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Research Article

A STUDY ON SPANNING 2-CONNECTED SUBGRAPHS IN RECTANGULAR GRID GRAPHS

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A R T I C L E I N F O

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A B S T R A C T

Graph theory is one of the mainly essential branches in mathematics and is helpful to locate relations between various associated systems. By way of the appearance of systems as the Grid, it is very important to study the path finding and partitioning problems to understand the analogous capabilities of assorted systems. This premise helps in grid computing such as advance efficacy by designating data and distribute it globally. In this paper we study the spanning 2- connected subgraphs of rectangular grid graphs.

INTRODUCTION

We introduce the classes of grid graphs which we call truncated rectangular grid graphs.

For $s \geq 3, t \geq 3, 0 \leq k \leq \min\{s-2, t-2\}$ and $0 \leq l \leq \min\{s-2, t-2\}$. we define a 1 – corner truncated rectangular grid graph $R(s,t)^{-1(k,l)}$ as the subgraph obtained from $R(s,t)$ by deleting $k \times l$ vertices from one corner in $V(s,t)$ together with their incident edges in a natural drawing. For illustration, consider $R(13,11)^{-1(3,2)}$ in figure.2.1(a). For $s \geq 6, t \geq 6, 1 \leq k \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ and $1 \leq l \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ we define a 2 – corner truncated rectangular grid graph $R\,(s,t)^{-2(k,l)}$ as the subgraph obtained from $R(s,t)$ by deleting $k \times l$ vertices from two opposite corners in $V(s,t)$ together with their incident edges in a natural drawing. For illustration, consider $R(13,11)^{-2(3,2)}$ in Figure 2.1 (b).

For $s \geq 6, t \geq 6, 1 \leq k \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ and $1 \leq l \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ we define a 3 – corner truncated rectangular grid graph $R(s,t)^{-3(k,l)}$ as the subgraph obtained from $R(s,t)$ by deleting $k \times l$ vertices from three corners in $V(s,t)$ together with their incident edges in a natural drawing. For illustration, consider $R(13,11)^{-3(3,2)}$ in Figure 2.1 (c). For $s \geq 6, t \geq 6, 1 \leq k \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ and $1 \leq l \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ we define a 4 – corner truncated rectangular grid graph $R(s,t)^{-4(k,l)}$ as the subgraph obtained from $R(s,t)$ by deleting $k \times l$ vertices from three corners in $V(s,t)$ together with their incident edges in a natural drawing. For illustration, consider $R(13,11)^{-4(3,2)}$ in Figure 1 (d).

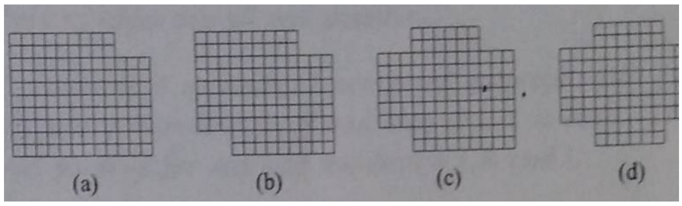

Figure 1 Truncated rectangular grid graphs

(a) $R(13,11)^{-1(3,2)}$
(b) $R(13,11)^{-2(3,2)}$
(c) $R(13,11)^{-3(3,2)}$
(d) $R(13,11)^{-4(3,2)}$

Spanning 2- connected subgraphs with a minimum number of edges for the 1-corner truncated rectangular grid graph $R(s,t)^{-1(k,k)}$ and for the 4- corner truncated rectangular grid graph $R(s,t)^{-4(k,k)}$ were studied in [2]. Subsequently, in [3] these results were generalized to $R(s,t)^{-1(k,l)}$ and $R(s,t)^{-4(k,l)}$. In [5] we considered the other truncated rectangular grid graphs. We summarize the results in [4] and [5] in the Theorem [1]. It characterizes which of the truncated rectangular grid graphs are Hamiltonian and guarantees the existence of a spanning 2- connected subgraph with at most three edges more than their number of vertices.

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Theorem [1]

Let

$R(s,t)^{-1(k,l)}, R(s,t)^{-2(k,l)}, R(s,t)^{-3(k,l)}$ *and* $R(s,t)^{-4(k,l)}$ denote the 1 – corner truncated rectangular grid graph, the 2 – corner truncated rectangular grid graph, the 3 – corner truncated rectangular grid graph and 4 – corner truncated rectangular grid graph as defined above respectively. Then:

1. $\mathrm{R(s,t)}^{-1(\mathrm{k,l})}$ contains a spanning 2 – connected subgraph with (at most) |V| + 1 edge and is hamiltonian if and only if both s.1 are even or both s.t and k.1 are odd.
2. $\mathrm{R(s,t)}^{-2(\mathrm{k,l})}$ contains a spanning 2 – connected subgraph with

- $|\mathrm{V}|$ edges if s.t is even and at least one of k and l is even if both s and t are even;
- $|\mathrm{V}| + 2$edges if s and t are even and k and l are odd.
- $|\mathrm{V}| + 1$ Edges in all other cases.

These number of edges are all best possible.
c) $\mathrm{R(s,t)}^{-3(\mathrm{k,l})}$ contains a spanning 2 – connected subgraph with

- $|\mathrm{V}|$ edges if both s.t and k.l are even;
- $|\mathrm{V}| + 2$ edges if all of s, t, k and l are odd;
- $|\mathrm{V}| + 1$ edges in all other cases.

These number of edges are all best possible.

d)$\mathrm{R(s,t)}^{-4(\mathrm{k,l})}$ contains a spanning 2 – connected subgraph with (at most)
$|\mathrm{V}| + 3$ edges and is hamiltonian if and only if s.t is even. The bound edges and
$|\mathrm{V}| + 3$ is best possible for any odd numbers s, t, k and l.

Main Results

Theorem 1: $R(s,t)^{-2(k,l)}$ contains no spanning 2-connected subgraph with at most $|V| + 1$ edges if both s and t are even and both k and l odd.

Proof. First, consider a bipartition (S,T) of $R(s,t)$ for some positive even integers s and t. Assume that one of the corner vertices is in S, whereas the two other corner vertices are in $|T|$ and that $|S| = |T|$. Thus can be proved by induction on s and t, and removing the cycle of the outer face if $s,t \geq 4$.

Secondly, consider a bipartition (S,T) of $R(k,l)$ for odd k and l. Assume that one of the corner vertices is in (if $s,t \geq 3$; otherwise consider an end vertex). Then we can show that all corner vertices (end vertices) are in S, and that $|S| = |T| + 1$. This can be proved by induction on s and t, and removing the cycle of the outer face if $s,t \geq 3$.

So if we remove the two opposite corner $R(k,l)'s$ from $R(s,t)$, we reduce $|S|$ by two more units then $|T|$, implying that $R(s,t)^{-2(k,l)}$ has a bipartition (S',T') with $|T'| = |S'| + 2$. In any spanning 2-connected subgraph G of $R(s,t)^{-2(k,l)}$ all vertices in T' have degree at least 2, hence

$|E(G)| \geq 2|T'| = |T'| + |S'| + 2 = |V(G)| + 2$.

□

Theorem 2: $R(s,t)^{-3(k,l)}$ contains no spanning 2-connected subgraph with at most $|V| + 1$ edge if all of s,t,k and l are odd.

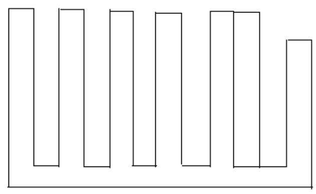
Proof. Consider a bipartition (S,T) of $R(s,t)$ for odd s and t. Assume that one of the corner vertices is in S. By the same arguments as in the proof of theorem 1, then all corner vertices are in. S,. And $|S| = |T| + 1$. The same holds for $R(k,l)$ if k and l are odd. So if we remove the three corner $R(k,l)'s$ from $R(s,t)$, we reduce $|S|$ by three more units than $|T|$, implying that $R(s,t)^{-3(k,l)}$ has a bipartition (S',T') with $|T'| = |S'| + 2$. In any spanning 2-connected subgraph G of $R(s,t)^{-3(k,l)}$ all vertices in T' have degree at least 2, hence $|E(G)| \geq 2|T'| = |T'| + |S'| + 2 = |V(G)| + 2$.

Theorem 3: $R(s,t)^{-4(k,l)}$ contains no spanning 2-connected subgraph with at most $|V| + 2$ edge if all of s,t,k and l are odd.

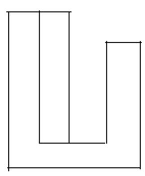
Proof. Consider a bipartition (S,T) of $R(s,t)$ for odd s and t. Assume that one of the corner vertices is in S. By the same arguments as in the proof of theorem 1, then all corner vertices are in S, and $|S| = |T| + 1$. The same holds for $R(k,l)$ if k and l are odd. So if we remove the four corner $R(k,l)'s$ from $R(s,t)$, we reduce $|S|$ by four more units than $|T|$, implying that $R(s,t)^{-4(k,l)}$ has a bipartition (S',T') with $|T'| = |S'| + 3$. In any spanning 2-connected subgraph G of $R(s,t)^{-4(k,l)}$ all vertices in T' have degree at least 2, hence $|E(G)| \geq 2|T'| = |T'| + |S'| + 3 = |V(G)| + 3$.

DISCUSSION

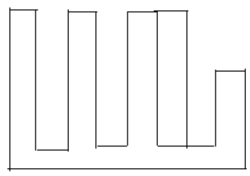
Here we observe certain cases of the above results.
Spanning 2-connected subgraph for $R(13,11)^{-1(2,3)}$ this pattern can be used for finding a spanning 2-connected subgraph with $|V| + 1$ edges for 1-corner truncated rectangular grid graph for any odd no's s and t for any ever number k & l.



Example: 1 $R(5,7)^{-1(2,2)}$ 2. $R(9,7)^{-1(2,3)}$

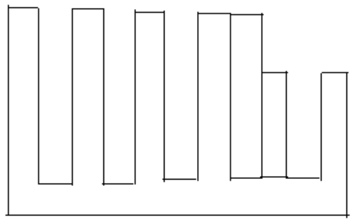


No. of vertices = 31
No. of edges = 32

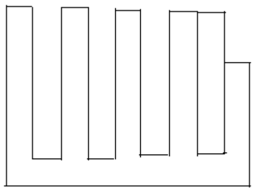


No. of vertices = 55
No. of edges = 56

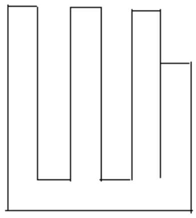
Spanning 2-connected subgraph for $R(12,11)^{-1(3,1)}$ this pattern can be used for finding a spanning 2-connected subgraph with $|V| + 1$ edges for 1-corner truncated rectangular grid graph for any ever no's s and t for any odd number k & l.



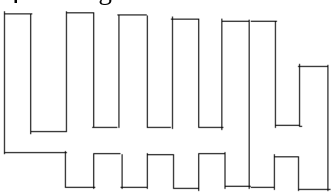
Example 1 $R(10,9)^{-1(1,3)}$



Example $R(7,10)^{-1(1,3)}$

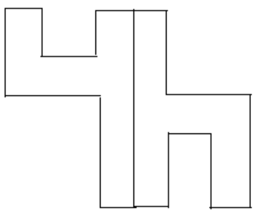


Spanning 2-connected subgraph for $R(13,11)^{-2(2,2)}$ this pattern can be used for finding a spanning 2-connected subgraph with $|V| + 1$ edges.

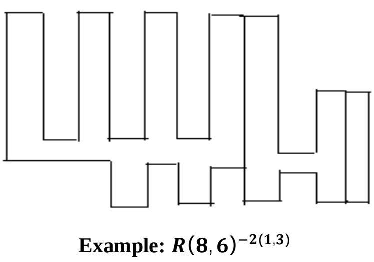


The Pattern can be used for finding a Spanning 2-connected subgraph with $|V| + 1$ edges for 2-corner truncated rectangular grid graph for any ever no's k & l and any odd numbers s and t.

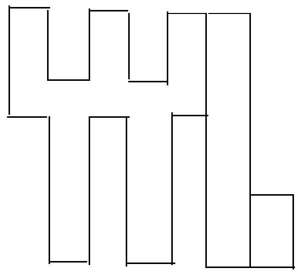
Example: $R(7,5)^{-2(2,2)}$



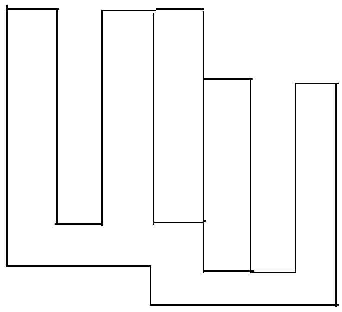
Spanning 2-connected subgraph for $R(12,10)^{-2(3,3)}$ this pattern can be used for finding a spanning 2-connected subgraph with $|V| + 2$ edges this pattern can be used for finding a spanning 2-connected subgraph with $|V| + 2$ edges for any ever numbers s and t for any odd number k & l.



Example: $R(8,6)^{-2(1,3)}$



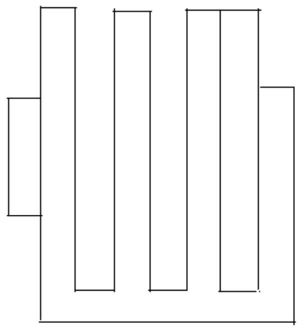
For the result $R(s,t)^{-2(k,l)}$ contains no spanning 2-connected subgraph with at most $|V| + 1$ edges if both s and t are ever and both k and l are odd.



Example: $R(8,6)^{-2(3,1)}$

No. of vertices = 42
No. of edges = 44

For the result $R(s,t)^{-3(k,l)}$ contains no spanning 2-connected subgraph with at most $|V| + 1$ edges if both s, t, k, l are odd.
Example: $R(9,11)^{-3(3,1)}$



No. of vertices = 90
No. of edges = 92

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