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Research Article

ON NON – HOMOGENEOUS SEXTIC EQUATION WITH FIVE UNKNOWNS

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A B S T R A C T

The sequences of integral solutions to the sextic Diophantine Equation 2(x+y)(x^3-y^3) = 31(z^2-w^2)P^4 with five unknowns are obtained.

Index Terms : Sextic equation having five unknowns with integral solutions.

INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems [1, 2, 3, 15]. Particularly in [4, 5] sextic equations with three unknowns are studied for their integral solutions[6 – 11] analyze sextic equation with five unknowns for their non-zero integer solutions and [12 – 14] deals with sextic equation with five unknowns. This communication analyze a sextic equation with five unknowns given by $2(x+y)(x^3-y^3) = 31(z^2-w^2)P^4$ infinitely many quintuples (x, y, z, w, T) Satisfying the above equation is obtained. Various interesting properties among the values of x, y, z, w and T are presented.

Notations

* $t_{m,n}$ – Polygonal number of rank n with size m.
* CP_n^{14} – Centered tetra decagonal pyramidal number of rank n.
* PP_n – Pentagonal Pyramidal number of rank n.
* SO_n – Stella Octangular number of rank n.
* CP_n^6 - Centered Hexagonal Pyramidal number of rank n.
* GnO_n – Gnomic number of rank n.
* P_A^5 – Pentagonal number of rank 5.
* OH_n – Octahedral number of rank n.
* Pr_n – Pronic number of rank n.
* FN_n^4 – Four Dimensional figurate number.

METHOD OF ANALYSIS

The Diophantine equation to be solved for its non-zero distinct integral solutions is given by,

$$2(x+y)(x^3-y^3) = 31(z^2-w^2)P^4 \quad (1)$$

Introducing the linear transformations,

$$x = u+v;\ y = u-v;\ z = u+2v;\ w = u-2v;\ u \neq v \neq 0 \quad (2)$$

In (1) leads to

$$v^2 + 3u^2 = 31P^4 \quad (3)$$

We obtain different choices of integral solutions to (1) through solving (3) which are illustrated as follows

Set : 1

Assume $P = a^2 + 3b^2$ (4)

where a and b are non-zero distinct integers.

Write 31 as, $31 = (2 + i3\sqrt{3})\,(2 - i3\sqrt{3})$ (5)

Substituting (4) and (5) in (3) and applying the method of factorization and equating positive factors, we get

$$(v + i\sqrt{3}\,u) = (2 + i3\sqrt{3})\,(a + i\sqrt{3}b)^4$$

Equating real and imaginary parts, we have

$$u = u(a,b) = 3a^4 + 27b^4 + 8a^3b - 24ab^3 - 54a^2b^2$$
$$v = v(a,b) = 2a^4 + 18b^4 - 36a^3b + 108ab^3 - 36a^2b^2 \quad (6)$$

Employing (2), the values of x, y, z, w and T are given by

$$x = x(a,b) = 5a^4 + 45b^4 - 90a^2b^2 - 28a^3b + 84ab^3$$
$$y = y(a,b) = a^4 + 9b^4 - 18a^2b^2 + 44a^3b - 132ab^3$$
$$z = z(a,b) = 7a^4 + 63b^4 - 126a^2b^2 - 64a^3b + 192ab^3$$
$$w = w(a,b) = -a^4 - 9b^4 + 18a^2b^2 + 80a^3b - 240ab^3$$
$$P = P(a,b) = a^2 + 3b^2$$

Properties

1. $x(a,1) - 60FN_a^4 + 56PP_a + 57Obl_a \equiv 45 \ (mod\ 141)$

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2. $7y(a,1) - z(a,1) - 186So_a \equiv 0 \ (mod\ 930)$
3. $y(a,1) - 12FN_a^4 - 88PP_a + 61Pr_a \equiv 9 \ (mod\ 71)$
4. $x(a,1) + y(a,1) - 6Biq_a - 16CP_a^6 + 4T_{4,4a} + 44Pr_a \equiv 54 \ (mod\ 4)$
5. $x(a,1) - 5y(a,1) - 248CP_a^6 \equiv 0 \ (mod\ 744)$
6. $x(a,a) - 4T_{4,2a^2} = 0$
7. $z(a,1) + 7w(a,1) - 248So_a \equiv 0 \ (mod\ 1240)$
8. $z(a,1) + 110t_{3,a} + 128PP_a - 84FN_a^4 \equiv 63 \ (mod\ 247)$
9. $w(a,1) - P_A^5 - 27CP_a^{14} + 12FN_a^4 \equiv 9 \ (mod\ 204)$

Set : 2

One may write (3) as

$$v^2 + 3u^2 = 31P^4 * 1 \quad (7)$$

Also write 1 as

$$1 = \frac{(1+i\sqrt{3})(1-i\sqrt{3})}{4} \quad (8)$$

Using (4), (5) and (8) in (7) and applying the method of factorization and equating positive factors we get

$$(v + i\sqrt{3}\,u) = \frac{(1+i\sqrt{3})(a+i\sqrt{3}b)^4(2+i3\sqrt{3})}{2} \quad (9)$$

Equating real and imaginary parts, we have

$$u = u(a,b) = \frac{1}{2}\ [5a^4 + 45b^4 - 90a^2b^2 - 28a^3b + 84ab^3] \quad (10)$$

$$v = v(a,b) = \frac{1}{2}\ [-7a^4 - 63b^4 + 126a^2b^2 - 60a^3b + 180ab^3] \quad (11)$$

As our aim is to find integral solutions choose a and b suitably so that the solutions are integers taking $a = 2A$, $b = 2B$

$$x = x(A,B) = -16A^4 - 144B^4 + 288A^2B^2 - 704A^3B + 2112AB^3$$

$$y = y(A,B) = 96A^4 + 864B^4 - 1728A^2B^2 + 256A^3B - 768AB^3$$

$$z = z(A,B) = -72A^4 - 648B^4 + 1296A^2B^2 - 1184A^3B + 3552AB^3$$

$$w = w(A,B) = 152A^4 + 1368B^4 - 2736A^2B^2 + 736A^3B - 2208AB^3$$

$$T = T(A,B) = 4A^2 + 12B^2$$

Properties

1. $x(A,1) + y(A,1) - 960FN_A^4 - 448CP_A^6 + 1360\ Obl_a \equiv 720 \ (mod\ 2704)$
2. $x(A,1) + 16Biq_A - 704CP_A^6 - 288\ Obl_a \equiv 144 \ (mod\ 1824)$
3. $y(A,1) + 6x(A,1) - 3968CP_A^6 \equiv 0 \ (mod\ 11904)$
4. $w(A,1) - 182FN_A^4 + T_{5170,A} - 318CP_A^{14} - 6CP_A^6 \equiv 1368 \ (mod\ 4367)$
5. $x(A,A) - 92Biq_A - 4T_{4,19A^2} = 0$
6. $z(A,1) + 864FN_A^4 + 592SO_A - 1224Pr_A \equiv 648 \ (mod\ 1736)$
7. $y(A,1) + z(A,1) - 288FN_A^4 - 520CP_A^6 + 816P_A^5 + 21GnO_A \equiv 0 \ (mod\ 3216)$
8. $z(A,1) + x(A,1) + 88Biq_A - 1888CP_A^6 - 3168T_{3,A} - 792GnO_A \equiv 0 \ (mod\ 2496)$
9. $w(A,1) - z(A,1) - 2688FN_A^4 + 7616P_A^5 - 8592\ OH_A \equiv 2016 \ (mod\ 8624)$

Equation (8) can also be written as

$$1 = \frac{(1+i4\sqrt{3})(1-i4\sqrt{3})}{49} \quad (12)$$

Proceeding as above, the different sets of integer solutions of (1) are illustrated below,

$$(v + i\sqrt{3}\,u) = \frac{(1+i4\sqrt{3})(a+i\sqrt{3}b)^4(2+i3\sqrt{3})}{7} \quad (13)$$

Equating real and imaginary parts, we have

$$v = v(a,b) = \frac{1}{7}\ [-34a^4 - 306b^4 + 612a^2b^2 - 132a^3b + 396ab^3] \quad u = u(a,b) = \frac{1}{7}\ [11a^4 + 96b^4 - 198a^2b^2 - 136a^3b + 408ab^3]$$

Choosing $a = 7A$, $b = 7B$, in the above equation, we obtain

$$u = u(A,B) = 3773A^4 + 33957B^4 - 67914A^2B^2 - 46648A^3B + 139944AB^3$$

$$v = v(A,B) = -11662A^4 - 104958B^4 + 209916A^2B^2 - 45276A^3B + 135828AB^3$$

Substituting the values of u, v in (2), we get the non-zero distinct integer solutions to (1) to be,

$$x = x(A,B) = -7889A^4 - 71001B^4 + 142002A^2B^2 - 91924A^3B + 275772AB^3$$

$$y = y(A,B) = 15435A^4 + 138915B^4 - 277830A^2B^2 - 1372A^3B + 4116AB^3$$

$$z = z(A,B) = -19551A^4 - 175959B^4 + 351918A^2B^2 - 137200A^3B + 411600AB^3$$

$$w = w(A,B) = 27097A^4 + 243873B^4 + 43904A^3B - 131712AB^3$$

$$P = P(A,B) = 49A^2 + 147B^2$$

Properties

1. $x(A,1) + 94668FN_A^4 - 91924CP_A^6 - 134113\ Obl_A \equiv 71001 \ (mod\ 141659)$
2. $y(A,A) - 4T_{5,150A^2} - 3073Biq_A = 0$
3. $z(B,1) + 19551Biq_B - 137200CP_B^6 - 351918Pr_B \equiv 175959 \ (mod\ 59682)$
4. $z(A,1) - w(A,1) + 46648Biq_A + 181004CP_A^6 + 1679328T_{3,A} \equiv 419832 \ (mod\ 296352)$
5. $y(A,1) - 15435Biq_A - 1372CP_A^6 + T_{555662,A} \equiv 138915 \ (mod\ 273713)$
6. $x(A,1) + y(A,1) - 90552FN_A^4 + 186592P_A^5 + 31986\ Obl_A \equiv 67914 \ (mod\ 247902)$
7. $z(A,1) + y(A,1) + 4116Biq_A - 138572CP_A^6 + 148176T_{3,A} + 37044GnO_A \equiv 0 \ (mod\ 267540)$
8. $w(A,1) + x(A,1) - 230496FN_A^4 - 48020CP_A^6 + 653072T_{3,A} \equiv 172872 \ (mod\ 470596)$

Remark

Instead of (2) one may also introduce another set of transformation as

$$x = u + v;\ y = u - v;\ z = 2uv + 1;\ w = 2uv - 1;\ u \neq v \neq 0 \quad (13)$$

Set : 3

By substituting the equation (4) and (6) in (13), we obtain the integral solutions to (1) are given by

$u = u(a,b) = 3a^4 + 27b^4 + 8a^3 - 24ab^3 - 54a^2b^2$
$v = v(a,b) = 2a^4 + 18b^4 - 36a^3b + 108ab^3 - 36a^2b^2$

From (13) the integer solutions of (1) are

$x = x(a,b) = 5a^4 + 45b^4 - 80a^2b^2 - 28a^3b + 84ab^3$
$y = y(a,b) = a^4 + 9b^4 - 16a^2b^2 + 44a^3b - 132ab^3$
$z = z(a,b) = 12(a^8 + 81b^8 + 630a^4b^4 - 84a^6b^2 - 756a^2b^6) + 184(27ab^7 - a^7b - 63a^3b^5 + 21a^5b^3) + 1$
$w = w(a,b) = 12(a^8 + 81b^8 + 630a^4b^4 - 84a^6b^2 - 756a^2b^6) + 184(27ab^7 - a^7b - 63a^3b^5 + 21a^5b^3) - 1$

Set : 4

And also by substituting the equation (10) and (11) in (13), we obtain the integral solutions to (1) are given by

$u = u(A,B) = -40A^4 + 360B^4 - 720A^2B^2 - 224A^3B + 672AB^3$
$v = v(A,B) = -56A^4 - 504B^4 + 1008A^2B^2 - 480A^3B + 1440AB^3$

From (13), the integer solutions of (1) are

$x = x(A,B) = -16A^4 - 144B^4 + 288A^2B^2 - 704A^3B + 2112AB^3$
$y = y(A,B) = 96A^4 + 864B^4 - 1728A^2B^2 + 256A^3B - 768AB^3$
$z = z(A,B) = 4480(-A^8 - 81B^8 - 630A^4B^4 + 84A^6B^2 + 756A^2B^6) - 13312(A^7B - 27AB^7 - 21A^5B^3 + 63A^3B^5) + 1$
$w = w(A,B) = 4480(-A^8 - 81B^8 - 630A^4B^4 + 84A^6B^2 + 756A^2B^6) - 13312(A^7B - 27AB^7 - 21A^5B^3 + 63A^3B^5) + 1$

CONCLUTION

In this paper, we have presented sets of infinitely many non-zero distinct integer solutions to the sextic equation with five unknowns given by $2(x+y)(x^3-y^3) = 31(z^2-w^2)P^4$. As Diophantine equations are rich in variety due to their definition. One may attempt to find integer solutions to higher degree Diophantine equation with multiple variables.

References

1. Dickson, L.E., History of theory of numbers, vol.2, *Chelsea publishing company*, *New York*(1952).
2. Carmichael, R.D., The theory of numbers and Diophantine analysis, *Dover publications, New York*(1959).
3. Mordell, L.J., Diophantine equations, *Academic press, London* (1969).
4. Telang, S.G., Number Theory, *Tata MC Graw Hill Publishing Company, New Delhi* (1996).
5. Gopalan, M.A., ManjuSomanath and Vanitha, N., "Parametric Solutions of $x^2 - y^6 = z^2$ ", *Acta Ciencia Indica XXXIII,* vol.3, 1083-1085, 2007.
6. [6] Gopalan, M.A., Sangeetha, G., "On the Sextic Equations with three unknowns $x^2 - xy + y^2 = (k^2+3)^n z^6$", Impact *J.Sci.tech*, Vol.4, No: 4, 89-93, 2010.
7. Gopalan, M.A., Vijayashankar, A., "Integral solutions of the Sextic Equation $x^4 + y^4 + z^4 = 2w^6$ ", *Indian journal of Mathematics and Mathematical Sciences,* Vol.6, No:2, 241-245, 2010.
8. Gopalan, M.A., Vidhyalakshmi, S., Vijayashankar, A., "Integral Solutions of Non-Homogeneous Sextic equation $xy + z^2 = w^6$", *Impact J.Sci.tech,* Vol.6, No: 1, 47-52, 2012.
9. [9] Gopalan, M.A., Vidhyalakshmi, S., Lakshmi, L., "On the Non-Homogeneous Sextic equation $x^4 + 2(x^2+w)x^2y^2 + y^4 = z^4$" , *IJAMA,* 4(2), 171-173, Dec (2012).
10. Gopalan, M.A., Vidhyalakshmi, S., Kavitha, A., "Observations on the Homogeneous Sextic Equation with four unknowns $x^3 + y^3 = 2(k^2+3)z^5w$ ", *International Journal of Innovative Research in Science, Engineering and Technology,* Vol.2, Issue: 5, 1301-1307, 2013.
11. Gopalan, M.A., Sumathi, G., Vidhyalakshmi, S., "Integral Solutions of Non- homogeneous Sextic Equation with four unknowns $x^4 + y^4 + 16z^4 = 32w^6$", *Antarctica J.Math,* 10(6), 623-629, 2013.
12. Gopalan, M.A., Sumathi, G., Vidhyalakshmi, S., "Gaussian Integer Solutions of Sextic Equations with four unknowns $x^6 - y^6 = 4z(x^4 + y^4 + w^4)$", *Archimedes, J.Math,* 3(3), 263-266, 2013.
13. Gopalan, M.A., Vidyalakshmi, S., Lakshmi, K., "Integral Solutions of Sextic Equation with Five unknowns $x^3 + y^3 = z^3 + w^3 + 3(x-y)t^5$" , *IJERST,* 1(10), 562-564, 2012.
14. Gopalan, M.A., Sumathi, G., Vidyalakshmi, S., "Integral Solutions of Sextic Non-Homogeneous Equation with Five unknowns $x^3 + y^3 = z^3 + w^3 + 6(x+y)t^5$", *International Journal of Engineering Research,* Vol.1, Issue.2, 146-150, 2013.
15. Gopalan, M.A., Aarthy Thangam, S., Kavitha, A., "On Non-homogeneous Sextic equation with five unknowns $2(x-y)(x^3+y^3) = 28(z^2-w^2)T^4$", *Jamal Academic Research Journal (JARJ) ,* special Issue, 291-295, ICOMAC- 2015.
16. Meena, K., Vidhyalakshmi, S., AarthyThangam, S., "On Non-homogeneous Sextic equation with five unknowns $(x+y)(x^3+y^3) = 26(z^2-w^2)T^4$", *Bulletin of Mathematics and Statistics Research,* Vol.5, Issue.2, 45-50, 2017.

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