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Research Article

A GENERALIZATION OF INDEPENDENT RESOLVING PARTITION OF A GRAPH

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A B S T R A C T

Let G = (V, E) be a simple connected graph. A partition $\pi = \{V_1, V_2, V_3 \ldots, V_k\}$ is called a resolving partition of G if for any $u \in V(G)$, the code of u with respect to π (denoted by $c_\pi(u)$) namely $(d(u,V_1), d(u, V_2), \ldots, d(u, V_k))$ is distinct for different $u \in V(G)$ where $d(u, V_i) = \min\{d(u, x) / x \in V_i\}$. The minimum cardinality of a resolving partition of a graph G is called the partition dimension of G and is denoted by pd (G)[2]. Several types of resolving partition have been considered like connected resolving partition [7], metric chromatic number of a graph (that is, independent resolving partition) [4], equivalence resolving partition [6] etc. A new type of resolving partition called isolate vertex resolving partition was introduced in [5].This partition is a generalization of an independent resolving partition. A detailed study of this partition is done in this paper.

INTRODUCTION

Definition: [2] Let G = (V, E) be a simple, finite, connected and undirected graph. A partition $\prod = \{V_1, V2, \ldots, V_k\}$ of V (G) is called a resolving partition of G if the code $c_\prod(u) = (d(u, V_1), d(u, V_2), \ldots, d(u, V_k))$ is different for different $u \in V(G)$ where $d(u, V_i) = \min\{d(u, x)/ x \in V_i\}$. The minimum cardinality of a resolving partition of a graph G is called the partition dimension of G and is denoted by pd (G).

Definition: [5] Let G = (V, E) be a simple, finite, connected and undirected graph. Let $\prod = \{V_1, V_2, \ldots, V_k\}$ be a partition of V(G). If each $\langle V_i \rangle$ contains an isolate and if ∏ is a resolving partition, then ∏ is called an isolate vertex resolving partition. The trivial partition namely $\prod = \{\{u_1\}, \{u_2\}, \ldots\ldots, \{u_n\}\}$ where $V(G)=\{u_1, u_2, \ldots, u_n\}$ is an isolate vertex resolving partition. The minimum cardinality of an isolate vertex resolving partition is called the isolate vertex partition dimension of G and is denoted by pd_{is} (G).

Definition: A double star is a graph obtained by taking two stars and joining the vertices of maximum degrees with an edge.

Remark: [5] Every independent resolving partition is an isolate vertex resolving partition. Therefore, $pd_{is}(G) \leq ipd(G) \leq pd(G)$.

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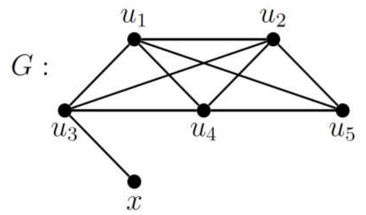
Characterizations

Lemma: Let G be a connected graph with $pd_{is}(G) = n = |V(G)|$. Let a pendent vertex x be attached at a single vertex of G. Let H be the resulting graph. Let $G = \langle V_1 \rangle + \langle V_2 \rangle$ where $\langle V_1 \rangle$ and $\langle V_2 \rangle$ are connected and diam($\langle V_1 \rangle$), diam($\langle V_2 \rangle$) less than or equal to 2. Let x be attached to $u_1 \in V_1$. Then $pd_{is}(H) = |V(H)| - 1$ if and only if any pd_{is} - partition ∏ of H containing a set W with $x \in W$ and $|W| \geq 3$, there exist $y, z \in W$ such that y and z are non-adjacent.

Proof: Suppose the condition of the hypothesis in the theorem is satisfied. Suppose $pd_{is}(H) = |V(H)|-1$. Then no pd_{is}- partition of H can contain a set W with $|W| \geq 3$. Conversely, suppose any pd_{is} - partition ∏ of H containing a set W with $x \in W$, and $|W| \geq 3$, then there exist $y, z \in W$ such that y and z are non-adjacent. Then $pd_{is}(H) \leq |V(H)| - 2$. Let $\prod = \{W_1, W_2, \ldots, W_r\}$ be a pd_{is} - partition of H, where $r \leq |V(H)| - 2$. Let $|W_i| \geq 3$. Let $x \in W_i$. Let $u_1, u_2 \in W_i \cap V(G)$ be non-adjacent. Then $\prod_1 = \{W_1, W_2, \ldots \{u_1, u_2\}$, all singletons omitting x}. $\{x, u_1, u_2\}$ is an element of ∏ and hence u_1, u_2 are resolved by some $W_i \subseteq V(G)$. Therefore $\prod_1$ is an isolate resolving partition of G. Therefore $pd_{is}(G) \leq |\prod_1| \leq n - 1$, a contradiction. Therefore, $pd_{is}(H) = |V(H)| - 1$. ★

Remark: The condition that there exist $y, z \in W$ with $|W| \geq 3$, $x \in W$ and y, z are non-adjacent cannot be dropped.
For,



Let $\prod$ = {{x, u_4, u_5}, {u_1}, {u_2}, {u_3}}.Then $c_{\prod}(x)$ = (0, 2, 2, 1), $c_{\prod}(u_4)$ = (0, 1, 1, 1), $c_{\prod}(u_5)$ = (0, 1, 1, 2).

Lemma: Let G be a connected graph with pd_{is} (G) = n = |V (G)|. Let a pendent vertex x be attached at a single vertex of G. Let H be the resulting graph. Let G = <V_1>+ <V_2>where <V_1>and <V_2 >are connected and diam(<V_1>), diam(< V_2 >) less than or equal to 2 and neither < V_1> nor < V_2> contains a K_3 with a pendent vertex. Let x be attached to $u_1 \in V_1$. Then $pd_{is}(H)$ = |V(H)|– 1 if and only if any pd_{is} - partition $\prod$ of H containing exactly two two-elements sets W_1, W_2 each with cardinality 2 such that $x \in W_1$ and x is adjacent with exactly one element, (say) u_3 of W_2={u_2, u_3} $\subseteq$ V(G), then either u_2 is adjacent with $u_1 \in W_1$– {x} or u_3 is adjacent with u_1 or both u_2 and u_3 are adjacent with u_1.

***Proof**.* Let $x \notin W_1 \cup W_2$. Then W_1, W2 $\subseteq$ V (G). Since $\prod$ contains exactly two two- element sets, { *x* } $\in \prod$. Since x is adjacent exactly one vertex of V (G), both W_1 andW_2 cannot be resolved by *x*. Therefore, atleast one of W_1, W_2 is resolved by a set $W_3 \subseteq$ V (G). Therefore, $\prod$ – {*x*} is an isolate vertex resolving partition of G. Therefore, $\prod$ - {*x*} ≤ n – 2, a contradiction.

Let $x \in W_1$. (similar proof if $x \in W_2$). Let W_1 = {x_1, u_1},
W_2 = {u_2, u_3}.

Case (i): *x* is not adjacent with u_2 as well as u_3.
Then either W_2 is resolved by u_1 or by any set in $\prod$ which contains only elements of V(G). In any case, $\prod$ - {*x*} is an isolate vertex resolving partition of G, a contradiction.

Case (ii): *x* is adjacent with exactly one of u_2, u_3 (say) u_3.

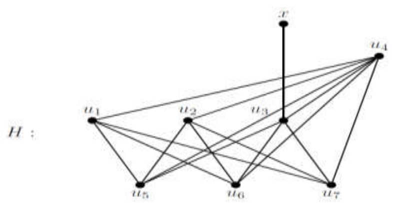
That is *x* is adjacent with u_3, *x* is not adjacent with u_2. By hypothesis, either u_1 adjacent with u_2 or adjacent with u_3 or both.

Subcase (i): u_1 is adjacent with u_2.

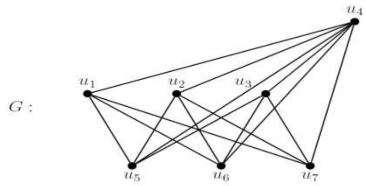
Then {u_2, u_3} is not resolved by {x, u_1}. Therefore there exist some set of $\prod$ containing only elements of G which resolves {u_2, u_3}. Therefore, $\prod$ - {*x*} is an isolate resolving partition of G, a contradiction.

Subcase (ii): u_1 is not adjacent with u_2. Then u_1 is adjacent with u_3. Therefore, W_1- {*x*} resolves W_2. Therefore, $\prod$ - {*x*} is an isolate resolving partition of G, a contradiction. ★

Remark: The condition that either u_2 is adjacent with $u_1 \in W_1$- {*x*} or u_3 is adjacent with u_1 or both u_2 and u_3 are adjacent with u_1 cannot be dropped.
For,



Let $\prod$ ={{u_1, x}, {u_2, u_3}, {u_4}, {u_5}, {u_6}, {u_7}}.Then $c_{\prod}(u_1)$ = (0, 2, 1,…), $c_{\prod}(x)$ = (0, 1,2,…), $c_{\prod}(u_2)$ = (2, 0, 1,…), $c_{\prod}(x)$ = (1, 0, 1, …).$\prod$ is an isolate resolving partition of H. Therefore, pd_{is} (H) ≤ |$\prod$| = 6 = 8 – 2 = |V (H)|– 2.



Let $\prod$ = {{u_1}, {u_2, u_3}, {u_4}, {u_5}, {u_6}, {u_7}}.Then $\prod$ is not an isolate resolving partition of G.

In fact, pd_{is} (G) = |V(G)| = 7. In this example, u_1 and u_3 are not adjacent with u_2.

Remark: Let G be a connected graph. If two independent vertices say x_1, x_2 are resolved by a vertex of G and for any two independent vertices say x_3, $x_4$with {x_3,x_4} ≠ {x_1,x_2}, $x_3$and x_4 are not resolved by any vertex of G, then $pd_{is}(G)$ ≤ n–1

Proof: Obvious.

Lemma: Let G be a tree. $pd_{is}(G)$ = n – 1 if and only if G = P_4.

Proof: Let G be a tree and let $pd_{is}(G)$ = n – 1 . Then diam(G) ≤ 3. If diam (G) = 1 then G = K_2 and $pd_{is}(G)$ = 2 , a contradiction. If diam(G) = 2, then G is a star and $pd_{is}(G)$ = |V(G)|, a contradiction. Let *diam*(G) = 3. Then G is a double star $D_{r,s}$. If r = s = 1, then G = P_4 and
$pd_{is}(G)$ = 3 = |V(G)| – 1. If r (or) s ≥ 2, then $pd_{is}(G)$ = 3 = |V(G)| –2, a contradiction. Therefore, if G is a tree and $pd_{is}(G)$ = n–1, then G = P_4.
The converse is obvious.
★

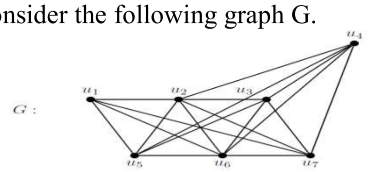
Lemma: Let G be a unicyclic graph. Then $pd_{is}(G)$ = n–1 if and only if G = $K_3$with one or more pendent vertices at a single vertex or C_4 with a pendent vertex.

Proof: Let G be a unicyclic graph with pd_{is} (G) = n – 1. Suppose diam (G) ≥ 4. Let v_1, v_2, v_3, v_4, v_5 be an induced path of length 4 in G. Then $\prod$ ={{v_1, v_3}, {v_2, v_4}, {v_5}, singletons} is an isolate vertex resolving partition of G. Therefore, $pd_{is}(G)$ ≤ n– 2, a contradiction. Therefore, diam (G) ≤ 3. If G contains C_n (n ≥ 8), then diam (G) ≥ 4, a contradiction. Suppose G contains C_7. Then there is no path attached at any vertex of C_7, since diam (C_7) = 3. If G = C_7, then $pd_{is}(G)$ ≤ 5, a contradiction. Suppose G contains C_6. Then also there is no path attached at any vertex of C_6. pd_{is} (C_6) ≤ 4. Suppose G contains C_5. If G = C_5, then pd_{is} (G) = 3, a contradiction. If G contains C_5 and a pendant vertex, then diam (G) = 3 and $pd_{is}(G)$ ≤ 4, a contradiction. Suppose G contains C_4. If G = C_4, then pd_{is} (G) = 4, a contradiction. If G contains C_4 and a pendent vertex, then diam (G) = 3 and pd_{is} (G) = 4. If G is C_4 with two pendent vertices one each at two vertices of C_4 or two or more pendent vertices at a single vertex of C_4, then diam (G) = 3 and pd_{is} (G) ≤ | V(G)| –2. Suppose G contains C_3. If G = C_3, then pd_{is} (G) = 3, a contradiction. If G is C_3 with one or more pendent vertices at a single vertex, then pd_{is} (G) = 3. If G is C_3 with a P_2 attached at a vertex, then diam (G) = 3 and pd_{is} (G) ≤ 3, a contradiction. If G is C_3 with two pendent vertices

attached one each at two vertices of C_3, then $pd_{is}(G) \leq 3$, a contradiction.
The converse is obvious. ★

Result: $pd_{is}(G) \leq n-1$ if and only if for any partition of V(G) into V_1, V_2 such that
$G = \langle V_1\rangle + \langle V_2\rangle$, if $\langle V_i\rangle$ is connected, $i \in \{1, 2\}$ then diam $(\langle V_i\rangle) \geq 3$ or if $\langle V_i\rangle$ is disconnected, then there exist an edge in $\langle V_i\rangle$ or $\langle V_i\rangle$ is connected and contains a K_3 with a pendent vertex as an induced subgraph.

For, Let us consider the following graph G.



Let $\prod = \{\{u_1, u_4\}, \{u_2\}, \{u_3\}, \{u_5\}, \{u_6\}, \{u_7\}\}$.Then $c_\prod(u_1) = (0, 1, 2, 1, 1, 1)$; $c_\prod(u_4) = (0, 1, 1, 1, 1, 1)$. $\prod$ is an isolate resolving partition of G. Therefore, $pd_{is}(G) \leq n-1$.

Theorem: Let G be a connected graph. Then $pd_{is}(G) = n-1$ if and only if either for any three vertices u_1, u_2, u_3 such that $\langle\{u_1, u_2, u_3\}\rangle$ is disconnected, $d(u_1, v) = d(u_2, v)$ for any $v \in V(G)$, $v \notin \{u_1, u_2, u_3\}$ or $d(u_2, v) = d(u_3, v)$ for every $v \in V(G)$, $v \notin \{u_1, u_2, u_3\}$ or $d(u_1, v) = d(u_3, v)$ for every $v \in V(G)$, $v \notin \{u_1, u_2, u_3\}$ or for any four vertices u_1, u_2, u_3, u_4 such that u_1 and u_2 are not adjacent, u_3 and u_4 are not adjacent and $d(u_1, v) = d(u_2, v)$ for every $v \in V(G)$, $v \neq u_1, u_2$ and $d(u_3, v) = d(u_4, v)$ for every $v \in V(G)$, $v \neq u_3, u_4$ and G is such that for any partition of V(G) into subsets V_1 and V_2, either $G \neq \langle V_1\rangle + \langle V_2\rangle$ or if $G = \langle V_1\rangle + \langle V_2\rangle$, then if $\langle V_i\rangle$, i = 1 or 2 is connected, then its diameter greater than or equal to 3 or if $\langle V_i\rangle$ is disconnected, then there exist an edge in $\langle V_i\rangle$.

Proof: If G satisfies the conditions in the theorem, $pd_{is}(G) \neq n$ and $pd_{is}(G) > n-2$. Therefore
$pd_{is}(G) = n-1$. If $pd_{is}(G) = n-1$, then the conditions of the theorem are obviously satisfied. ★

Paths and Cycles

Theorem: $pd_{is}(G) = 2$ if and only if $G = P_2$.

Proof: Let $pd_{is}(G) = 2$. Let $= \{V_1, V_2\}$ be an isolate vertex resolving partition of V (G). Suppose $|V(G)| \geq 3$. Let $V_1 = \{u_1, u_2, \ldots, u_k\}$ and $V_2 = \{v_1, v_2, \ldots, v_r\}$. Since is $\prod$ an isolate vertex resolving partition, $d(u_i, V_2)$ is different for every i and $d(V_1, u_j)$ is different for every j. since $|V(G)| \geq 3$, at least one of V_1, V_2 has at least two elements. Let $|V_1| \geq 2$. Then there exist a vertex $u \in V_1$ such that $d(u, V_2) \geq 2$. Let $d(u, V_2) = r \geq 2$. Let $u, w_1, w_2, \ldots, w_{r-1}, v_j$ be the shortest path from u to V_2. Then $w_1, w_2, \ldots, w_{r-1} \in V_1$. $d(v_j, V_1) = 1$.Let x be an isolate of V_1.Then $d(x, V_2) = 1$.That is there exist $y \in V_2$ such that $d(x, y) = 1$.Clearly, $x \notin \{u_1, w_1, w_2, .., w_{r-1}\}$.Therefore, $d(v_j, V_1) = d(y, V_1) = 1$. If $v_j \neq y$, then v_j and y are not resolved. If $v_j = y$,then x and w_{r-1} are not resolved, a contradiction. Therefore $|V(G)| \leq 2$. Clearly, $|V(G)| = 2$. That is $G = P_2$. The converse is obvious. ★

Theorem 3.2. $pd_{is}(P_n) = \begin{cases} 2 \text{ if } n = 2 \\ 3 \text{ if } n \geq 3 \end{cases}$

Proof: Obviously $pd_{is}(P_2) = 2$, $pd_{is}(P_3) = 3 = pd_{is}(P_4)$.
Let $n \geq 5$. Let $V(P_n) = \{u_1, u_2, \ldots, u_n\}$.
Let $\prod = \{\{u_1, u_4, u_6, u_8, \ldots\}, \{u_2, u_5, u_7 \ldots\}, \{u_3\}\}$. Clearly, $\prod$ is an isolate vertex resolving partition of P_n. Therefore $pd_{is}(P_n) \leq 3$. If $pd_{is}(P_n) = 2$, then n = 2, a contradiction by previous theorem. Therefore, $pd_{is}(P_3) = 3$. ★

Theorem: Let $n \geq 3$. Then $pd_{is}(C_n) = \begin{cases} 3 \text{ if } n \neq 4 \\ 4 \text{ if } n = 4 \end{cases}$

Proof. It can be seen that, $pd_{is}(C_3) = 3$, $pd_{is}(C_4) = 4$.
When n = 5, $\prod = \{\{1, 3, 4\}, \{2\}, \{5\}\}$ is an isolate vertex resolving partition of G. Therefore,
$pd_{is}(C_5) \leq 3$. But $pd_{is}(G) = 2$ if and only if $G = P_2$. Therefore, $pd_{is}(C_5) = 3$.
Let $n \geq 6$.
Case (i): When $n = 6k$, $k \geq 1$.

Subcase(i): k is even
Let $\prod = \{\{1, 4, 6, 8, \ldots, 3k, 3k+2, 3k+3, \ldots, 6k-1\}, \{2, 5, 7, 9, \ldots, 3k+1, 3k+4, 3k+6, \ldots. 6k\}, \{3\}\}$. Then $\prod$ is an isolate vertex resolving partition of G. Therefore, $pd_{is}(C_{6k}) \leq 3$. But $pd_{is}(G) = 2$ if and only if $G = P_2$. Therefore, $pd_{is}(C_{6k}) = 3$.
Subcase(ii): k is odd.
Let $\prod = \{\{1, 4, 6, 8, \ldots, 3k+1, 3k+3, 3k+4, 3k+6, \ldots, 6k-1\}, \{2, 5, 7, 9, \ldots, 3k, 3k+2, 3k+5, \ldots, 6k\}, \{3\}\}$. Then $\prod$ is an isolate vertex resolving partition of G. Therefore, $pd_{is}(C_{6k}) \leq 3$. But $pd_{is}(G) = 2$ if and only if $G = P_2$. Therefore, $pd_{is}(C_{6k}) = 3$.
Case (ii): When $n = 6k+1$, $k \geq 1$.

Subcase(i): k is even.
Let $\prod = \{\{1, 2, 5, 7, \ldots, 3k+3, 3k+5, \ldots, 6k+1\}, \{3, 6, 8, 10, \ldots, 3k+4, 3k+6, \ldots, 6k\}, \{4\}\}$. Then $\prod$ is an isolate vertex resolving partition of G. Therefore, $pd_{is}(C_{6k+1}) \leq 3$. But $pd_{is}(G) = 2$ if and only if $G = P_2$. Therefore, $pd_{is}(C_{6k+1}) = 3$.
Subcase(ii): k is odd.
Let $\prod = \{\{1, 2, 5, 7, \ldots, 3k+4, 3k+6, \ldots, 6k+1\}, \{3, 6, 8, 10, \ldots, 3k+3, 3k+5, \ldots, 6k\}, \{4\}\}$. Then $\prod$ is an isolate vertex resolving partition of G. Therefore, $pd_{is}(C_{6k+1}) = 3$. But $pd_{is}(G) = 2$ if and only if $G = P_2$. Therefore, $pd_{is}(C_{6k+1}) = 3$.
Case (iii): When $n = 6k+2$, $k \geq 1$.

Subcase(i): k is even.
Let $= \{\{1, 4, 6, 8, \ldots, 3k+4, 3k+5, 3k+7, \ldots, 6k+1\}, \{2, 5, 7, 9, \ldots, 3k+3, 3k+6, \ldots, 6k+2\}, \{3\}\}$. Then $\prod$ is an isolate vertex resolving partition of G. Therefore, $pd_{is}(C_{6k+2}) \leq 3$. But $pd_{is}(G) = 2$ if and only if $G = P_2$. Therefore, $pd_{is}(C_{6k+2}) = 3$.
Subcase(ii): k is odd.

Let $\prod = \{\{1, 4, 6, 8, \ldots, 3k+3, 3k+4, 3k+6, \ldots, 6k+1\}, \{2, 5, 7, 9, \ldots, 3k+2, 3k+5, \ldots, 6k+2\}, \{3\}\}$. Then $\prod$ is an isolate vertex resolving partition of G. Therefore, $pd_{is}(C_{6k+2}) \leq 3$. But $pd_{is}(G) = 2$ if and only if $G = P_2$. Therefore, $pd_{is}(C_{6k+2}) = 3$.
Case (iv): When $n = 6k+3$, $k \geq 1$.
Subcase(i): k is even.

Let $\prod = \{\{1, 4, 6, 8, \ldots, 3k+4, 3k+6, \ldots, 6k+2\}, \{2, 5, 7, 9, \ldots, 3k+3, 3k+5, \ldots 6k+3\}, \{3\}\}$.Then $\prod$ is an isolate vertex resolving partition of G. Therefore, $pd_{is}(C_{6k+3}) \leq 3$.
But $pd_{is}(G) = 2$ if and only if $G = P_2$. Therefore, $pd_{is}(C_{6k+3}) = 3$.
Subcase(ii): k is odd.

Let $\prod = \{\{1, 4, 6, 8, \ldots, 3k+3, 3k+5, \ldots, 6k+2\}, \{2, 5, 7, 9, \ldots, 3k+4, 3k+6, \ldots, 6k+3\}, \{3\}\}$. Then $\prod$ is an isolate vertex resolving partition of G. Therefore, $pd_{is}(C_{6k+3}) \leq 3$. But $pd_{is}(G) = 2$ if and only if $G = P_2$. Therefore, $pd_{is}(C_{6k+3}) = 3$.

Case (v): When $n = 6k+4$, $k \geq 1$.
Subcase(i): k is even.

Let $\prod$=_{1,4,6,8,… , 3k+2,3k+4,3k+5,3k+7,…. 6k + 3},{2,5,7,9,… , 3k+3,3k+6,3k+8, …, 6k+4},{3}}. Then $\prod$ is an isolate vertex resolving partition of G.Therefore, $pd_{is}(C_{6k+4}) \leq 3$. But $pd_{is}(G) = 2$ if and only if $G = P_2$. Therefore, $pd_{is}(C_{6k+4}) = 3$.
Subcase(ii): k is odd.

Let $\prod$={{1,4,6,8,… ,3k+3,3k+5,3k+6,3k+8,….. 6k + 3},{2,5,7,9,… ,3k+2, 3k+4,3k+7, …,6k+4}, {3}}. Then $\prod$ is an isolate vertex resolving partition of G.Therefore, $pd_{is}(C_{6k+4}) \leq 3$. But $pd_{is}(G) = 2$ if and only if $G = P_2$. Therefore, $pd_{is}(C_{6k+4}) = 3$.
Case (vi): When $n = 6k+5$, $k \geq 1$.
Subcase(i): k is even.
Let $\prod$ = {{1, 4, 6, 8,… , 3k+4, 3k+6, …., 6k + 4},{2, 5, 7, 9,…, 3k+5, 3k+7,…, 6k+5}, {3}}.Then $\prod$ is an isolate vertex resolving partition of G. Therefore, $pd_{is}(C_{6k+5}) \leq 3$. But $pd_{is}(G) = 2$ if and only if $G = P_2$. Therefore, $pd_{is}(C_{6k+5}) = 3$.
Subcase (ii): k is odd.
Let $\prod$={{1,4,6,8,… ,3k+3, 3k+5, 3k+7, …., 6k + 4}, {2,5,7, 9,… , 3k+2, 3k+4, 3k+6,…, 6k+5}, {3}}. Then $\prod$ is an isolate vertex resolving partition of G. Therefore, $pd_{is}(C_{6k+5}) \leq 3$. But $pd_{is}(G) = 2$ if and only if $G = P_2$. Therefore, $pd_{is}(C_{6k+5}) = 3$. ★

Let **H** = {Connected graphs G of order $n \geq 3$ such that $H = G - \{v\}$ is a complete multipartite graph for some vertex v of G}. Let **F** = {G ∈ **H** satisfying one of the following properties (i) For every integer i, with $1 \leq i \leq k$, $a_i \in \{0, n_i\}$ and there are at least two distinct integers j, j′, $1 \leq j, j' \leq k$ for which $a_j = a_{j'} = 0$ (ii) There is exactly one integer j with $1 \leq j \leq k$ such that $0 < a_j < n_j$ and $a_j = n_j - 1$, for this integer j. Let **G** = {$G = G_n + 2k_2$ where G_n is a complete multipartite graph of order $n - 4 \geq 1$}.
In [3], Graphs of order n containing an induced complete multipartite subgraph of order n – 1 are used to characterize all connected graphs of order $n \geq 4$ with locating chromatic number n – 1.

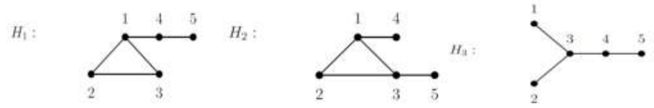
Theorem: $pd_{is}(G) = n - 1$ if and only if either G ∈ **G** or G is obtained from a complete multipartite graph H with k-partite sets $k \geq 2$ and joining a vertex v to all but one vertex of H and there exist two vertices in the partite set of H which contains the unique vertex non-adjacent with v.

Proof: Suppose $pd_{is}(G) = n - 1$. But $pd_{is}(G) \leq ipd(G) \leq pd(G)$. Therefore ipd (G) = n or n - 1. If ipd (G) = n, then G is a complete bipartite graph. Then $pd_{is}(G) = n$, a contradiction. Therefore, ipd(G) = n – 1. Therefore, G ∈ **H** ∪ **G**.

Conversely, suppose G ∈ **H** ∪ **G**. If G ∈ **G**, then $pd_{is}(G) = n - 1$. Suppose G ∈ **F**. If the defining property (i) for graphs in **F** is satisfied by G, then $pd_{is}(G) < n - 1$, a contradiction. Therefore G is a graph in **F** for which the condition (ii) is satisfied with the additional constraint that there exist 2 vertices in the partite set of H which contains the unique vertex non-adjacent with v. ★

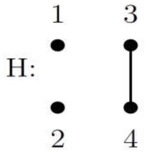
Bounds on Isolate Vertex Resolving Partition

Theorem: Let G be a connected graph of order $n \geq 5$ containing an induced subgraph H ∈ {$2K_1 \cup K_2$, $P_2 \cup P_3$, $P_2 \cup K_3$, P_5, C_5, $C_5 + e$, H_1, H_2, H_3} where

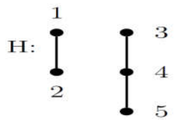


Then $pd_{is}(G) \leq n - 2$.

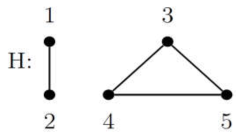
Proof: Suppose $H = 2K_1 \cup K_2$



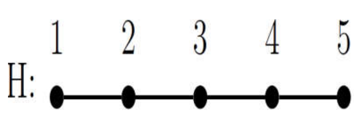
Let Π = {{1, 3}, {2, 4},…., {n}}.Then $c_\Pi(1) = (0, d_2, d_3, d_4,…,d_n)$, $c_\Pi(3) = (0,1,d'_3,d'_4,……,d'_n)$ $c_\Pi(2) = (d''_1,0,d''_3,d''_4,……,d''_n)$, $c_\Pi(4) = (1,0,d'''_3,d'''_4,……,d'''_n)$. Therefore, Π is an isolate vertex resolving partition. Therefore, $pd_{is}(G) \leq |\Pi| = n - 2$.
Let $H = P_2 \cup P_3$.



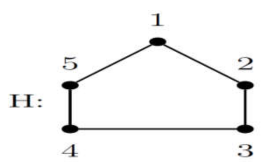
Let Π = {{1, 3}, {2, 5}, {4}, …, {n}}.Then $c_\Pi(1) = (0, 1, d_3, d_4, d_5, …, d_n)$, $c_\Pi(3) = (0,2,d'_3,d'_4,……,d'_n)$, $c_\Pi(2) = (1,0,d''_3,d''_4,……,d''_n)$, $c_\Pi(5) = (2,1,0,d'''_4,d'''_5,……,d'''_n)$. Therefore, Π is an isolate vertex resolving partition. Therefore, $pd_{is}(G) \leq |\Pi| = n - 2$.
Let $H = P_2 \cup K_3$.



Let Π = {{1, 3, 5}, {2}, {4},…., {n}}.Then $c_\Pi(1) = (0, 1, d_3, d_4, d_5, …d_n)$, $c_\Pi(3) = (0,d'_2,1,……,d'_n)$, $c_\Pi(5) = (0,d''_2,1,……,d''_n)$. Therefore Π is an isolate vertex resolving partition. Therefore $pd_{is}(G) \leq |\Pi| = n - 2$.

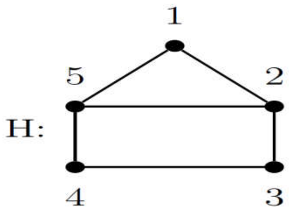


Let Π = {{1, 3}, {2, 5}, {4},…, {n}}.Then $c_\Pi(1) = (0, 1, 3, d_4,…,d_n)$, $c_\Pi(3) = (0,1,1,d''_4,……,d''_n)$, $c_\Pi(2) = (1,0,2,d''_4,……,d''_n)$, $c_\Pi(5) = (2,0,1,d'''_4,……,d'''_n)$. Therefore, Π is an isolate vertex resolving partition. Therefore, $pd_{is}(G) \leq |\Pi| = n - 2$.
Let $H = C_5$.

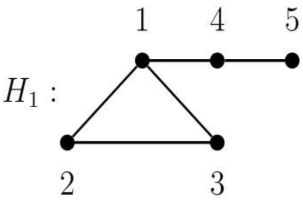


Let $\Pi = \{\{1, 3, 4\}, \{2\}, \{5\},\ldots,\{n\}\}$.Then $c_\Pi(1) = (0, 1, 1, d_4, \ldots, d_n)$, $c_\Pi(3) = (0,1,2,d'_4,\ldots\ldots,d'_n)$, $c_\Pi(4) = (0,2,1,d''_4,\ldots\ldots,d''_n)$. Therefore, Π is an isolate vertex resolving partition. Therefore, $pd_{is}(G) \le |\Pi| = n - 2$.

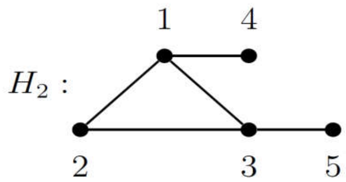
Let $H = C_5 + e$



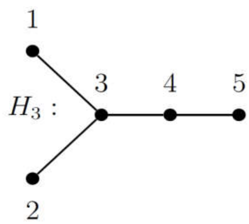
Let $H = C_5 + e$. Let $\Pi = \{\{1, 3, 4\}, \{2\}, \{5\},\ldots, \{n\}\}$.Then, $c_\Pi(1) = (0, 1, 1, d_4,\ldots, d_n)$, $c_\Pi(3) = (0,1,2,d'_4,\ldots\ldots,d'_n)$, $c_\Pi(4) = (0,2,1,d''_4,\ldots\ldots,d''_n)$ .Therefore, Π is an isolate vertex resolving partition. Therefore, $pd_{is}(G) \le |\Pi| = n - 2$.
Let $H = H_1$.



Let $\Pi = \{\{1, 2, 5\}, \{3\}, \{4\}, \ldots, \{n\}\}$.Then $c_\Pi(1) = (0, 1, 1, d_4,\ldots,d_n)$, $c_\Pi(2) = (0,1,2,d'_4,\ldots\ldots,d'_n)$, $c_\Pi(5) = (0,3,1,d''_4,\ldots\ldots,d''_n)$ .Therefore, Π is an isolate vertex resolving partition. Therefore, $pd_{is}(G) \le |\Pi| = n - 2$.
Let $H = H_2$.



Let $\Pi = \{\{1, 2, 5\}, \{3\}, \{4\},\ldots, \{n\}\}$.Then $c_\Pi(1) = (0, 1, 1, d_4, \ldots, d_n)$, $c_\Pi(2) = (0,1,2,d'_4,\ldots\ldots,d'_n)$, $c_\Pi(5) = (0, 3, 1, d''_4, \ldots, d''_n)$. Therefore, Π is an isolate vertex resolving partition. Therefore, $pd_{is}(G) \le |\Pi| = n - 2$.
Let $H = H_3$.



Let $\Pi = \{\{1, 3, 5\}, \{2\}, \{4\},\ldots, \{n\}\}$.Then $c_\Pi(1) = (0, 2, 2, d_4, \ldots,d_n)$, $c_\Pi(2) = (0,1,1,d'_4,\ldots\ldots,d'_n)$, $c_\Pi(5) = (0,3,1,d''_4,\ldots\ldots,d''_n)$. Therefore, Π is an isolate vertex resolving partition. Therefore, $pd_{is}(G) \le |\Pi| = n - 2$. ★

Definition: [3] Let G be a connected graph of order atleast three such that $H = G - v$ is a complete multipartite graph for some vertex v of G. Let $V_1, V_2, \ldots, V_k$, $k \ge 2$ denote the partite sets of H. Let $|V_i| = n_i$, $1 \le i \le k$ and let a_i, $(1 \le i \le k)$ denote the number of vertices in V_i which are adjacent in G with v. Define $\sigma(G)$ by $\sigma(G) = \sum_{i=1}^{k} \max\{a_i, n_i - a_i\}$.

Result: There are graphs with $G - v$ a complete multipartite graph for some $v \in V(G)$ such that $pd_{is}(G) = \sigma(G) + 1$. Let H be a complete multipartite graph with partite sets $V_1, V_2, \ldots,V_k$ and $|V_i| = n_i \ge 1$. Let $n_i \ge 2$ for atleast one i, $1 \le i \le k$. Add a new vertex v to H and make v adjacent with exactly one vertex of each V_i, $1 \le i \le k$. Let G be the resulting graph. Let $V_1, V_2, \ldots,V_t$ have cardinality 1 and the remaining partite sets have cardinality atleast 2. $\sigma(G) = 1 + 1 + 1+\ldots\ldots+1$ (t – times) + $\sum_{i=t+1}^{k} n_i - 1 = t + n_{t+1} + \ldots.+n_k - (k - t) = n - 1 - k + t$. Let $\Pi = \{u_{t+1}, \ldots,u_k, v\}, \{x\}\}$ where x runs over $V(G) - \{u_{i+1}, \ldots, u_k, v\}$. Clearly, Π is a minimum isolate vertex resolving partition of G. Therefore, $pd_{is}(G) = n - (k - t + 1) + 1 = n - k + t = \sigma(G) + 1$.

Lemma: Let G be a connected graph such that $G - v$ is a complete multipartite graph for some vertex $v \in V(G)$. Then $pd_{is}(G) \le \sigma(G) + 1$.

Proof: It has been proved in [3] that $ipd(G) \le \sigma(G) + 1$. But $pd_{is}(G) \le ipd(G)$.
Therefore $pd_{is}(G) \le \sigma(G) + 1$. ★

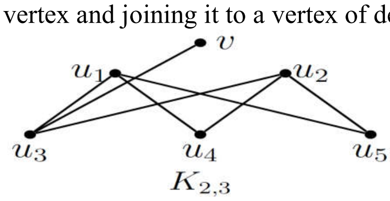
Lemma: Given a positive integer k, there exist a graph G such that $pd_{is}(G) = \sigma(G) - k$.

Proof: Let H be a complete multipartite graph with partite sets $V_1, V_2, \ldots, V_{k+2}$, $|V_i| \ge 2$ for all i. Add a vertex v to H and make it adjacent with exactly one vertex of H.
Let $|V_i| = n_i$ $(1 \le i \le k + 2)$. $\sigma(G) = n - 2$. Let $\Pi = \{\{v, u_{11}, u_{21}, \ldots, u_{k+2,1}\}$, singletons$\}$. Therefore, $\Pi = n -(k + 2 + 1) + 1 = n - k - 2$.

Suppose, $pd_{is}(G) \le n - k - 3$. Suppose Π' is a pd_{is} partition of G such that one of the sets in the partition is $\{v\}$. Then there exist one set of the partition containing two elements (namely the adjacent vertex of v and the non-adjacent vertex of v in the set if exist). Therefore, $\Pi' = 1 + 1 + n - 3 = n - 1$. Therefore $n - 1 \le n - k - 3$. $k \le -2$, a contradiction.
Suppose, one of the sets say S, of Π' contains v as well as other elements from H. Then S cannot contain the unique adjacent vertex of v in H. It can contain exactly one non-adjacent vertex from each of the partite sets. Therefore, $|S| \le 1 + k + 2 = k + 3$. Further the remaining sets of Π' must be singletons. Therefore, $|\Pi'| \ge 1 + n - (k + 3) = n - k - 2$. But $|\Pi'| \le n - k - 3$. Therefore, $n- k - 2 \le |\Pi'| \le n - k - 3$, a contradiction. Therefore, $pd_{is}(G) \ge n - k - 2$. Therefore, $pd_{is}(G) = n - k - 2 = \sigma - k$.

★

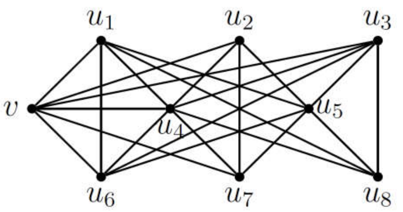
Illustration: Let G be obtained from $K_{2,3}$ by adding a new vertex and joining it to a vertex of degree 2 in $K_{2,3}$.



Let $\Pi = \{\{v, u_1, u_4\}, \{u_2\}, \{u_3\}, \{u_5\}\}$. Π is an isolate resolving partition of G. Therefore, $pd_{is} \le 4$. Suppose $pd_{is} = 3$.

Let $\Pi' = \{V_1, V_2, V_3\}$ be a pd_{is} – partition of G. Let $v \in V_1$ (say). V_1 can contain at most two elements one from the partite sets of $K_{2,3}$. The remaining elements which are atleast 3 in number must be accommodated in V_2 and V_3. Therefore, either V_2 or V_3 contains atleast two elements from $K_{2,3}$. Suppose V_2 contains atleast two elements from $K_{2,3}$. If $|V_2| = 3$, then $V_2 = \{u_3, u_4, u_5\}$. Then u_4 and u_5 cannot be resolved by V_1 and V_3. Therefore $|V_2| = 2$. Since elements of V_2 are resolved by V_1 or V_3, V_2 can contain only u_3 and u_4. If V_3 contains two elements then it should be u_1 and u_2, since V_3 has an isolate. But u_1 and u_2 cannot be resolved by any element. Therefore, V_3 contains one element. In this case, V_1 contains three elements. But V_1 can contain only v, u_1, u_4 a contradiction. (since $u_4 \in V_2$). Therefore, $pd_{is}(G) \neq 3$. $pd_{is}(G) \neq 1, 2$ (since $pd_{is}(G) = 1$ if and only if $G = K_1$, $pd_{is}(G) = 2$ if and only if $G = K_2$). Therefore, $pd_{is}(G) = 4$. $\sigma(G) = 4$. Therefore, $pd_{is}(G) = \sigma(G) = 4$.

Illustration: Let us consider the following graph G.



G

Now $\sigma(G) = 3 + 1 + 2 + 6$. There are two isolate vertex resolving partition of G namely
$\Pi_1 = \{\{v, u_5, u_8\}, \{u_1\}, \{u_2\}, \{u_3\}, \{u_4\}, \{u_6\}, \{u_7\}\}$ and $\Pi_2 = \{\{v\}, \{u_4, u_5\}, \{u_7, u_8\}, \{u_1\}, \{u_2\},\{u_3\}, \{u_6\}\}$.Therefore $|\Pi_1| = |\Pi_2| = 7$. It can easily verified that $pd_{is}(G) = 7$. That is $pd_{is}(G) = \sigma(G) + 1$.

Illustration: Let H be a complete multipartite graph. Add a vertex v to H and join v to every vertex of H. Let G be the resulting graph. The graph G is a complete multipartite graph and therefore $pd_{is}(G) = |V(G)|$ and $\sigma(G) = |V(H)|$. Therefore, $pd_{is}(G) = \sigma(G) + 1$.

Theorem: Let H be a complete multipartite graph with k-partite sets, $k \geq 2$. Join a vertex v to H and join v to all but one vertex of H. There exist atleast two vertices in the partite set which contains a non-adjacent vertex of v. Then $pd_{is}(G) = n - 1$.

Proof: Let $\Pi = \{\{v, u_{11}\}, \text{singletons}\}$, where u_{11} is the unique vertex not adjacent with v. $|\Pi| = n - 1$.Therefore, $pd_{is}(G) \leq n - 1$. In any isolate vertex resolving partition of G, the set containing v, cannot contain two more elements. Also any set in the partition other than the set containing v cannot contain two elements if the set containing v contains two elements. Therefore, there exist exactly one set in the partition containing two elements. Therefore, $pd_{is}(G) = n - 1$. ★

Theorem. Let G be a graph obtained from a complete multipartite graph H by adding a vertex (say) v. Let $V_1, V_2, ..., V_k$ be the partitite set of H with $|V_i| = n_i$ $(1 \leq i \leq k)$. Let v be joined with a_i, vertices of V_i $(1 \leq i \leq k)$. Let $a_i = 0$ for atleast two partite sets $a_i = n_i$, for the remaining partite sets. When $a_i = 0$, then the partite set contains atleast two elements. Then $pd_{is}(G) < n - 1$.

Proof: Let Without loss of generality $a_1 = a_2 = ... a_t = 0$, $t \geq 2$ and $a_i = n_i$, $t + 1 \leq i \leq k$.

Then $\Pi = \{\{v, u_{11}, u_{21}\}, \text{singletons}\}$ is an isolate vertex resolving partition of G, where $u_{11} \in V_1$ and $u_{21} \in V_2$. Therefore, $pd_{is}(G) \leq |\Pi| = n - 2 < n - 1$. ★

Lemma 4.11. Let G be a connected graph of the form $H + 2K_2$, where H is a complete multipartite graph of order $n - 4 \geq 1$. Then $pd_{is}(G) = n - 1$.

Proof: Let $V(2K_2) = \{\{u_1, u_2, u_3, u_4\}\}$, where u_1 and u_2 are adjacent and u_3 and u_4 are adjacent. Let $\Pi = \{\{u_1, u_3\}, \text{singletons}\}$. Clearly, Π is an isolate vertex resolving partition of G. Therefore, $pd_{is}(G) \leq n - 1$. Suppose, $pd_{is}(G) \leq n - 2$.Then there exist a pd_{is} partition $\Pi_1 = \{\{V_1, V_2, \ldots, V_k\}\}$, $k \leq n - 2$. Any V_i cannot contain two vertices of H. Therefore, vertices of H must appear as singletons. Suppose V_1 contains u_1, u_3, u_4. Since V_1 has an isolate, V_1 cannot contain any vertex of H. Therefore $V_1 = \{u_1, u_3, u_4\}$. But $c_{\Pi 1}(u_3) = c_{\Pi 1}(u_4)$, a contradiction. Therefore, either V_1 contains u_1 and u_3 or u_1 and u_4 or u_2 and u_3 or u_2 and u_4. Therefore, $|V_1| = 2$. Suppose $V_1 = \{u_1, u_3\}$ and $V_2 = \{u_2, u_4\}$. Therefore, $c_{\Pi 1}(u_1) = c_{\Pi 1}(u_3)$, a contradiction. Therefore, only one of V_1, V_2 is a doubleton. Therefore, $|\Pi_1| = n - 1$, a contradiction. Therefore, $pd_{is}(G) = n - 1$. ★

Lemma: Suppose $G = \langle V_1\rangle + \langle V_2\rangle$. If $\langle V_1\rangle$ and $\langle V_2\rangle$ are connected and diameter of either one or both of $\langle V_1\rangle$ and $\langle V_2\rangle$ is 3, then $pd_{is}(G) = n - 1$ if and only if any P_4 in $\langle V_2\rangle$ does not contain a pendent vertex attached with an internal vertex of P_4 and $\langle V_2\rangle$ does not contain an induced subgraph H which is obtained from a complete graph H_1 by attaching two pendent vertices one at each two vertices of H_1 and removing one or more edges at a vertex other than the vertices at which a pendent is attached, leaving at least one edge.

Proof: Suppose $G = \langle V_1\rangle + \langle V_2\rangle$. Let $\langle V_1\rangle$ and $\langle V_2\rangle$ be connected and let diameter of either one or both of $\langle V_1\rangle$ and $\langle V_2\rangle$ be 3. Let $diam(\langle V_2) \rangle = 3$. Clearly, $pd_{is}(G) \leq n - 1$.
Suppose, P_4 in $\langle V_2\rangle$ contains a pendent vertex attached with an internal vertex of P_4. Let x_1, x_2, x_3, x_4 be the vertices of P_4 and y be a pendent attached with x_2. Let $\Pi = \{\{x_4, y\}, \{x_1, x_3\}, \{x_2\}, \text{all other singletons}\}$. Then $c_\Pi(x_1) = (2, 0, 1, ……)$, $c_\Pi(x_2) = (1, 1, 0,……)$, $c_\Pi(x_3) = (1, 0, 1,…….)$, $c_\Pi(x_4) = (0, 0, 2,……)$, $c_\Pi(y) = (0, 2, 1,……)$.
Therefore, $pd_{is}(G) \leq n - 2$.
If P_4 in $\langle V_2\rangle$ does not contain a pendent vertex attached with an internal vertex of P_4. Then, $pd_{is}(G) \geq n - 2$.

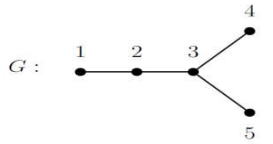
Let x_1, x_2, x_3, x_4 be a diametrical path of $\langle V_2\rangle$. Let $\Pi = \{\{x_1, x_3\}, \text{all other singletons}\}$ Then x_1, x_3 are resolved by x_4. Suppose $pd_{is}(G) \leq n - 2$. Suppose x, y, z belong to V_2 such that $\langle \{x, y, z\}\rangle$ is not connected. Let $\Pi = \{\{x, y, z\}, \text{all other singletons}\}$. Suppose x and y are adjacent and z is not adjacent with x, as well as y. Then d(x, z) or d(y, z) = 2. Suppose d(y, z) = 2. Let y, z_1, z be the path between y and z. Then y and z are at equal distance from any vertex other than x. Therefore, Π is not resolving. Suppose $x_1, x_2, x_3, x_4 \in V_2$ such that x_1 and x_3 are independent and x_2 and x_4 are independent. Then V_2 contains an induced subgraph H which is obtained from a complete graph H_1 by attaching two pendent vertices one each a two vertices of H_1 and removing one or more edges at a vertex other than vertices at which a pendent is attached, leaving atleast one edge. Then there exist an isolate vertex resolving partition Π such that Π contains two doubletones.

Then $pd_{is}(G) \leq n - 2$. Therefore, if G satisfies the hypothesis then $pd_{is}(G) = n - 1$.

Conversely, $pd_{is}(G) = n - 1$. Then clearly the conditions are satisfied. ★

Result: $pd_{is}(G) \leq n - 2$ if G is a double star $D_{r, s}$ where $r, s \geq 2$.

Proof: When r = 1, s = 2 we have



$\Pi = \{\{1, 4\}, \{2, 5\}, \{3\}\}$.Now, $c_\Pi(1) = (0, 1, 2)$, $c_\Pi(4) = (0, 2, 1)$, $c_\Pi(2) = (1, 0, 1)$, $c_\Pi(5) = (2, 0, 1)$. Therefore, $pd_{is}(G) \leq 3$.
Let r and s ≥ 2. Let $u_1, u_2, \ldots, u_r$ be the pendents at the center u and $v_1, v_2, \ldots, v_s$ be the pendents at the centre v. Then $\Pi = \{\{u_1, v_1\}, \{u_2, v_2\}, \{x_i\}\}$, where $3 \leq i \leq s + r - 2$ is an isolate vertex resolving partition. Therefore, $pd_{is}(G) \leq n - 2$.

Lemma: Let G be a connected graph with order greater than or equal to 4. Let u_1, u_2, u_3, u_4 be four vertices of G such that u_1, u_2 are non-adjacent, u_3 and u_4 are non-adjacent and there exist a vertex v, whose distances from u_1 and u_2 are not equal and there exist a vertex w, whose distance from u_3 and u_4 are not equal. Then $pd_{is}(G) \leq n - 2$.

Proof: Let $\Pi = \{\{u_1, u_2\}, \{u_3, u_4\}, \{v\}, \{w\}, \text{singletons}\}$. v resolves u_1 and $u_2$and w resolves u_3 and u_4. Therefore Π is an isolate vertex resolving partition of G. Therefore, $pd_{is}(G) \leq n - 2$. ★

Lemma: Let G be a connected graph with order greater than or equal to 4. Let u_1, u_2, u_3 be three vertices such that $< \{u_1, u_2, u_3\} >$ is disconnected. If there exist vertices v_1,v_2, v_3 such that $d(u_1, v_1) \neq d(u_2, v_1)$, $d(u_2, v_2) \neq d(u_3, v_2)$ and $d(u_1, v_3) \neq d(u_3, v_3)$, then $pd_{is}(G) \leq n - 2$.
Proof. Obvious. ★

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