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Research Article

FINSLER HYPERSURFACE ADMITTING TORSE - FORMING VECTOR FIELD

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A B S T R A C T

In this paper we investigate Torse – forming infinitesimal transformation of Cartan's curvature on FinslerianHypersurface. Further, we obtain the Cartan curvature generate non-flat space under a Torse - forming infinitesimal transformation...

INTRODUCTION

Finsler geometry is a kind of differential geometry and byoriginated P. Finsler (1894-1970) in 1918.It is usually considered as a generalization of the Riemannian geometry. In fact, B.Riemann, inhis lecture in 1854, already suggested a possibility of studyingmore general geometry than Riemannian geometry, but he said thegeometrical meaning of quantities appearing in such a generalized space will not be clear and it can not produce any contribution tothe geometry.Consquently all people had neglected for about 60 yearsto study such ageometry.

Differential geometry of the total space of a manifold's tangent bundle has its roots in various problems like Differential Equations, Calculus of Variations, Mechanics, Theoretical Physics and Biology. Nowadays, it is a distinct domain of differential geometry and has important applications in the theory of physical fields and special problems from Mathematical Biology. This significance, has led to the creation of new concepts and geometric structures, which are specific to TM, such as systems of Second Order Differential Equations (SODE), metric structures, semi sprays and nonlinear connections. Actually, investigating these concepts can be regarded as a powerful device for the study of the geometric properties of the tangent bundle.

The theory of a subspace of a Fisler space has been introduced by H.Rund [2]. As an m- dimensional subspace F^m of Finsler space F^n represented parametrically by the set of equation $x^i = x^i(u^\alpha)$.

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The authors G.M. Brown, Moor, C.Shibata, M.Matsumoto B.Y. C hen, C.S. Bagewadi, L.M.Abatangelo, Dragomir and S.Hojo have studied different properties of subspaces of Finsler, Kahler and Riemannian spaces. The authors S.K.Narasimhamurthy and C.S.Bagewadi have studied and published C- Conformal special Finsler spaces admitting a parallel vector field "and "Infinitesimal C – Conformal motions of special Finsler spaces etc..

Torse-forming infinitesimal transformation $\bar{x}^i = x^i + v^i(x)dt$, ${V_{/j}}^i = v^i\mu_j + \alpha\delta^i_j$ and torse – forming curvature inheritance in a Finsler space have been studied by Mishra and Lodhi.

Let us consider an n-dimensional an affinely connected Finsler space F^n with symmetric connection parameter $\Gamma^{*i}_{jk}(x,\dot{x})$.The covariant derivative of $T^i_j(x,\dot{x})$ with respect to xk in the sense of Cartan's is given by

$$T^i_{j/k} = \partial_k T^i_j - \dot{\partial}_m T^i_j G^m_k + T^m_j \Gamma^{*i}_{mk} - T^i_m \Gamma^{*m}_{jk} \quad (1)$$

$$T^i_{l/j} = \partial_j T^i_l + T^r_l G^i_{rj} - \dot{T}^m_j \Gamma^{*i}_{mk} - T^i_r G^r_{lj} \quad (2)$$

$$T^i_{l/j} = \partial_j T^i_l + T^r_l V^i_{rj} - T^i_r V^r_{lj} \quad (3)$$

In (1.2) /j denote the h-covariant differentiation and in (3) /j denote the v- covariant differentiation.

Cartan's connection coefficient $\Gamma^{*i}_{jk}(x,\dot{x})$ satisfy the following relations

$$\dot{\partial}_h \Gamma^{*i}_{jk}\,\dot{x}^h = 0 \quad (4)$$

$$\dot{\partial}_h \Gamma^{*i}_{jk}\,\dot{x}^h = \dot{\partial}_j \Gamma^{*i}_{hk} \quad (5)$$

These notations involving commutation formula of Cartan's covariant derivative

$$2T^i_{j/[hk]} = -\dot{\partial}_r T^i_j K^r_{hk} + T^s_j K^i_{shk} - T^i_s K^s_{jhk} \quad (6)$$

Where,

$$K^i_{jhk}(x,\dot{x}) = 2\{\partial_{[k}\Gamma^{*i}_{j]h} + \dot{\partial}_s \Gamma^{*i}_{h[j} G^s_{k]j} + \Gamma^{*s}_{h[j}\Gamma^{*i}_{k]s}\} \quad (7)$$

The commutation formula for a tensor T^i_j we show the role of curvature tensor and torsion tensor as follows

$$T^i_{l/k_{/j}} - T^i_{l/j_{/k}} = T^h_l R^i_{hkj} - T^i_{l/h} R^h_{kj} \quad (8)$$

Where R^i_{hkj} is called Cartan third curvature tensor.

$$T^i_{l/k_{/j}} - T^i_{l/j_{/k}} = T^h_l S^i_{hkj} - T^i_h S^h_{lkj} \quad (9)$$

Where S^i_{hkj} is called Cartan first curvature tensor.

Hypersurface F^{n-1} of the Finsler SPACE F^n

Finsler hyper surface $F^{n-1} = (M^{n-1}, L(u,v))$ of a Finsler space $F^n = M^n, L(x,y))$ $(n \geq 4)$ may parametrically represented by the equation $x^i = x^i(u^\alpha)$, $(\alpha = 1,2,\dots n-1)$, where u^α) an the Gaussian coordinates of F^{n-1}. It will be assumed throughout the paper that the functions x^i are at least of class C^3 and the matrix of the projection parameters $B^i_\alpha = \frac{\partial x^i}{\partial u^\alpha}$ has rank m.

The fundamental metric tensor $g_{\alpha\beta}$ and Cartan's C- tensor $C_{\alpha\beta\gamma}$ of F^{n-1} are given by

1. $g_{\alpha\beta}(u,v) = g_{ij}(x,\dot{x}) B^i_\alpha B^j_\beta$
2. $C_{\alpha\beta\gamma} = C_{ijk} B^i_\alpha B^j_\beta B^k_\gamma$

The following notations will be used here

$$B^i_\alpha = \frac{\partial x^i}{\partial u^\alpha}, \quad B^i_{\alpha\beta} = \frac{\partial^2 x^i}{\partial u^\alpha \partial u^\beta}, B^j_{\alpha\beta} = v^\alpha B^i_{\alpha\beta}, B^{ijk\dots}_{\alpha\beta\gamma\dots} = B^i_\alpha, B^j_\beta, B^k_\gamma$$

1. $g^{\alpha\beta} = g^{ij} B^{\alpha\beta}_{ij}$
2. $B^\alpha_i = g^{\alpha\beta} g_{ij} B^j_\beta$
3. $C_\alpha = B^i_\alpha C_i$, $\quad C^\alpha = B^\alpha_i C^i$
4. $C^\alpha_{\beta\gamma} = B^\alpha_i C^i_{jk} B^{jk}_{\beta\gamma}$
5. $h\beta_{\alpha\beta} = g_{\alpha\beta} - l_\alpha l_\beta$ and $\quad h_{\alpha\beta} = h_{ij} B^{ij}_{\alpha\beta}$
6. $l_\alpha = B^i_\alpha l_i$

Finsler Hypersurface Admitting Torse-Forming Vector Field

Let F^n be an Fnisler space with a fundamental function $L(x,y)$

Where $y = \dot{x}$ and equipped with the Cartan connection $C\Gamma = (F^i_{jk}, N^i_k, C^i_{jk})$.

Definition: A vector field V_i in F^n is called torse-forming if it satisfies the condition

$$V^i_{/j} = V^i_j + \alpha\delta^i_j \quad (10)$$

Where α is a non zero scalar function and j being any non null vector field. The scalar function appearing in (10) is a point function and satisfy following

1. $\dot{\partial}_j(\alpha) = 0$
2. $\alpha_{/h} = \alpha_h$ (11)

Definition: We consider an infinitesimal transformation of the form

$$\bar{x}^i = x^i + v^i(x)dt, \quad (12)$$

$$V_{/j}{}^i = v^i\mu_j + \alpha\delta^i_j \quad (13)$$

Such a transformation is called a torse- forming infinitesimal transformation.

Let F^{n-1} be an hypersurface of a Finsler space F^n and define a vector field $X^\alpha = X^i B^\alpha_i$ in F^{n-1}.

Transvecting equation (3.1) by $B^\alpha_i B^j_\beta$

$$B^\alpha_i B^j_\beta V^i_{/j} = B^\alpha_i B^j_\beta V^i_j + B^\alpha_i B^j_\beta \alpha\delta^i_j \quad V^\alpha_{/\beta} = V^\alpha_\beta + \alpha\delta^\alpha_\beta \quad (14)$$

Differentiating (3.1) covariantly with respect to γ we get

$$V^\alpha_{/\beta/_\gamma} = V^\alpha_{\gamma\beta} + V^\alpha_{\beta/\gamma} + \alpha_\gamma\delta^\alpha_\beta \quad (15)$$

Or

$$V^\alpha_{/\beta/_\gamma} = (V^\alpha_\gamma + \alpha\delta^\alpha_\gamma)_\beta + V^\alpha_{\beta/\gamma} + \alpha_{/\gamma}\delta^\alpha_\beta \quad (16)$$

$$V^\alpha_{/\beta/_\gamma} = V^\alpha{}_{\gamma\beta} + \alpha_\beta\delta^\alpha_\gamma + V^\alpha{}_{\beta/\gamma} + \alpha_{/\gamma\beta}\delta^\alpha_\beta \quad (17)$$

Interchanging γ and β in equation (3.6) we get

$$V^\alpha_{/\gamma/_\beta} = V^\alpha{}_{\beta\gamma} + \alpha_\gamma\delta^\alpha_\beta + V^\alpha{}_{\gamma/\beta} + \alpha_{/\beta}\delta^\alpha_\gamma \quad (18)$$

Subtracting (3.6) and (3.7) we get

$$V^\alpha_{/\beta/_\gamma} - V^\alpha_{/\gamma/_\beta} = \alpha({}_\beta\,\delta^\alpha_\gamma \; -{}_\gamma\delta^\alpha_\beta) + V^\alpha{}_{(\beta/\gamma-\gamma/\beta)} + \alpha_{/\gamma}\delta^\alpha_\beta - \alpha_{/\beta}\delta^\alpha_\beta \quad (19)$$

Using equation (3.2) (b) and in (3.8) we get

$$V^\alpha_{/\beta/_\gamma} - V^\alpha_{/\gamma/_\beta} = \alpha({}_\beta\,\delta^\alpha_\gamma \; -{}_\gamma\delta^\alpha_\beta) + V^\alpha{}_{(\beta/\gamma-\gamma/\beta)} + \alpha_\gamma\delta^\alpha_\beta - \alpha_\beta\delta^\alpha_\gamma \quad (20)$$

We obtain to the commutation formula (1.6)

$$V^\alpha_{/\beta/_\gamma} - V^\alpha_{/\gamma/_\beta} = \dot{\partial}_\rho\,\gamma V^\alpha K^\rho_{\beta\gamma} + V^\eta\, V^\alpha_{\eta\beta\gamma} \quad (21)$$

In view of (3.9) and (3.10) we get

$$\alpha({}_\beta\,\delta^\alpha_\gamma \; -{}_\gamma\delta^\alpha_\beta) + V^\alpha{}_{(\beta/\gamma-\gamma/\beta)} + \alpha_\gamma\delta^\alpha_\beta - \alpha_\beta\delta^\alpha_\beta = -\dot{\partial}_\rho\,\gamma V^\alpha K^\rho_{\beta\gamma} + V^\eta\, K^\alpha_{\eta\beta\gamma} \quad (22)$$

Or

$$V^\eta K^\alpha_{\eta\beta\gamma} = \dot{\partial}_\eta V^\alpha\, K^\rho_{\beta\gamma} + \alpha({}_\beta\,\delta^\alpha_\gamma \; -{}_\gamma\delta^\alpha_\beta) + V^\alpha{}_{(\beta/\gamma-\gamma/\beta)} + \alpha_\gamma\delta^\alpha_\beta - \alpha_\beta\delta^\alpha_\gamma \quad (23)$$

Or

$$V^\eta K^\alpha_{\eta\beta\gamma} = \dot{\partial}_\rho V^\alpha\, K^\rho_{\beta\gamma} + \alpha A + \beta_\gamma V^\alpha + C \quad (24)$$

Where a) $\quad A = ({}_\beta\,\delta^\alpha_\gamma \; -{}_\gamma\delta^\alpha_\beta)$

1. $\beta_\gamma = \beta_{/\gamma} - \gamma/_\beta$
2. $\alpha_\gamma\delta^\alpha_\beta - \alpha_\beta\delta^\alpha_\gamma$

Thus we have

Theorem 1: In Finsler Hypersurface F^{n-1} the torse-forming transformation generates non- flat space.

In view of (3.13)(b) if βγ is symmetric, then we have

$$\beta_\gamma = \beta_{/\gamma} - \gamma/_\beta = 0 \quad (25)$$

Introducing (3.14) in (3.12) we get

$$V^\eta K^\alpha_{\eta\beta\gamma} = \dot{\partial}_\eta V^\alpha\, K^\rho_{\beta\gamma} + \alpha({}_\beta\,\delta^\alpha_\gamma \; -{}_\gamma\delta^\alpha_\beta) + \alpha_\gamma\delta^\alpha_\beta - \alpha_\beta\delta^\alpha_\gamma \quad (26)$$

Contracting (3.15) with respect to the indices α and γ we get

$$V^\eta K_{\eta\beta} = \dot{\partial}_\rho V^\alpha\, K^\rho_{\beta\gamma} + \alpha(n_{\beta-\beta}) + \alpha_\beta - n\alpha_\beta$$

$$\beta = \frac{V^{\eta}K_{\eta\beta} - \partial_{\rho}V^{\alpha}\,K^{\rho}_{\beta\alpha} + (n-1)\alpha_{\beta}}{(n-1)\alpha} \qquad (27)$$

Hence we state

Theorem 2: In a Finsler hypersurface F^{n-1} equipped with torse –forming transformation in view of Cartan curvature tensor $K^{\alpha}_{\delta\beta\gamma}$ this is non null vector field β is given by (3.16) if βγ is symmetry

Torse-forming curvature $R^{\alpha}_{\delta\beta\gamma}$ on F^{n-1}.

Introducing equation (3.4) in (1.8) we get

$$V^{\alpha}_{/\beta/\gamma} - V^{\alpha}_{/\gamma/\beta} = V^{\alpha}_{/\delta}\,R^{\delta}_{\gamma\beta} - V^{\delta}R^{\alpha}_{\delta\gamma\beta} \qquad (28)$$

Using equation (3.9) in (4.1) we get

$$\alpha(_{\beta}\,\delta^{\alpha}_{\gamma} - _{\gamma}\delta^{\alpha}_{\beta}) + V^{\alpha}{}_{(\beta/\gamma - \gamma/\beta)} + \alpha_{\gamma}\delta^{\alpha}_{\beta} - \alpha_{\beta}\delta^{\alpha}_{\gamma} = V^{\alpha}_{/\delta}R^{\delta}_{\gamma\beta} - V^{\delta}R^{\alpha}_{\delta\gamma\beta} \qquad (29)$$

In view of (3.4) and (4.2) we get

$$\alpha(_{\beta}\,\delta^{\alpha}_{\gamma} - _{\gamma}\delta^{\alpha}_{\beta}) + V^{\alpha}{}_{(\frac{\beta}{\gamma} - \frac{\gamma}{\beta})} + \alpha_{\gamma}\delta^{\alpha}_{\beta} - \alpha_{\beta}\delta^{\alpha}_{\gamma} = (V^{\alpha}{}_{\delta} + \alpha\,\delta^{\alpha}_{\delta})R^{\delta}_{\gamma\beta} - V^{\delta}R^{\alpha}_{\delta\gamma\beta} \qquad (30)$$

$$\alpha(_{\beta}\,\delta^{\alpha}_{\gamma} - _{\gamma}\delta^{\alpha}_{\beta}) + V^{\alpha}{}_{(\beta/\gamma - \gamma/\beta)} + \alpha_{\gamma}\delta^{\alpha}_{\beta} - \alpha_{\beta}\delta^{\alpha}_{\gamma} = (V^{\alpha}{}_{\delta} + R^{\alpha}_{\delta})\,R^{\delta}_{\gamma\beta} - V^{\delta}R^{\alpha}_{\delta\gamma\beta} \qquad (31)$$

$$V^{\delta}R^{\alpha}_{\delta\gamma\beta} = V^{\alpha}{}_{\delta}\,R^{\delta}{}_{\gamma\beta} + \alpha R^{\alpha}_{\gamma\beta} + \alpha_{\gamma}\delta^{\alpha}_{\beta} - \alpha_{\beta}\delta^{\alpha}_{\gamma} + V^{\alpha}_{\gamma/\beta} - V^{\alpha}_{\beta/\gamma} + \alpha_{\beta}\delta^{\alpha}_{\gamma} - \alpha_{\gamma}\delta^{\alpha}_{\beta} \qquad (32)$$

Hence we state

***Theorem 3*:** In a Finsler hypersurface F^{n-1} equipped with torse-forming transformation (2.4) the torse-forming vector v_i satisfies the relation (30)

Contracting the (4.5) equation with respect to the indices α and β we get

$$V^{\delta}R^{\alpha}_{\delta\gamma\alpha} = V^{\alpha}{}_{\delta}\,R^{\delta}{}_{\gamma\alpha} + \alpha R^{\alpha}_{\gamma\alpha} + \alpha_{\gamma}\delta^{\alpha}_{\alpha} - \alpha_{\alpha}\delta^{\alpha}_{\gamma} + V^{\alpha}_{\gamma/\alpha} - V^{\alpha}_{\alpha/\gamma} + \alpha_{\alpha}\delta^{\alpha}_{\gamma} - \alpha_{\gamma}\delta^{\alpha}_{\alpha} \qquad (33)$$

Or $V^{\delta}R^{\alpha}_{\delta\gamma\alpha} = \delta(V^{\alpha}R^{\delta}_{\gamma\alpha}) + \alpha R^{\alpha}_{\gamma\alpha} + \alpha(n_{\gamma-\gamma}) + V^{\alpha}_{\gamma/\alpha} - V^{\alpha}_{\alpha/\gamma} + \alpha_{\gamma}\delta_{\alpha\gamma}$ (34)

Or

$$V^{\delta}R^{\alpha}_{\delta\gamma\alpha} = \delta(V^{\alpha}R^{\delta}_{\gamma\alpha}) + \alpha R^{\alpha}_{\gamma\alpha} + \alpha(n-1)_{\gamma} + V^{\alpha}_{\gamma/\alpha} - V^{\alpha}_{\alpha/\gamma} + (n-1)\alpha_{\gamma} \qquad (35)$$

$$V^{\alpha}_{\frac{\gamma}{\alpha}} - V^{\alpha}_{\frac{\alpha}{\gamma}} + V^{\alpha}_{\delta}R^{\delta}_{\gamma\alpha} + \alpha(n-1)_{\gamma} + \alpha R^{\alpha}_{\gamma\alpha} - (n-1)\alpha_{\gamma} - V^{\delta}R_{\delta\gamma} = 0 \qquad (36)$$

Thus we state

Theorem 4: In Finsler hypersurface F^{n-1} equipped with torse-forming transformation (3.4) the non null vector field β satisfies differential equation (4.9).

Introducing (3.4) in commutation formula (1.9) we get

$$V^{\alpha}_{/\gamma/\beta} - V^{\alpha}_{/\beta/\gamma} = V^{\delta}S^{\alpha}_{\delta\gamma\beta} \qquad (37)$$

Or

$$V^{\alpha}_{/\beta/\gamma} - V^{\alpha}_{/\gamma/\beta} = -V^{\delta}S^{\alpha}_{\delta\gamma\beta} \qquad (38)$$

Using equation (3.8) in (4.11)

$$\alpha(_{\beta}\,\delta^{\alpha}_{\gamma} - _{\gamma}\delta^{\alpha}_{\beta}) + V^{\alpha}{}_{(\beta/\gamma - \gamma/\beta)} + \alpha_{\gamma}\delta^{\alpha}_{\beta} - \alpha_{\beta}\delta^{\alpha}_{\gamma} = -V^{\delta}S^{\alpha}_{\delta\gamma\beta} \qquad (39)$$

$$V^{\delta}S^{\alpha}_{\delta\gamma\beta} = \alpha(_{\gamma}\,\delta^{\alpha}_{\beta} - _{\beta}\delta^{\alpha}_{\gamma}) + V^{\alpha}{}_{(\gamma/\beta - \beta/\gamma)} + \alpha_{\beta}\delta^{\alpha}_{\gamma} - \alpha_{\gamma}\delta^{\alpha}_{\beta} \qquad (40)$$

Hence we state

Theorem 5: In Finsler hypersurface F^{n-1} the cartan first curvature tensor $S^{\alpha}_{\delta\gamma\beta}$ with torse-forming transformation (3.4) generates non flat space. Contracting α and β in equation (4.13) we get

$$V^{\delta}S^{\alpha}_{\delta\gamma\beta} = \alpha(_{\gamma}\,\delta^{\alpha}_{\alpha} - _{\alpha}\delta^{\alpha}_{\gamma}) + V^{\alpha}{}_{(\gamma/\alpha - \alpha/\gamma)} + \alpha_{\alpha}\delta^{\alpha}_{\gamma} - \alpha_{\gamma}\delta^{\alpha}_{\alpha} \qquad (41)$$

Or

$$V^{\delta}S^{\alpha}_{\delta\gamma\alpha} + \alpha(\delta_{\gamma-\gamma}) + V^{\alpha}_{\gamma/\alpha} - V^{\alpha}_{\alpha/\gamma} + \alpha_{\gamma} - n\alpha_{\gamma} \qquad (42)$$

$$V^{\delta}S_{\delta\gamma} + (n-1)\alpha_{\gamma} - V^{\alpha}_{\gamma/\alpha} - V^{\alpha}_{\alpha/\gamma} - \alpha(n_{\gamma} - \gamma) = 0 \qquad (43)$$

$$V^{\delta}S_{\delta\gamma} - V^{\alpha}_{\gamma/\alpha} + V^{\alpha}_{\alpha/\gamma} - \alpha(n-1)_{\gamma} + (n-1)\alpha_{\gamma} = 0 \qquad (44)$$

Hence we state

Theorem 6: In Finsler hypersurface F^{n-1} equipped with torse-forming transformation (3.4) the non null vector field β satisfies the relation (4.17).

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