International Journal of Current Advanced Research
ISSN: O: 2319-6475, ISSN: P: 2319-6505, Impact Factor: 6.614
Available Online at www.journalijcar.org
Volume 7; Issue 5(K); May 2018; Page No. 12954-12956
DOI: http://dx.doi.org/10.24327/ijcar.2018.12956.2296

Research Article

ON COMPLEMENT OF INTERVAL VALUED FUZZY GRAPHS

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A R T I C L E I N F O

Article History:

Received 10th February, 2018
Received in revised form 6th March, 2018 Accepted 24th April, 2018
Published online 28th May, 2018

Key words:

Interval Valued Fuzzy Graph (IVFG), complement of IVFG, classic IVFG, non-classic IVFG, self-complementary IVFG, perfect edge, imperfect edge.

A B S T R A C T

Complement of an IntervalValued Fuzzy Graph (IVFG) was defined byTalebi and Rashmanlou [7]. We observed that their definition fails in some cases and we reformulated the notion of complement in such a way that it applies to all IVFG's. We also introduced the concepts of Classic and Non-classic IVFG's, and Perfect and Imperfect edges of an IVFG and state some theorems regarding these concepts.

INTRODUCTION

Graph theory has numerous applications to problems in computer science, electrical engineering, operations research, economics, networking routing, transportation, etc. Formally, a*graph* (or a *crisp graph*) is defined as a pair,$G^* = (V, E)$ consisting of a non-empty finite set V of elements called *vertices* and a finite set E of pairs of vertices called *edges*. In 1965, L. A. Zadeh[8] introduced the notion of fuzzy set :"A *fuzzy setA* on a set X is characterized by a mapping $\mathfrak{M} : X \rightarrow [0, 1]$, which is called the membership function and fuzzy set A on X is denoted by $A = \{(x, \mathfrak{M}(x)) : x \in X\}$". In 1975, Rosenfeld [4] introduced the notion of fuzzy graph as : " A *fuzzy graph*$G_F = (V, \sigma, \mu)$ consists of a non-empty set V together with a pair of functions $\sigma : V \rightarrow [0, 1]$ and $\mu : V \times V \rightarrow [0, 1]$ such that for all $A, B \in V$, $\mu(A, B) \leq min\{\sigma(A), \sigma(B)\}$. Here $\sigma(A)$ and $\mu(A, B)$ represent the membership values of the vertex A and of the edge (A, B) in$G_F$respectively". He also proposed definitions of paths, cycles, connectedness, etc. Zadeh[9] also introduced the notion of *interval valued fuzzy sets*, as an extension of *fuzzy sets*, in which the values of the membership degree are intervals of numbers instead of fixed numbers. In 2009, Hongmei and Lianhua [2] defined *interval valued fuzzy graphs* and in 2011, Akram and Dudek [1] defined some operations on them. Talebi and Rashmanlou[7] studied properties of isomorphism and complement on intervalvalued fuzzy graphs. [5]and[6] are some recent works in this area.

We observe that the definition of complement of an IVFG given by Talebi and Rashmanlou fails in some cases. We modified the definition in such a way that it applies to all IVFG's. The new definition gives the same complement for IVFG's where the former definition applies. These observations motivated the notions of classic and non-classic IVFG's and perfect and imperfect edges.

Some Basic Concepts

In the following discussion, for any given set$X$, $\mathcal{P}(X)$denotes the *power set* of X. That is, the collection of all subsets of$X$.

Definition[9]. An ***interval valued fuzzy set(IVFS)****A* on X is characterized by an interval-valued function$\imath : X \rightarrow \mathcal{P}[0,1]$ such that $\imath(x) = [a_x^-, a_x^+]$ where $0 \leq a_x^- \leq a_x^+ \leq 1$. For each $x \in X$, $\imath(x)$ is called the *interval number*of x. An IVFS A on X is denoted by $A = \{(x, \imath(x)) : x \in X\}$.

Definition[2]. An ***interval valued fuzzy graph (IVFG)***$G = (V, \sigma, \mu)$ consists of a non-empty set V together with a pair of interval valued functions $\sigma : V \rightarrow [0, 1]$ and $\mu : V \times V \rightarrow [0, 1]$ where

$\sigma(A) = [\sigma_A^-, \sigma_A^+]$, $0 \leq \sigma_A^- \leq \sigma_A^+ \leq 1$
and$\mu(AB) = [\mu_{AB}^-, \mu_{AB}^+]$, $0 \leq \mu_{AB}^- \leq \mu_{AB}^+ \leq 1$
represent the interval number of the vertex A and of the edge AB in G respectively satisfying
$\mu_{AB}^- \leq \min\{\sigma_A^-, \sigma_B^-\}and\mu_{AB}^+ \leq \min\{\sigma_A^+, \sigma_B^+\}$
for all $A, B \in V$.

Definition[3]. Let $G = (V, \sigma, \mu)$ and $G' = (V', \sigma', \mu')$ be two IVFG's. Then G and G' are said to be ***isomorphic***, written as $G \cong G'$, if there exist a bijection $h : V \rightarrow V'$ such that

1. $\sigma_A^- = \sigma'^-_{h(A)}$, $\sigma_A^+ = \sigma'^+_{h(A)}$ for every vertex $A \in V$.

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2. $\mu^-_{AB} = \mu'^-_{h(A)h(B)}$, $\mu^+_{AB} = \mu'^+_{h(A)h(B)}$ for every edge AB in G.

Definition[7]. The ***complement of IVFG*** $G = (V,\sigma,\mu)$ is an IVFG $\bar{G} = (V,\sigma,\bar{\mu})$ where $\bar{\mu}(AB) = [\bar{\mu}^-_{AB}, \bar{\mu}^+_{AB}]$ where
$\bar{\mu}^-_{AB} = \min\{\sigma^-_A, \sigma^-_B\} - \mu^-_{AB}$, $\bar{\mu}^+_{AB} = \min\{\sigma^+_A, \sigma^+_B\} - \mu^+_{AB}$
for every $A, B \in V$.

Example

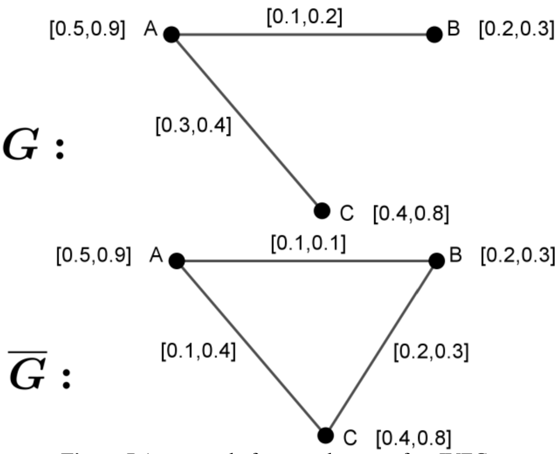


Figure I An example for complement of an IVFG.

In the following example, we show that the construction of complements, in the above sense, fails for some IVFG's.

Example

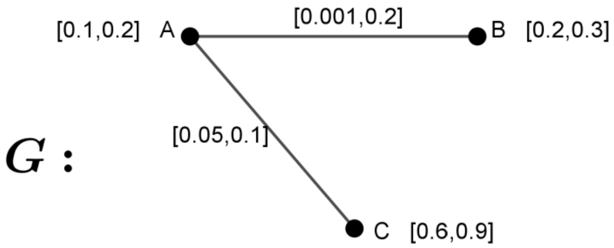


Figure ii An example of an IVFG where the construction of complement fails.

Here, $[\bar{\mu}^-_{AB}, \bar{\mu}^+_{AB}]$ = [0.099,0], which is not an interval. So we cannot construct an IVFG $\bar{G}$
The definition of complement stated below applies to all IVFG's.

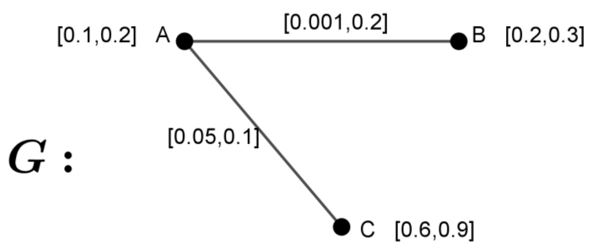
Definition. The ***complement of IVFG*** $G = (V,\sigma,\mu)$ is an IVFG $\bar{G} = (V,\sigma,\bar{\mu})$ where $\bar{\mu}(AB) = [\bar{\mu}^-_{AB}, \bar{\mu}^+_{AB}]$

$$= \begin{cases} [\min\{\sigma^-_A, \sigma^-_B\} - \mu^-_{AB}, \min\{\sigma^+_A, \sigma^+_B\} - \mu^+_{AB}], \text{if } \min\{\sigma^-_A, \sigma^-_B\} - \mu^-_{AB} \le \min\{\sigma^+_A, \sigma^+_B\} - \mu^+_{AB} \\ [\min\{\sigma^+_A, \sigma^+_B\} - \mu^+_{AB}, \min\{\sigma^+_A, \sigma^+_B\} - \mu^+_{AB}], \text{if } \min\{\sigma^-_A, \sigma^-_B\} - \mu^-_{AB} > \min\{\sigma^+_A, \sigma^+_B\} - \mu^+_{AB} \end{cases}$$

for all $A, B \in V$.

Now, using this *definition.*, we can draw complement, of above graph, which is given as the next example.

Example



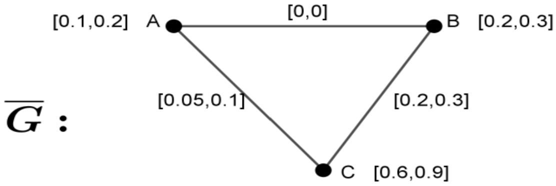


Figure iii An example of complement of IVFG.

Classic and Non-Classic IVFG's

Definition. An IVFG $G = (V,\sigma,\mu)$ is called a classic IVFG if all its edges satisfy the condition

$$\min\{\sigma^-_A, \sigma^-_B\} - \mu^-_{AB} \le \min\{\sigma^+_A, \sigma^+_B\} - \mu^+_{AB}.$$

Otherwise we call it as a ***non-classic IVFG.***

Definition. Let $G = (V,\sigma,\mu)$ be an IVFG. Then edges AB in G satisfying
$\min\{\sigma^-_A, \sigma^-_B\} - \mu^-_{AB} \le \min\{\sigma^+_A, \sigma^+_B\} - \mu^+_{AB}$
are called ***perfect edges*** and all other edges AB for which

$$\min\{\sigma^-_A, \sigma^-_B\} - \mu^-_{AB} > \min\{\sigma^+_A, \sigma^+_B\} - \mu^+_{AB}$$

are called ***imperfect edges***.

Remark

1. All edges of an IVFG are perfect iffthe IVFG is classic.
2. If edge AB is an imperfect edge, then $\bar{\mu}(AB)$ is always a real number in [0,1).

We now state some theorems on classic IVFG's

Theorem: For any IVFG $G = (V, \sigma, \mu)$, $\bar{G}$ is always classic.

Theorem. Let G and H be any two IVFGs such that $G \cong H$. Then any isomorphism $h: G \to H$ maps perfect edges to perfect edges and imperfect edges to imperfect edges.

Theorem.Let $G = (V,\sigma,\mu)$ be any IVFG. Then G is classic iff $\bar{\bar{G}} \cong G$.

Proof.*Suppose G is classic*. We shall prove that the identity

Definition[7]. An IVFG G is said to be ***self - complementary***if $G \cong \bar{G}$.

Example

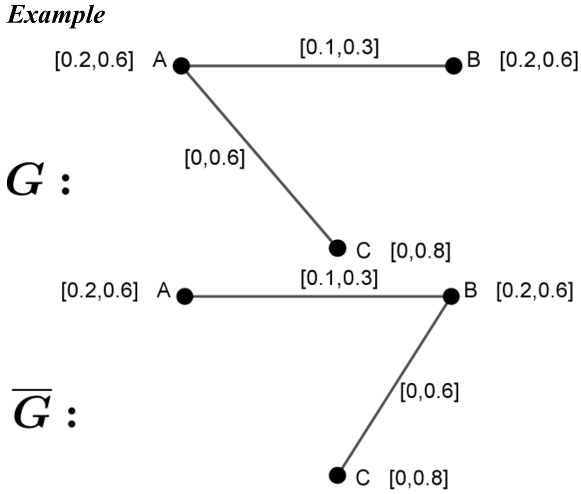


Figure iv An example of a self-complementary IVFG.

Theorem. $G$is a self – complementary IVFG ⇒ G is a classic IVFG.

Proof. $G$is a self – complementary IVFG⇒ $G \cong \bar{G}$.
We already stated that $\bar{G}$ is always classic
⇒all edges of $\bar{G}$ are perfect. (1)

We also know that any isomorphism maps perfect edges to perfect edges. (2)
Since $G \cong \bar{G}$, from (1) and (2), it is clear that, all edges of G are perfect.
$\Rightarrow G$ is classic ■

Remark. The IVFG in *figure(i)* is classic. But it is not self-complementary. Hence converse of *theorem* above is not true.

CONCLUSION

We observed that the definition of complement of an IVFG given by Talebi and Rashmanlou fails in some cases. We modified the definition in such a way that it applies to all IVFG's. The new definition gives the same complement for IVFG's where the former definition applies. These observations motivated the notions of classic and non-classic IVFG's and perfect and imperfect edges. We also stated some theorems regarding this and prove that an IVFG is self-complementary implies G is classic, but the converse is not true.

Acknowledgement

Part of the paper was presented and published in the International Conference on Science and Technology in Cochin University of Science and Technology (CUSAT) in April, 2018

Reference

1. Akram, M. and Dudek, W.A., Interval valued fuzzy graphs. *Computers Math. Appl.,* Vol.61(2), 2011, 289-299.
2. JuHongmei and Wang Lianhua, Interval-valued Fuzzy Subsemigroups and subgroups Associated by Interval-valued Fuzzy Graphs, Global congress on Intelligent Systems, 2009, 484-487.
3. Rashmanlou and Madhumangal Pal, *Some Properties of Highly Irregular Interval Valued Fuzzy Graphs*, World *Applied Sciences Journal*, 2013, 1756-1773.
4. Rosenfeld, A., Fuzzy graphs, Fuzzy sets and their applications to cognitive and decision processes, 1975, 77-95.
5. Souriar Sebastian and Ann Mary Philip, On total regularity of the join of two interval valued fuzzy graphs, *International Journal of Scientific and Research Publications*, Vol.6(12), 2016, 45-55.
6. Souriar Sebastian and Ann Mary Philip, Regular and edge regular interval valued fuzzy graphs, *Journal of Comp. and Math. Sci.,* Vol.8(7), 2017, 309-322.
7. Talebi, A. A., and H. Rashmanlou., Isomorphism on interval-valued fuzzy graphs, Annals of Fuzzy Mathematics and Informatics, Vol.6(1), 2013, 47-58.
8. Zadeh L. A., Fuzzy sets, *Information and control*, Vol.8, 1965, 338-353.
9. Zadeh, L.A., The concept of a linguistic and application to approximate reasoning. *Information Sci.,* Vol.8(3), 1975, 199-249.

How to cite this article:

Deepthi Mary Tresa S *et al* (2018) 'On Complement of Interval Valued Fuzzy Graphs', *International Journal of Current Advanced Research,* 07(5), pp. 12954-12956. DOI: http://dx.doi.org/10.24327/ijcar.2018.12956.2296

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