermodynamics, and in this sense, QFT ultimately is a thermal field theory (Blasone *et al.*, 2011).

For our aims the most relevant point is that each SSB corresponds to a local phase coherence domain among some oscillating fields of the QV, that constitute one only dynamic system with the relative particles, i.e., a quantum system entangled with its thermal bath. The quanta of these coherent modes of field oscillations, necessarily appearing in the equations, and experimentally observed and measured (Goldstone *et al.*, 1962), are the so-called Nambu-Goldstone bosons (NGB): despite they follow the same statistic distribution of the gauge bosons of the fundamental forces, nevertheless they don’t mediate any energy exchange. Indeed, NGB are quanta of the phase coherent modes of whichever force field, i.e., either of interaction force fields (gauge bosons), or of material force fields (fermions), by which whichever energy exchange might occur. The corresponding interpretation of quantum information is not depending on the distinction between energy and information: differently from the QM interpretation, where the information

notion and measure, unlike the energy ones, are observer-related, in the thermal QFT interpretation information is a physical magnitude, expressing the NGB condensate value from which a phase coherence domain directly depends.

Now, because thermal QFT, differently from QM, can work on phase transitions, the notion of “phase coherence domains”, as far as based on the presence of long distance correlations among the quantum fields, implies an intrinsic change of scale. In this way, in condensed matter physics, we are faced with mesoscopic, macroscopic and even cosmological order parameters, that are parameters characterizing the new emerging level of matter organization, having their proper explanation at the quantum microscopic level. This is not a reductionist view. Actually, the characterizing properties of condensed matter are structures and functions with their own laws, emerging over the microscopic dynamics generating it. Of course, the emergence process is related also to a scale change, whose dynamic regime is, however, of a quantum nature. The resultant system is then a quantum mesoscopic/macroscopic/cosmological system, that avoids reduction to its quantum microscopic constituents.

Through the SSB principle in the QV ground state, infinitely QV ground state conditions there exist: within thermal QFT the classical Stone-Von Neumann Theorem does not hold. This theorem states that, for system with a finite number of degrees of freedom, which is always the case in QM because of the man-made choice by the observer of the orthonormal finite basis of the Hilbert space for representing a quantum system, the representations of the canonical commutation relations are all unitarily equivalent as to each other. On the contrary, in QFT systems, because we are faced with infinitely many phase transitions, the number of the degrees of freedom is not finite, so that infinitely many unitarily inequivalent representations of the canonical commutation (bosons) and anti-commutation (fermions) relations exist, as the fundamental Haag’s Theorem demonstrated in the late 50’s of the last century (Earman and Fraser, 2006). This holds not only in the relativistic (microscopic) domain, but it applies also to non-relativistic many-body systems in condensed matter physics, i.e., in the macroscopic and even in the cosmological domain, through the Principle of the QV Foliation at the ground state (Blasone *et al.*, 2011, pp. 53–96). That is, different SSBs and hence phase coherence domains at the QV ground state can be “stratified” over each other, so as to originate the so-called complex physical systems. The Principle of the QV Foliation is a “robust principle of construction and of memory used by nature for generating ever more complex systems” (Basti *et al.*, 2017, p. 39). It is important to recall the role played by the notion of stored energy in such a multi-level organization in coherent domains and sub-domains. Indeed, “this completes the classical thermodynamic picture of L. Szilard and L. Brillouin according to which the ‘Maxwell demon’, for getting information so as to compensate the entropic decay of the living body, must consume free energy from the environment. This means an increasing of the global entropy according to the dictate of the Second Law. However, this has to be completed in QFT with the evidence coming from the Third Law” (Basti, 2017, pp. 181–182).

In (Basti *et al.*, 2017) the authors give a complete justification of a coalgebraic interpretation of the thermal QFT mathematical formalism, demonstrating the possibility of

designing a new class of universal quantum computing architectures based on this coalgebraic formulation of QFT. At least two points of such a justification are important to emphasize here.

Firstly, the necessary condition to be satisfied in order that a coalgebras category¹ for some endofunctor can be interpreted as a computational system, is that it satisfies the formal notion of State Transition System (STS). Generally, a STS is an abstract pragmatic machine characterized as a pair (S, \rightarrow) , where S is a set of states, and $(\rightarrow) \subseteq S \times S$ is a transition binary relation over S . If p, q belongs to S , and (p, q) belongs to (\rightarrow) , then $(p \rightarrow q)$. I.e., there is a transition over S . For allowing that a dynamic/computational system be represented as a STS on a functorial coalgebra for some functor it is necessary that the functor admits a final coalgebra (Venema, 2007, p. 389). Roughly speaking, it is necessary that all the coalgebras in the category have a unique identifier, labelling it univocally. This final coalgebra requirement has a very intriguing realization into the final coalgebra associated with a particular abstract machine, the so-called Infinite State Black-Box Machine (Venema, 2007, p. 395). It is characterized by the fact that the Machine internal states, x_0, x_1, \dots , cannot be directly observed, but only some their values, the so-called “colors”, c_n , associated with a state transition μ . I.e.: $\mu(x_0) = (c_0, x_1), \mu(x_1) = (c_1, x_2), \dots$. In this way, the only observable of this dynamics is the infinite sequence of behaviors or “stream” $beh(x_0) = (c_0, c_1, c_2, \dots) \in C^\omega$ of value combinations or “words” over the data set C . The collection C^ω forms a labeled STS for the functor $C \times I$, where I is the set of all the identity functions (labels). In this way, we can endow C^ω with a transition structure γ splitting a stream $u = c_0 c_1 c_2, \dots$ into its head $h(u) = c_0$, and its tail $t(u) = c_1 c_2 c_3 \dots$. If we pose $\gamma(u) = (h(u), t(u))$, it is possible to demonstrate that the behavior map $x \mapsto beh(x)$ is the unique homomorphism from the Infinite State Black-Box

¹Any algebra can be defined as a structure defined on sets characterized by an endofunctor, projecting all the possible combinations, i.e., Cartesian products, of the subsets of the carrier set, on which the algebra is defined, onto the set itself. Any coalgebra can be defined as a structure defined on sets, whose endofunctor, projects from the carrier set onto the coproducts of this same set. Thermal QFT provides a systematic use of the dual categories of q -deformed Hopf coalgebras and algebras. A Hopf algebra is a bialgebraic structure equipped with an antiautomorphism that is simultaneously an unital associative algebra and a counital coassociative coalgebra. In the special case we are speaking about, q is a thermal parameter linked to the Bogoliubov transformation’s angle: as it is well known, the angle, with the frequency and the amplitude, is a parameter characterizing the phase of a given waveform. Algebras and coalgebras are in this case characterized by non-commutative products and coproducts. The q -deformation parameter, characterizing each pair of q -deformed Hopf coalgebra-algebra, physically is a thermal parameter. Duality is induced by the contravariant functor related to the Bogoliubov transformation (Basti *et al.*, 2017). Within the framework of Category Theory (CT) a category is a relational structure, namely a collection of morphisms, compositions of morphisms (arrows) and objects, i.e., identity (reflexive) morphisms, that preserves the structure up to isomorphism. A functor is a morphism between categories: generally, a functor is covariant, that is, it preserves arrows directions and composition order. Even if a functor connecting two categories is contravariant, i.e., reverses all the arrows directions and all the composition orders, the categories are homomorphic up to isomorphism (Awodey, 2006; Abramsky and Tzevelekos, 2011).

Machine to this coalgebra $\langle C, \gamma \rangle$, that is the final coalgebra in the category $\mathbf{Coalg}(C \times I)^2$.

The application of the Infinite State Black-Box Machine for characterizing the QFT dynamics as computing dynamics- that is, effectively, the QV endowed with SSB transition structure as a Quantum-Infinite-State-Black-Box-Machine-occurs as follows. We interpret the functor T characterizing the category of q -deformed Hopf coalgebras as a functor able to associate the observable C of each word (phase coherence domain, corresponding to a SSB of the QV) of the QFT infinite dataset C (the infinite canonical commutation relations characterizing the QV), with the correspondent I_c , so that $T = (C \times I)$.

Indeed, each I_c corresponds in the QFT formalism to the NGB condensate numerical value identifying univocally each phase coherence domain, i.e., a word, a single canonical commutation relation of the QV "language". In this way, the QV, because endowed with the SSB state-transition - effectively a phase-transition structure γ , selecting every time one canonical commutation relation (head) as to the rest of the others (tail)- corresponds to the final coalgebra of the category of q -deformed Hopf coalgebras for the functor T .

The dynamics of the Quantum-Infinite-State-Black-Box-Machine is thermo-dynamic, i.e., its state (phase) transition is "moved" by the Second Principle of energy equipartition, in a way that must satisfy, on one hand, the energy arrow contravariance related to the First Principle, and, on the other hand, without consuming all the QV energy reservoir, as requested by the Third Principle. A condition elegantly satisfied in the QFT formalism by the fractal structure of the systems phase space (Vitiello, 2012) and, therefore, by the chaotic character of the macroscopic trajectories (phase transitions) defined on it. All this implies the necessity of doubling the behavior map and all the related objects and structures, so as to finally satisfy the dual equivalence characterizing the QFT categorical formalism. In logical terms, this means that the semantics (coalgebra) induces its own syntax (algebra), so as to justify, the computer scientist's interest toward a coalgebraic approach to quantum computation for managing streams (Bruno *et al.*, 2011; Abramsky, 2013; Roumen, 2014). On the other hand, it demonstrates that the QFT interpretation of this approach is the more promising one. In fact, it determines an observer-independent dynamic choice of the structure (syntax) of the composed (system-thermal bath) Hilbert space of a QFT system. As far as it is based on the dual equivalence (semantics) of one pair of the q -deformed Hopf coalgebras and algebras categories for each fixed q , representing a QFT open system.

Finally, because via the so-called Gelfand-Naimark-Segal constructions in the C^* -algebra formalism topologies of the Stone spaces - with which C^* -algebras share the same topologies - are associated with Hilbert spaces, it is possible to demonstrate that the topological spaces of QFT satisfy the condition of a Stone Space, so as to include also thermal QFT

coalgebras into the general duality between between the category of Stone Coalgebras for the Vietoris functor and the category of Modal Boolean Algebras (Kupke *et al.*, 2004; Abramsky, 2005). In this way, it is possible to give a final foundation to the notion of QFT computing.

All this is related with the second, final observation, justifying the interpretation of the maximal entropy in a QFT doubled system as a semantic measure of information, i.e., as a statistical measure of maximal local truth in the CT coalgebraic logic. In the QFT mathematical formalism this maximum of the entropy measure is formally obtained when the far from equilibrium energy balance between a system (algebra) and its thermal bath (coalgebra) is dynamically, i.e., automatically satisfied. This means that we are allowed to interpret the QFT qubit of such a natural computation as an evaluation function in the semantic sense. Indeed, in the QFT composed Hilbert space including also the thermal bath degrees of freedom, \tilde{A} , i.e., $H_{A, \tilde{A}} = H_A \otimes H_{\tilde{A}}$, for calculating the static and dynamic entropy associated with the time evolution generated by the free energy, i.e., $|\phi(t)\rangle, |\psi(t)\rangle$, of the qubit mixed states $|\phi\rangle, |\psi\rangle$, one needs to double the states by introducing the tilde states $|\tilde{0}\rangle$ and $|\tilde{1}\rangle$, relative to the thermal bath, i.e., $|0\rangle \rightarrow |0\rangle \otimes |\tilde{0}\rangle$, and $|1\rangle \rightarrow |1\rangle \otimes |\tilde{1}\rangle$. This means that such a QFT version of a qubit implements effectively the CNOT (controlled NOT) logical gate, which flips the state of the qubit, conditional on a dynamic control of an effective input matching [18].

CONCLUSION

In this Short Communication we discussed the fundamental role played by quantum computing within the emerging morphological paradigm in the unconventional natural computing. We introduced the main reasons why thermal QFT computing, satisfying a principle of Coalgebraic Universality in computation, overcomes the trivializing argument against the so-called pancomputationalism exposed in (Müller and Hoffmann, 2017).

In conclusion, we stress the point that Coalgebraic Universality gives a higher degree of generality to QM computations, since any QTM can be interpreted in terms of STS, but not vice versa. On the contrary, the Coalgebraic Universality in computation is essential for QFT computing because Turing Universality does not hold in it. Indeed, as we have seen, the maximal entropy function acquires the role of a first-order local truth evaluation function as a dynamic measure of the reached optimal matching system-thermal bath. This justifies the assert according to which such computing devices are outside the Turing Universality in quantum computation (Basti *et al.*, 2017). This paradigm, indeed, because requiring second-order truth evaluation functions, as exemplified by Gödel's second incompleteness theorem, and then by Turing Theorem about the non-terminating character of the Universal Turing Machine (UTM) computations, requires also that the truth evaluation is performed by the human programmer. In this sense, following this paradigm, TMs properly process syntactic and not semantic information, just as QM computing devices do, so as to justify their proper definition as QTM

² In parenthesis, in the Infinite State Black-Box Machine the general coalgebraic principle of the observational (or behavioral) equivalence among states holds (Rutten, 2000; Sangiorgi, 2012). All the scholars agree that this has an immediate meaning for logic and mathematics of quantum systems, as a further justification for a coalgebraic interpretation of quantum systems logic and mathematics.

(Deutsch, 1985). These limitations justify the growing interest of scholars toward the coalgebraic paradigm in computation for quantum systems, in general (Bruno *et al.*, 2011; Abramsky, 2013; Roumen, 2014), and especially toward QFT systems (Basti *et al.*, 2017).

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References

- Abramsky, S. 2005. A Cook's Tour of the Finitary Non-Well-Founded Sets (original lecture: 1988). In *We Will Show Them! Essays in Honor of Dov Gabbay*, 1st ed.; Artemov, S. *et al.*, Eds.; Imperial College P: London, United Kingdom; Volume 1, pp. 1-18, ISBN: 9781904987116.
- Abramsky, S. 2013. Coalgebras, Chu Spaces, and Representations of Physical Systems. *J. Phil. Logic*, 42, pp. 551-574, DOI: 10.1007/s10992-013-9276-4.
- Abramsky, S. and Tzevelekos, N. 2011. Introduction to Categories and Categorical Logic. In *New Structures for Physics*, 1st ed.; Coecke, B., Ed.; Springer: Berlin-New York, German-United States, pp. 3-94, ISBN: 978-3642128202.
- Awodey, S. 2006. *Category Theory*, 1st ed.; Oxford UP: New York, United States, ISBN: 978-0-19-856861-2.
- Basti, G. 2017. The Quantum Field Theory (QFT) Dual Paradigm in Fundamental Physics and the Semantic Information Content and Measure in Cognitive Sciences. In *Representation of Reality: Humans, Animals and Machine*, 1st ed.; Dodig-Crnkovic, G.; Giovagnoli, R., Eds.; Springer: Cham, Switzerland, pp. 177-211, ISBN: 978-3-319-43782-8.
- Basti, G. *et al.* 2017. Quantum Field Theory and Coalgebraic Logic in Theoretical Computer Science. *Prog. in Biophys. & Mol. Bio.*, 130, pp. 39-52, DOI: 10.1016/j.pbiomolbio.2017.04.006.
- Blasone, M. *et al.* 2011. *Quantum Field Theory and Its Macroscopic Manifestations. Boson Condensation, Ordered Patterns and Topological Defects*, 1st ed.; Imperial College Press: London, United Kingdom, 2011, ISBN: 1-84816-280-4.
- Bogolyubov, N.N. jr. 2011. Perturbation Theory. *Encyclopedia of Mathematics*. Available online: http://www.encyclopediaofmath.org/index.php?title=Perturbation_theory&oldid=11676 (Accessed 3 June 2018).
- Bruno, A. *et al.* 2011. Geometric Phase and Gauge Theory Structure in Quantum Computing. *J. Phys. Conf.*, 306, 012065, DOI: 10.1088/1742-6596/306/1/012065.
- Davies, P. 2010. Universe from Bit. In *Information and the Nature of Reality*, 1st ed.; Davies, P.; Gregersen, N.H., Eds.; Cambridge UP: New York, United States, pp. 65-81, ISBN: 1139490966.
- Deutsch, D. 1985. Quantum Theory, the Church-Turing Principle and the Universal Quantum Computer. *Proc. R. Soc. Lond.*, 400, pp. 97-117, DOI: 10.1098/rspa.1985.0070.
- Dodig-Crnkovic, G. 2017. Nature as a Network of Morphological Infocomputational Processes for Cognitive Agents. *Eur. Phys. J. Spec. Top.*, 226, pp. 181-195, DOI: 10.1140/epjst/e2016-60362-9.
- Earman, J. and Fraser, D. 2006. Haag's Theorem and Its Implications for the Foundations of Quantum Field Theory. *Erkenntnis*, 64, pp. 305-344, DOI: 10.1007/s10670-005-5814-y.
- Feynmann, R. 1985. *The Character of Physical Law*, 1st ed.; MIT Press: Cambridge, United States, ISBN: 0 262 06016 7.
- Goldstone, J. *et al.* 1962. Broken Symmetries. *Phys. Rev.*, 127, pp. 965-970, DOI: 10.1103/PhysRev.127.965.
- Goyal, P. 2012. Information Physics-Towards a New Conception of Physical Reality. *Information*, 3, pp. 567-594, DOI: 10.3390/info3040567.
- Kupke, C. *et al.* 2004. Stone Coalgebras. *Theor. Comp. Sci.*, 327, pp. 109-134. DOI: 10.1016/j.tcs.2004.07.023.
- Müller, V.C. and Hoffmann, M. 2017. What Is Morphological Computation? On How the Body Contributes to Cognition and Control. *ALIFE*, 23, pp. 1-24, DOI: 10.1162/ARTL_a_00219.
- Roumen, F. 2014. Coalgebraic Quantum Computation. *Elec. Proceed. Theor. Comp. Sci.*, 158, pp. 29-38, DOI: 10.4204/EPTCS.158.3.
- Rutten, J.J.M. 2000. Universal Coalgebra: A Theory of Systems. *Theor. Comput. Sci.*, 249(1), pp. 3-80, DOI: 10.1016/S0304-3975(00)00056-6.
- Sangiorgi, D. 2012. Origins of Bisimulation and Coinduction. In *Advanced Topics in Bisimulation and Coinduction*, 1st ed.; Sangiorgi, D., Rutten, J.J.M., Eds.; Cambridge UP: Cambridge, United Kingdom, pp. 1-37. ISBN: 9781107004979.
- Venema, Y. 2007. Algebras and Coalgebras. In *Handbook of Modal Logic*, 1st ed.; Blackburn, P.; van Benthem, J.F.A.K.; Wolter, F., Eds.; Elsevier: Amsterdam, Netherlands, pp. 331-426, ISBN: 978-0444516909.
- Vitiello, G. 2012. Fractals, Coherent States and Self-Similarity Induced Noncommutative Geometry. *Phys. Lett. A*, 2527. DOI: 10.1016/j.physleta.2012.06.035.

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