International Journal of Current Advanced Research
ISSN: O: 2319-6475, ISSN: P: 2319-6505, Impact Factor: 6.614
Available Online at www.journalijcar.org
Volume 7; Issue 3(J); March 2018; Page No. 11118-11125
DOI: http://dx.doi.org/10.24327/ijcar.2018.11125.1916

Research Article

ARITHMETIC LABELING OF $W_4 \times P_n$ AND $K_{m,n}$

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ARTICLE INFO

Article History:

Received 11th December, 2017
Received in revised form 20th January, 2018 Accepted 14th February, 2018
Published online 28th March, 2018

Key words:

Cartesian product, Labeling, Arithmetic Labeling.

ABSTRACT

Acharya.B.D. and Hedge.S.M.was introduced the concept of arithmetic labeling and many research articles have published in this topic. In this paper, we have proved that the Cartesian product of $W_4 \times P_n$ and $K_{m,n}$ for every m,n > 0 are arithmetic graphs. Also we established a general formula to label the vertices of the graph G.

INTRODUCTION

In 1989 Acharya.B.D. and Hedge.S.M. Introduced a new version of sequential graph known as arithmetic graph and is defined as follows: Let G be a graph with q edges a and d are the positive integers, the labeling f of G is said to be (a, d) – arithmetic if the vertices are labeled by distinct nonnegative integers and the edge labels are induced by $f(x) + f(y)$ for each edge xy are in the form of $a, a + d, a + 2d, \dots, a + (q - 1)d$. A graph is called arithmetic if it is an (a, d) - arithmetic for some a and d.

Definition

A graph G is an ordered pair $(V(G), E(G))$ consisting of a non empty set $V(G)$ of vertices and a set $E(G)$, disjoint from $V(G)$ of edges, together with an incidence function ψ_G that associates with each edge of G is an unordered pair of vertices of G.

Definition

Walk is an alternating sequence of vertices and edges starting and ending with vertices.

A walk in which all the vertices are distinct is called a path.

Definition

A ***wheel*** graph is a graph formed by connecting a single vertex to all vertices of a cycle. A wheel graph with n vertices is denoted by W_n. That is $W_n = K_1 + C_{n-1}$ for every $n \geq 4$.

Definition

A complete bipartite graph is a bipartite graph (that is a set of graph vertices decomposed into two disjoint sets such that no two graph vertices within the same sets are adjacent) such that every pair of graph vertices in the distinct sets are adjacent.

Definition

The Cartesian product of the graphs G and H is denoted by $G \times H$ and defined the vertex set of $G \times H$ is the cartesian product $V(G) \times V(H)$ and any two vertices (u, u') and (v, v') are adjacent in $G \times H$ if and only if either $u = v$ and u' is adjacent with v' in H or $u' = v'$ and u is adjacent with v in G.

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Definition

Graph labeling is an assignment of labels, by integers to the edges or vertices or both of a graph.

A labeling or valuation of a graph G is an assignment of labels to the vertices of G that induces for each edge xy a label depending on the vertex labels $f(x)$ and $f(y)$.

Definition

A graph is said to be an arithmetic if its vertices can be assigned distinct non negative integers in such a way that the value of the edges are obtained as sum of the values assigned in an arithmetic progression.

Theorem

Let the graph G be the cartesian product of W_4 and P_n that is $W_4 \times P_n$
Where $n > 0$ is an (a,1) - arithmetic graph.

Proof

Let G = $W_4 \times P_n$ where $n > 0$. Then the graph is represented in (Figure: 1)as below.

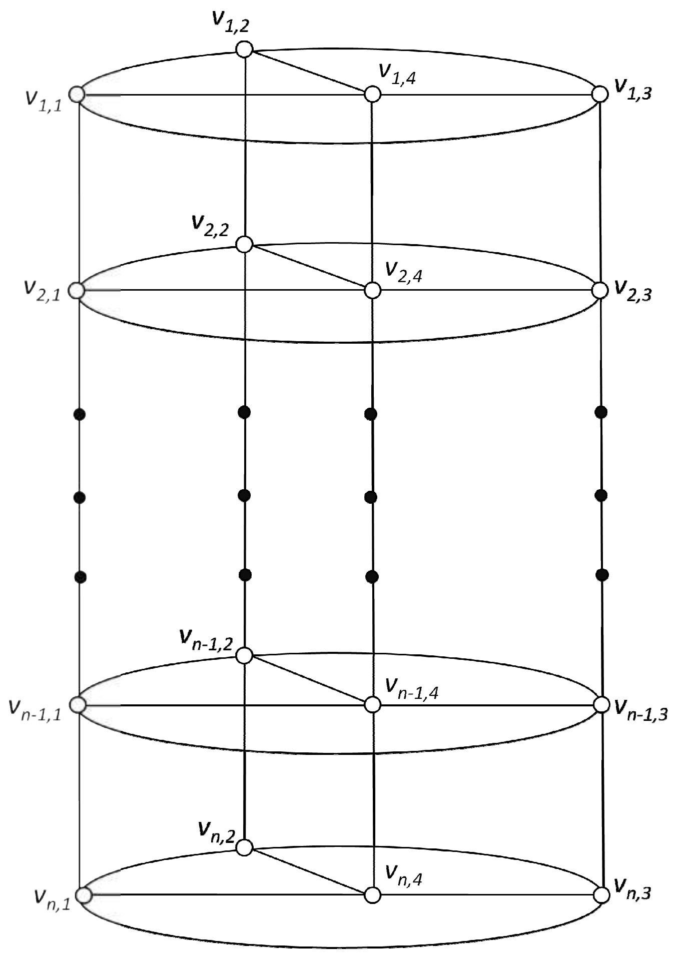

Figure 1

Let V be the vertex set of G and is denoted by $V(G) = \{V_{ij}/1 \leq i \leq n; 1 \leq j \leq m\}$

Define $f: V(G) \rightarrow N$. Now we are giving the label to the vertices of G as below

If 'i' is Odd

$f(v_{i,j}) = 5i + j - 4$ for all i where $1 \leq i \leq n$ *for all odd* n (or)

$1 \leq i \leq n-1$ *for all even* n; $j = 1,2,3.$ …….. (1)

$f(v_{i,j}) = 5(i-1)$ for all i where $1 \leq i \leq n$ *for all odd* n (or)

$1 \leq i \leq n-1$ *for all even* n; $j = 4$ …….. (2)

If 'i' is even

$f(v_{i,j}) = 2\left[3i - \left(\frac{i-2}{2}\right) - j\right] - 1$ for all i where $2 \le i \le n-1$ $for\ all\ odd\ n$
$(or) 2 \le i \le n$ $for\ all\ even\ n;$ $j = 1,2,3$ ………. (3)

$f(v_{i,j}) = ij + (i-2)$ for all i where $2 \le i \le n-1$ $for\ all\ odd\ n$

$(or) 2 \le i \le n$ $for\ all\ even\ n;$ $j = 4$ ……… (4)

The edge labels induced by $f(uv) = f(u) + f(v)$ are as follows

$f(v_{k,j}v_{k+1,j}) = 10k + 2 - j$ $where$ $1 \le k \le n;$ $j = 1,2,3,4$ ………*(5)*

If 'i' is Odd

$f(v_{i,r}v_{i,t}) = r + t + 2 + (i-1)\,10$ for all i where $1 \le i \le n$ $for\ all\ odd\ n$
$(or) 1 \le i \le n-1$ $for\ all\ even\ n;$ $r = 1,2;$ $t = 2,3$

$and\ \ r \ne t$ ……… (6)

$f(v_{i,r}v_{i,t}) = 10i + r - 9$ for all i where $1 \le i \le n$ $for\ all\ odd\ n$

$(or) 1 \le i \le n-1$ $for\ all\ even\ n;$ $r = 1,2,3;$ $t = 4$ ……… (7)

If 'i' is even

$f(v_{i,r}v_{i,t}) = 2[5i + 1 - (r+t)]$ for all i where $2 \le i \le n-1$ $for\ all\ odd\ n$

$(or) 2 \le i \le n$ $for\ all\ even\ n;$ $r = 1,2;$ $t = 2,3$ $and\ \ r \ne t$ ……… (8)
$f(v_{i,r}v_{i,t}) = (t-r) + [t - (r-1)] + 10(i-1)$ for all i

where $2 \le i \le n-1$ $for\ all\ odd\ n$ $(or) 2 \le i \le n$ $for\ all\ even\ n;$

$r = 1,2,3;$ $t = 4$ ………… (9)

Clearly the edges are labeled as $f(E(G)) = \{a, a+d, a+2d, \dots, a+(q-1)d\}$.

Therefore, f is an arithmetic labeling.

Hence the graph $G = W_4 \times P_n$ is an $(a, 1)$- arithmetic graph.

Example

Consider the graph $G = W_4 \times P_5$

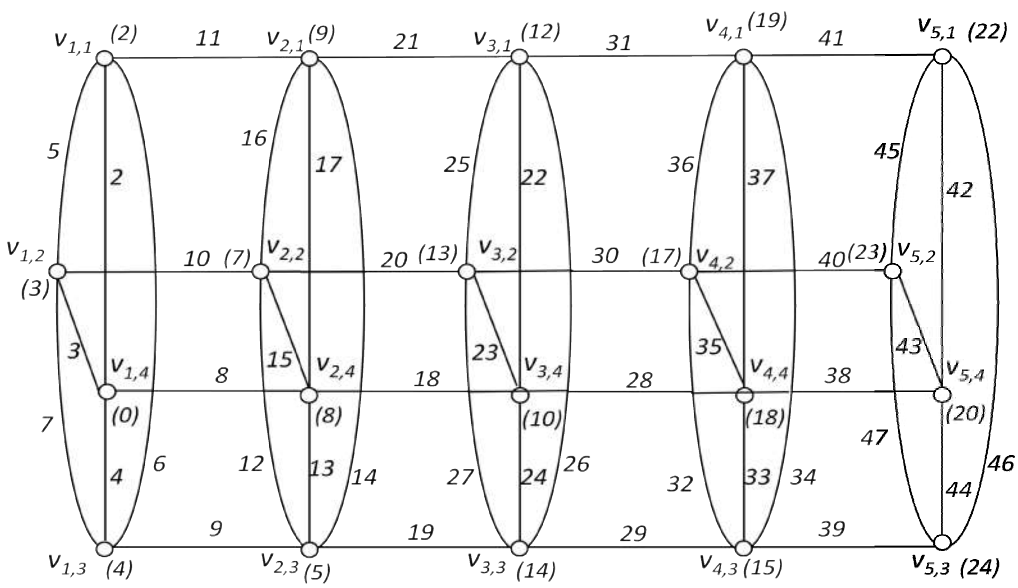

Figure 2

Here, n = 5; q = 46
The vertex label are as given below.

Equation (1) $\Rightarrow f(v_{i,j}) = 5i + j - 4$ ∀ odd i where i = 1,3,5,…,n; $j = 1,2,3$
When $i = 1,3,5;$ $j = 1,2,3 \Rightarrow f(v_{1,1}) = 2;$ $f(v_{1,2}) = 3;$ $f(v_{1,3}) = 4;$ $f(v_{3,1}) = 12;$
$f(v_{3,2}) = 13;$ $f(v_{3,3}) = 14;$ $f(v_{5,1}) = 22;$ $f(v_{5,2}) = 23;$ $f(v_{5,3}) = 24.$
Equation (2) $\Rightarrow f(v_{i,j}) = 5(i-1)$ ∀ odd i where i = 1,3,5,…,n; $j = 4$

When $i = 1,3,5;\ j = 4 \Rightarrow f(v_{1,4}) = 0;\ f(v_{3,4}) = 10;\ f(v_{5,4}) = 20.$
Equation (3) $\Rightarrow f(v_{i,j}) = 2\left[3i - \left(\frac{i-2}{2}\right) - j\right] - 1$ ∀ even i where i = 2,4,…,(n-1); $j = 1,2,3$
When $i = 2,4;\ j = 1,2,3 \Rightarrow f(v_{2,1}) = 9;\ f(v_{2,2}) = 7;\ f(v_{2,3}) = 5;\ f(v_{4,1}) = 19;$
$f(v_{4,2}) = 17;\ f(v_{4,3}) = 15.$
Equation (4) $\Rightarrow f(v_{i,j}) = ij + (i-2)$ ∀ even i where i = 2,4,…,(n-1); $j = 4$
when $i = 2,4;\ j = 4 \Rightarrow f(v_{2,4}) = 8;\ f(v_{4,4}) = 18.$
The edge labels are given as below
Equation (5) $\Rightarrow f(v_{k,j}\, v_{k+1,j}) = 10k + 2 - j$ where k = 1,2,3,…,n; j = 1,2,3,4.
When k = 1,2,3,4,5; j = 1,2,3,4 $\Rightarrow f(v_{1,1}\, v_{2,1}) = 11; f(v_{2,1}\, v_{3,1}) = 21;\ f(v_{3,1}\, v_{4,1}) = 31;$
$f(v_{4,1}\, v_{5,1}) = 41;\ f(v_{1,2}\, v_{2,2}) = 10;\ f(v_{2,2}\, v_{3,2}) = 20;\ f(v_{3,2}\, v_{4,2}) = 30;$
$f(v_{4,2}\, v_{5,2}) = 40;\ f(v_{1,3}\, v_{2,3}) = 9;\ f(v_{2,3}\, v_{3,3}) = 19;\ f(v_{3,3}\, v_{4,3}) = 29;$
$f(v_{4,3}\, v_{5,3}) = 39;\ f(v_{1,4}\, v_{2,4}) = 8;\ f(v_{2,4}\, v_{3,4}) = 18;\ f(v_{3,4}\, v_{4,4}) = 28;$
$f(v_{4,4}\, v_{5,4}) = 38.$
Equation (6) $\Rightarrow f(v_{i,r}\, v_{i,t}) = r + t + 2 + (i-1)10$ ∀ odd i where i = 1,3,5,…,n; r = 1,2;
t = 2,3 and $r \neq t$
When i = 1,3,5; r = 1,2 and t = 2,3 $\Rightarrow f(v_{1,1}\, v_{1,2}) = 5; f(v_{1,2}\, v_{1,3}) = 7;\ f(v_{1,1}\, v_{1,3}) = 6;$
$f(v_{3,1}\, v_{3,2}) = 25;\ f(v_{3,2}\, v_{3,3}) = 27;\ f(v_{3,1}\, v_{3,3}) = 26;\ f(v_{5,1}\, v_{5,2}) = 45;$
$f(v_{5,2}\, v_{5,3}) = 47;\ f(v_{5,1}\, v_{5,3}) = 46.$
Equation (7) $\Rightarrow f(v_{i,r}\, v_{i,t}) = 10i + r - 9$ ∀ odd i where i = 1,3,5,…,n; r = 1,2,3; t = 4.
When i = 1,3,5; r = 1,2,3 and t = 4 $\Rightarrow f(v_{1,1}\, v_{1,4}) = 2;\ f(v_{1,2}\, v_{1,4}) = 3;\ f(v_{1,3}\, v_{1,4}) = 4;$
$f(v_{3,1}\, v_{3,4}) = 22;\ f(v_{3,2}\, v_{3,4}) = 23;\ f(v_{3,3}\, v_{3,4}) = 24;\ f(v_{5,1}\, v_{5,4}) = 42;$
$f(v_{5,2}\, v_{5,4}) = 43;\ f(v_{5,3}\, v_{5,4}) = 44.$
Equation (8) $\Rightarrow f(v_{i,r}\, v_{i,t}) = 2[5i + 1 - (r+t)]$ ∀ even i where i = 2, 4,…, (n-1); r =1, 2;
t = 2,3 and $r \neq t$
When i = 2,4; r = 1,2; t = 2,3 $\Rightarrow f(v_{2,1}\, v_{2,2}) = 16; f(v_{2,2}\, v_{2,3}) = 12;\ f(v_{2,1}\, v_{2,3}) = 14;$
$f(v_{4,1}\, v_{4,2}) = 36;\ f(v_{4,2}\, v_{4,3}) = 32;\ f(v_{4,1}\, v_{4,3}) = 34.$
Equation (9) $\Rightarrow f(v_{i,r}\, v_{i,t}) = (t-r) + [t - (r-1)] + 10(i-1)$ ∀ even i where
i = 2,4,…,(n-1); r = 1,2,3; t = 4
When i = 2,4; r = 1,2,3 and t = 4 $\Rightarrow f(v_{2,1}\, v_{2,4}) = 17; f(v_{2,2}\, v_{2,4}) = 15;\ f(v_{2,3}\, v_{2,4}) = 13;$
$f(v_{4,1}\, v_{4,4}) = 37;\ f(v_{4,2}\, v_{4,4}) = 35;\ f(v_{4,3}\, v_{4,4}) = 33.$
In this graph $a = 2$ and $d = 3 - 2 = 1$.

The edge labels are in the arithmetic progression

$a = 2, a + d = 3, a + 2d = 4, a + 3d = 5, a + 4d = 6, a + 5d = 7,$
$a + 6d = 8, a + 7d = 9, a + 8d = 10, a + 9d = 11, a + 10d = 12,$
$a + 11d = 13, a + 12d = 14, a + 13d = 15, a + 14d = 16,$
$a + 15d = 17, a + 16d = 18, a + 17d = 19, a + 18d = 20,$
$a + 19d = 21, a + 20d = 22, a + 21d = 23, a + 22d = 24,$
$a + 23d = 25, a + 24d = 26, a + 25d = 27, a + 26d = 28,$
$a + 27d = 29, a + 28d = 30, a + 29d = 31, a + 30d = 32,$
$a + 31d = 33, a + 32d = 34, a + 33d = 35, a + 34d = 36,$
$a + 35d = 37, a + 36d = 38, a + 37d = 39, a + 38d = 40,$
$a + 39d = 41, a + 40d = 42, a + 41d = 43, a + 42d = 44,$
$a + 43d = 45, a + 44d = 46,\quad a + (q-1)d = a + 45d = 47.$

Then the graph $G = W_4 \times P_5$ is an (2,1)-arithmetic graph.

Theorem

The graph G = $k_{m,n}$ where m, n > 0 is an (a,2) –arithmetic graph.

Proof

Let G = $k_{m,n}$ where m, n > 0.
Then the graph is given in (Figure : 3) as below

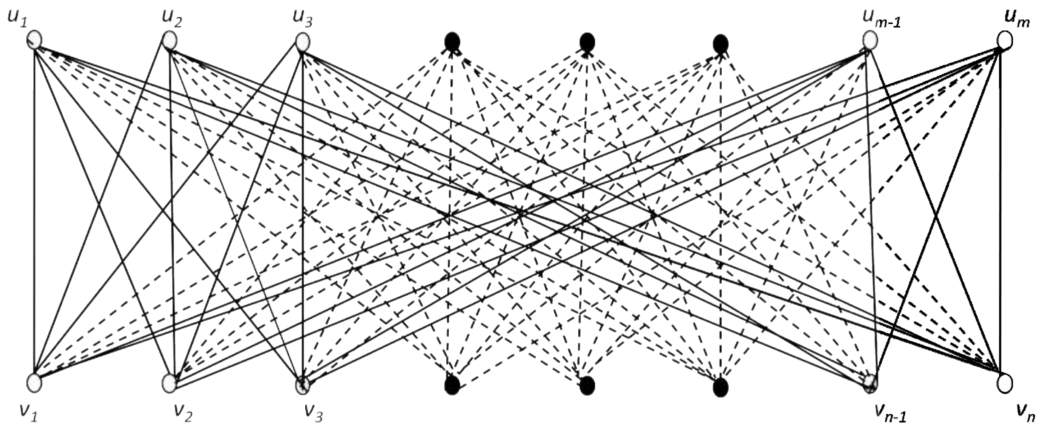

Figure 3

Let U and V be the vertex set of G and is denoted by,
$U(G) = \{U_i/1 \leq i \leq m\}$ and
$V(G) = \{V_j/1 \leq j \leq n\}$
Define $f: U(G) \to N$ and $f: V(G) \to N$.
Now we are giving the label to the vertices of G as below.

$f(u_i) = 2n(i-1) + 1$ where $1 \leq i \leq m$. …………. (i)

$f(v_j) = 2j$ where $1 \leq j \leq n$. …………. (ii)

The edge labels induced by $f(uv) = f(u) + f(v)$ are as follows.

$f(u_i v_j) = 2n(i-1) + 2j + 1$ where $1 \leq i \leq m;\ 1 \leq j \leq n$. …………. (iii)

Clearly the edges are labeled as

$f(E(G)) = \{a, a+d, a+2d, \ldots, a+(q-1)d\}$

Therefore, *f* is an arithmetic labeling.
Hence, the graph G = $K_{m,n}$ is an (a,2) - arithmetic graph.

Example

Consider the graph G = $k_{4,5}$

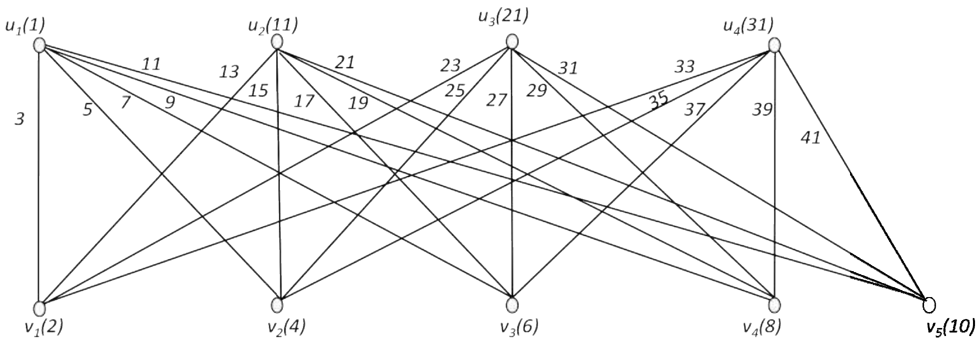

Figure 4

Here, m = 4; n = 5 and q = 20.
The vertex labels are given below.
Equation (i) $\Rightarrow f(u_i) = 2n(i-1) + 1$ where $1 \leq i \leq m$.
When i = 1,2,3,4 $\Rightarrow f(u_1) = 1; f(u_2) = 11; f(u_3) = 21; f(u_4) = 31$.
Equation (ii) $\Rightarrow f(v_j) = 2j$ where $1 \leq j \leq n$.
When j = 1,2,3,4,5 $\Rightarrow f(v_1) = 2; f(v_2) = 4; f(v_3) = 6; f(v_4) = 8; f(v_5) = 10$.
The edge labels are given below
Equation (iii) $\Rightarrow f(u_i v_j) = 2n(i-1) + 2j + 1$ where $1 \leq i \leq m; 1 \leq j \leq n$.
When i = 1,2,3,4 and j = 1,2,3,4,5 $\Rightarrow f(u_1v_1) = 3;\ f(u_1v_2) = 5;\ f(u_1v_3) = 7;$
$f(u_1v_4) = 9;\ f(u_1v_5) = 11;\ f(u_2v_1) = 13;\ f(u_2v_2) = 15;\ f(u_2v_3) = 17;$
$f(u_2v_4) = 19;\ f(u_2v_5) = 21;\ f(u_3v_1) = 23;\ f(u_3v_2) = 25;\ f(u_3v_3) = 27;$
$f(u_3v_4) = 29;\ f(u_3v_5) = 31;\ f(u_4v_1) = 33;\ f(u_4v_2) = 35;\ f(u_4v_3) = 37;$
$f(u_4v_4) = 39;\ f(u_4v_5) = 41$.
In this graph $a = 3$ and $d = 5 - 3 = 2$.

The edge labels are in the arithmetic progression
$a = 3, a + d = 5, a + 2d = 7, a + 3d = 9, a + 4d = 11, a + 5d = 13,$
$a + 6d = 15, a + 7d = 17, a + 8d = 19, a + 9d = 21,$
$a + 10d = 23, a + 11d = 25, a + 12d = 27, a + 13d = 29,$
$a + 14d = 31, a + 15d = 33, a + 16d = 35,$
$a + 17d = 37, a + 18d = 39, a + (q - 1)d = a + 19d = 41.$
Then the graph $G = K_{4,5}$ is an $(a, 2)$-arithmetic graph.

Theorem

The graph G = $k_{m,n}$ where m, n > 0 is an (m,1) – arithmetic graph.

Proof

Let G = $k_{m,n}$ where m, n > 0. Then the graph is given in (figure: 5) as below

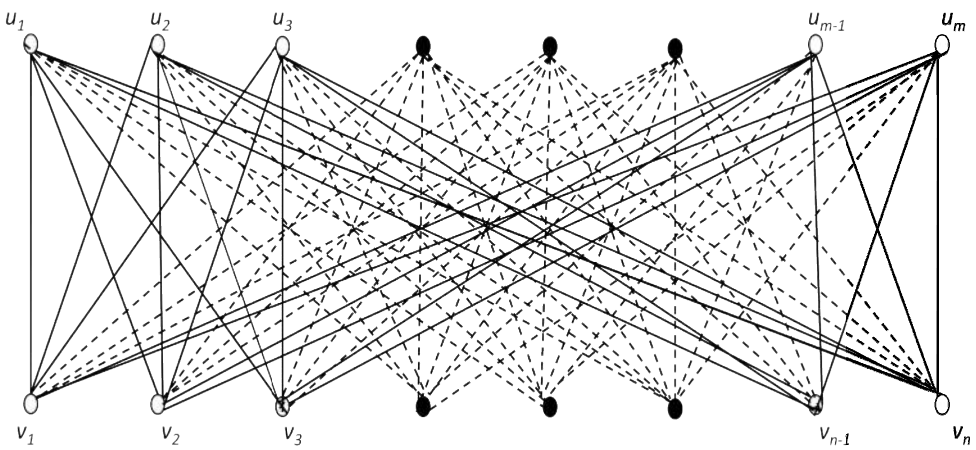

Figure 5

Let U and V be the vertex set of G and is denoted by
$U(G) = \{U_i/1 \leq i \leq m\}$ and
$V(G) = \{V_j/1 \leq j \leq n\}$
Define $f: U(G) \to N$ and $f: V(G) \to N.$
Now we are giving the label to the vertices of G as below.

$f(u_i) = (i - 1)$ where $1 \leq i \leq m.$ ……….. (α)
$f(v_j) = mj$ where $1 \leq j \leq n.$ ……….. (β)
The edge labels induced by $f(uv) = f(u) + f(v)$ are as follows.
$f(u_i v_j) = i + mj - 1$ where $1 \leq i \leq m;\ \ 1 \leq j \leq n$ ……….(γ)
Clearly the edges are labeled as
$f(E(G)) = \{a, a + d, a + 2d, \ldots, a + (q - 1)d\}$
Therefore, *f* is an arithmetic labeling.
Hence, the graph G is an (m,1)-arithmetic graph.

Example

Consider the graph G = $k_{3,5}$

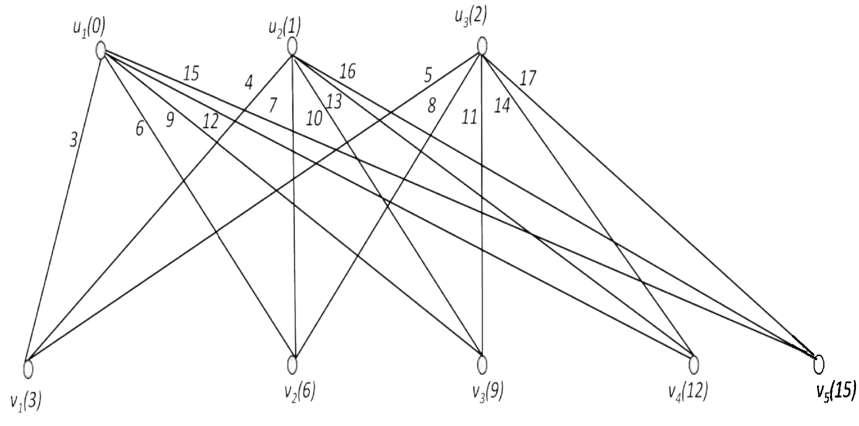

Figure 6

Here, m = 3; n = 5 and q = 15
The vertex labels are given below.
Equation $(\alpha) \Rightarrow f(u_i) = (i-1)$ where $1 \le i \le m$.
When i =1,2,3 $\Rightarrow f(u_1) = 0; f(u_2) = 1; f(u_3) = 2$.
Equation $(\beta) \Rightarrow f(v_j) = mj$ where $1 \le j \le n$.
When j = 1,2,3,4,5 $\Rightarrow f(v_1) = 3; f(v_2) = 6; f(v_3) = 9; f(v_4) = 12;\ f(v_5) = 15$.
The edge labels are given below
Equation $(\gamma) \Rightarrow\ \ f(u_i v_j) = i + mj - 1$ where $1 \le i \le m; 1 \le j \le n$.
When i=1,2,3; j=1,2,3,4,5 $\Rightarrow f(u_1v_1) = 3;\ f(u_1v_2) = 6;\ f(u_1v_3) = 9;$
$f(u_1v_4) = 12;\ f(u_1v_5) = 15;\ f(u_2v_1) = 4;\ f(u_2v_2) = 7;\ f(u_2v_3) = 10;$
$f(u_2v_4) = 13;\ f(u_2v_5) = 16;\ f(u_3v_1) = 5;\ f(u_3v_2) = 8;\ f(u_3v_3) = 11;$
$f(u_3v_4) = 14;\ f(u_3v_5) = 17.$
In this graph $a = 3$ and $d = 4 - 3 = 1$.

The edge labels are in the arithmetic progression
$a = 3, a + d = 4, a + 2d = 5, a + 3d = 6, a + 4d = 7, a + 5d = 8,$
$a + 6d = 9, a + 7d = 10, a + 8d = 11, a + 9d = 12, a + 10d = 13,$
$a + 11d = 14, a + 12d = 15, a + 13d = 16, a + (q-1)d = a + 14d = 17.$
Hence $G = K_{3,5}$ is an $(m, 1) = (3,1)$-arithmetic graph.

Corollary

Let $k_{m,n}$ be a complete bipartite graph (X,Y) with m, n vertices. The graph G is obtained from $k_{m,n}$ by joining the end vertices of same partition with an edge then G is an (m-1, 1) – arithmetic graph.

Proof

Let $k_{m,n}$ be a complete bipartite graph (X, Y) with m, n vertices. The graph G is obtained from $k_{m,n}$ by joining the end vertices of same partition with an edge.

Then the graph G is given in (Figure: 7) as below

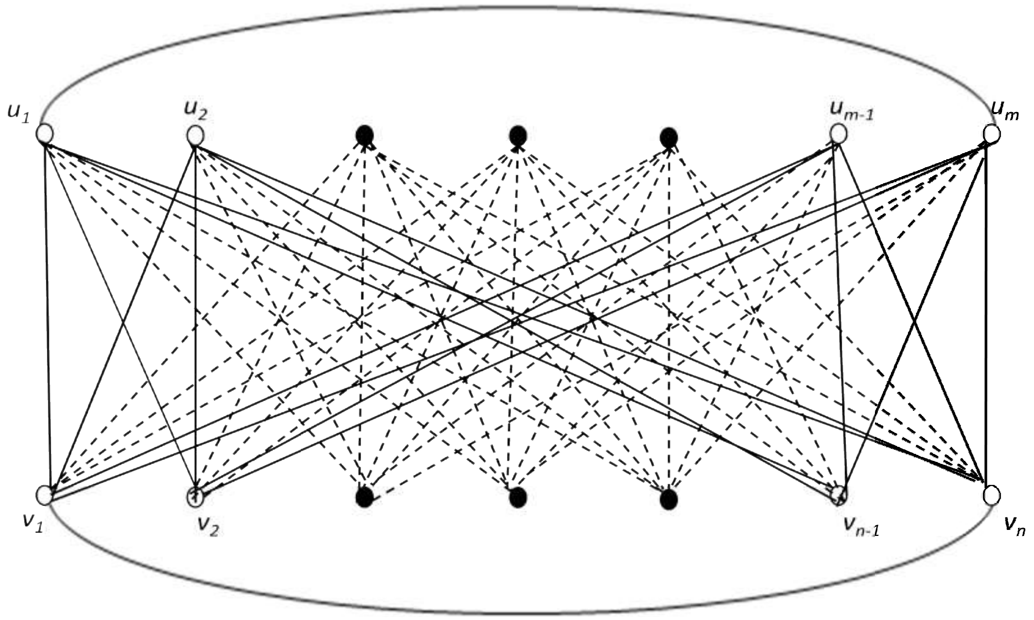

Figure 7

Let U and V be the vertex set of G and is denoted by
$U(G) = \{U_i / 1 \le i \le m\}$ and
$V(G) = \{V_j / 1 \le j \le n\}$
Define $f: U(G) \to N$ and $f: V(G) \to N$.
Now we are giving the label to the vertices of G as below.
$f(u_i) = (i-1)$ where $1 \le i \le m$. ……….. (A)
$f(v_j) = mj$ where $1 \le j \le n$. ……….. (B)
The edge labels induced by $f(uv) = f(u) + f(v);\ f(u_1u_n) = f(u_1) + f(u_n);$
$f(v_1v_n) = f(v_1) + f(v_n)$ are as follows.
$f(u_iv_j) = i + mj - 1$ where $1 \le i \le m;\ \ 1 \le j \le n$. ……….. (C)
$f(u_1u_m) = m - 1$ where m > 0 ……….. (D)
$f(v_1v_n) = mn + m$ where m, n > 0 ……….. (E)

Clearly the edges are labeled as
$f(E(G)) = \{a, a+d, a+2d, \dots, a+(q-1)d\}$
Therefore, f is an arithmetic labeling.
Hence, G is an (m-1,1)-arithmetic graph.

Example

Consider the graph G is a $k_{4,3}$ complete bipartite graph with 7 vertices and the end vertices which are connected by the same partition.

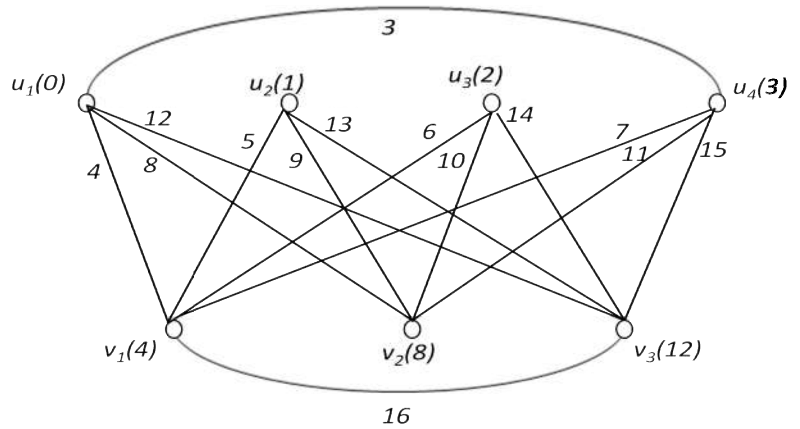

Figure 8

Here, m = 4; n = 3 and q = 14
The vertex labels are given below.
Equation (A) $\Rightarrow$ $f(u_i) = (i-1)$ where $1 \le i \le m$.
When i = 1,2,3,4 $\Rightarrow f(u_1) = 0;\ f(u_2) = 1;\ f(u_3) = 2;\ f(u_4) = 3$.
Equation (B) $\Rightarrow$ $f(v_j) = mj$ where $1 \le j \le n$.
When j = 1,2,3 $\Rightarrow f(v_1) = 4;\ f(v_2) = 8;\ f(v_3) = 12$.
The edge labels are given below
Equation (C) $\Rightarrow$ $f(u_i v_j) = i + mj - 1$ where $1 \le i \le m;\ 1 \le j \le n$.
When i = 1,2,3,4 and j = 1,2,3 $\Rightarrow f(u_1v_1) = 4;\ f(u_1v_2) = 8;\ f(u_1v_3) = 12;$
$f(u_2v_1) = 5;\ f(u_2v_2) = 9;\ f(u_2v_3) = 13;\ f(u_3v_1) = 6;\ f(u_3v_2) = 10;$
$f(u_3v_3) = 14;\ f(u_4v_1) = 7;\ f(u_4v_2) = 11;\ f(u_4v_3) = 15.$
Equation (D) $\Rightarrow$ $f(u_1u_m) = m - 1$ where n > 0.
When m = 4 $\Rightarrow f(u_1u_4) = 3$.
Equation (E) $\Rightarrow$ $f(v_1v_n) = mn + m$ where m, n > 0
When m = 4; n = 3 $\Rightarrow f(v_1v_3) = 16$
In this graph $a = 3$ and $d = 4 - 3 = 1$.
The edge labels are in the arithmetic progression
$a = 3, a+d = 4, a+2d = 5, a+3d = 6, a+4d = 7, a+5d = 8,$
$a+6d = 9, a+7d = 10, a+8d = 11, a+9d = 12, a+10d = 13,$
$a+11d = 14, a+12d = 15, a+(q-1)d = a+13d = 16.$
Therefore G is an (m-1, 1) = (3, 1) - arithmetic graph.

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How to cite this article:

Stephen John. B and Jovit Vinish Melma.G (2018) 'Arithmetic Labeling of W4× Pn AND Km,n', *International Journal of Current Advanced Research,* 07(3), pp. 11118-11125. DOI: http://dx.doi.org/10.24327/ijcar.2018.11125.1916