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Research Article

PBIB-DESIGNS AND ASSOCIATION SCHEMES VIA MINIMUM NONSPLIT DOMINATING SETS OF SOME CIRCULANT GRAPHS

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A B S T R A C T

A dominating set D of a graph $G = (V, E)$ is a nonsplit dominating set if the induced subgraph $\langle V - D\rangle$ is connected. The nonsplit domination number $\gamma_{ns}(G)$ is the G is the minimum cardinality of a nonsplit dominating set of G. The set of vertices is a γ_{ns}-set if it is nonsplit dominating set with $\gamma_{ns}(G)$. In this paper, we obtain the total number of γ_{ns}-sets, the Partially Balanced Incomplete Block (PBIB)-Designs on minimum γ_{ns}-sets of Circulant graphs with m-association schemes for $1 \le m \le \lfloor \frac{p}{2} \rfloor$.

INTRODUCTION

All graphs considered in this paper are finite, undirected and connected with no loops and multiple edges. As usual $p = | V |$ and $q = | E |$ denote the number of vertices and edges at a graph G, respectively. For any undefined terms in this paper, we refer to [7] and [11].

For a given positive integer p, let $s_1, s_2, \ldots, s_t$ be a sequence of integers where $0 < s_1 < s_2 < \ldots < s_t < \frac{p+1}{2}$. The Circulant graph $C_p(S)$ where $S = s_1, s_2, \ldots, s_t$ is the graph on p vertices labelled as $v_1, v_2, \ldots, v_p$ with vertex v_i adjacent to each vertex $v_{i \pm sj \, (\text{mod } p)}$ and the values s_t are called jump sizes.

Circulant graphs have a vast number of uses and applications in telecommunication network, a return is possible to the physical sciences with an application of Circulant to the evolution of destiny fluctuation and Circulant graph are an important sub field of graph theory, see [16].

Bose and Nair introduced a class of binary, equi-replicate and proper designs, which are called Partially Balanced Incomplete Block (PBIB)- Designs. In these designs, all the elementary contrasts are not estimated with the same variance. The variances depend on the type of association between the treatments. For details on PBIB designs and its related concepts, we refer to [5] and [6].

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There are many applications of PBIB- Designs may be used to contract (2, n) visual cryptographic schemes for black and white images with small pixel expansion, see [1].

Given ν elements (objects or vertices), a relation satisfying the following conditions is called a PBIB with m-associate classes:

1. Any two treatments are either first associates or second associates,. . . or m^{th} associates, the relation of associations being symmetric.
2. Each object has exactly n_k, k^{th} associates, the number n_k independent of x.
3. If two objects x and y are k^{th} associates, then the number of objects which are i^{th} associates of x and j^{th} associates of y is p^k_{ij} and is independent of the k^{th} associates x and y. Also $p^k_{ij} = p^k_{ji}$.

With the association scheme on ν objects, a PBIB–Design is an arrangement of ν objects into b sets (called blocks) of size g where $g < \nu$ such that

1. Every element is contained in exactly r blocks.
2. Each block contains g distinct elements.
3. Any two elements which are m^{th} associates occur together in exactly λ_m blocks.

The number ν, b, g, r, $\lambda_1, \lambda_2, \ldots, \lambda_m$ are called the parameters of the first kind, whereas the numbers $n_1, n_2, \ldots, n_m$, p^k_{ij} (i, j, k =1, 2, . . .,m) are called the parameters of the second kind, see [4].

A subset $D \subseteq V$ is said to be a dominating set of a graph G, if every vertex in $V - D$ is adjacent to some vertex in D. The minimum cardinality of vertices in such a set is called the domination number $\gamma(G)$. For complete review on theory of domination, we refer to [8], [9], [10], [12] and [13].

A dominating set D of a graph $G = (V, E)$ is a nonsplit dominating set if the induced subgraph $\langle V - D\rangle$ is connected. The nonsplit domination number $\gamma_{ns}(G)$ is the minimum cardinality of a nonsplit dominating set. The minimum nonsplit dominating set D with $|D| = \gamma_{ns}(G)$ is called γ_{ns}-set. This concept was introduced by Kulli and Janakiram, see [14].

Slater [17] has introduced the concept of the number of dominating sets of G, which he denoted by $HED(G)$ in honor of Steve Hedetniemi. In this paper, $\tau_{ns}(G)$ is used to denote the minimum number of γ_{ns}-sets of G. The PBIB-Design associated with domination related parameters are studied by [2], [3] and [18].

2.Circulant graph C_p (1)

The jump size of Circulant graph is one, known as cycle C_p with $p \geq 4$ vertices. That is, $C_p(1) \cong C_p$; $p \geq 4$.

Proposition 2.1: [14] For any Circulant graph C_p (1) with $p \geq 4$ vertices,

$$\gamma_{ns}\left(C_p(1)\right) = p - 2.$$

Theorem 2.1: The collection of all γ_{ns}-sets of a Circulant graph C_p (1); $p \geq 4$ vertices form a PBIB-Designs with $\lfloor \frac{p}{2} \rfloor$ -association scheme and parameters are $v = p$, $b = p$, $g = p - 2$, $r = p - 2$ and $\lambda_m = \begin{cases} p-3, & if\ m = 1 \\ p-4, & otherwise. \end{cases}$

Proof. Let C_p (1) be a Circulant graph with $p \geq 4$ vertices and labelled as $v_1, v_2, \ldots, v_p$. By Proposition 2.1, we have $\gamma_{ns}\left(C_p(1)\right) = p - 2$. Further, C_p (1) with $p \geq 4$ have p blocks of γ_{ns}-set, it implies $b = \tau_{ns}\left(C_p(1)\right) = p$. By Proposition 2.1, we have $g = \gamma_{ns}\left(C_p(1)\right) = p - 2$, where g is the number of elements contained exactly in a block.
By virtue of the above facts, we have $r = p - 2$.
To obtain the m – associates for the elements, where $1 \leq m \leq \lfloor \frac{p}{2} \rfloor$.

Two distinct elements are first associates, if they have jump size 1 and they are k^{th} – associates ($2 \leq k \leq \lfloor \frac{p}{2} \rfloor$), if they have k jump sizes. These associates are as shown in Table 1 along with their matrix representations.

By Table 1, the parameters of second kind are given by $n_i = 2$ for $1 \leq i \leq \frac{p-1}{2}$ or $1 \leq i \leq \frac{p}{2} - 1$ or $n_{\frac{p}{2}} = 1$.

With the association scheme for the Table 1, we have the matrix representation of the Circulant graph $C_{P(S_1, S_2, \ldots, S_t)}$ is

$$P^k = \begin{pmatrix} p_{11}^k & p_{12}^k & \cdots & p_{1\frac{p-1}{2}}^k \\ p_{21}^k & p_{22}^k & \cdots & p_{2\frac{p-1}{2}}^k \\ \vdots & \vdots & \vdots & \vdots \\ p_{(\frac{p-1}{2})1}^k & p_{(\frac{p-1}{2})2}^k & \cdots & p_{(\frac{p-1}{2})(\frac{p-1}{2})}^k \end{pmatrix} \text{or}$$

$$P^k = \begin{pmatrix} p_{11}^k & p_{12}^k & \cdots & p_{1\frac{p}{2}}^k \\ p_{21}^k & p_{22}^k & \cdots & p_{2\frac{p}{2}}^k \\ \vdots & \vdots & \cdots & \vdots \\ p_{\frac{p}{2}1}^k & p_{\frac{p}{2}2}^k & \cdots & p_{\frac{p}{2}\frac{p}{2}}^k \end{pmatrix}$$

The possible values of k in the matrix P^k are given below :
If $k = 1$, then

1. $p_{ij}^1 = 1$ for $1 \leq i \leq \frac{p-1}{2} - 1$ and $1 \leq i \leq \frac{p}{2} - 1$, $j = i + 1$,
2. $p_{ij}^1 = 1$ for $1 \leq j \leq \frac{p-1}{2} - 1$ and $1 \leq j \leq \frac{p}{2} - 1$, $i = 1 + j$,
3. $p_{ij}^1 = 1$ for $1 = \frac{p-1}{2}$, $i = \frac{p-1}{2}$,

If $2 \leq k \leq \frac{p-3}{2}$ and $2 \leq k \leq \frac{p}{2} - 1$, then

1. $p_{ij}^k = 1$ for $1 \leq i \leq \frac{p-3}{2}$ and $1 \leq i \leq \frac{p}{2} - 1$, $i + j = k$, $j = k + i$ and $i + j = p - k$.
2. $p_{ij}^k = 1$ for $1 \leq j \leq \frac{p-3}{2}$ and $1 \leq j \leq \frac{p}{2} - 1$, $i = k + j$ and $i + j = p - k$.

If $k = \frac{p-1}{2}$ and $k = \frac{p}{2}$, then

1. $p_{ij}^k = 1$ for $1 \leq i \leq \frac{p-3}{2}$, $j = \frac{p-1}{2} - i$,
2. $p_{ij}^k = 1$ for $1 \leq i \leq \frac{p-1}{2}$, $j = \frac{p+1}{2} - i$,
3. $p_{ij}^k = 2$ for $1 \leq i \leq \frac{p}{2} - 1$, $j = k - i$

With other entries are all zero.
Hence the parameters of first kind are given by $v = p$, $b = p$, $g = p - 2$, $r = p - 2$, $\lambda_m = p - 3$, where $m = 1$; otherwise, $\lambda_m = p - 4$.

Table 1 Association schemes of $C_{P(S_1, S_2, \ldots, S_t)}$.

Elements	Association scheme						
	First	Second	. .	k	. .	$\frac{p-1}{2}$	$\frac{p}{2}$
$v1$	$vp, v2$	$vp\text{-}1, v3$	. . .	$v(p-(k-1))(\text{mod } p)$, $v(1+k)(\text{mod } p)$	. . .	$v_{1+\frac{p-1}{2}}, v_{1+\frac{p-1}{2}+1}$	$v_{1+\frac{p}{2}}$
$v2$	$v1, v3$	$vp, v4$	. . .	$v(p-(k-2))(\text{mod } p)$, $v(2+k)(\text{mod } p)$	. . .	$v_{2+\frac{p-1}{2}}, v_{2+\frac{p-1}{2}+1}$	$v_{2+\frac{p}{2}}$
$v3$	$v2, v4$	$v1, v5$	. . .	$v(p-(k-3))(\text{mod } p)$, $v(3+k)(\text{mod } p)$	. . .	$v_{3+\frac{p-1}{2}}, v_{3+\frac{p-1}{2}+1}$	$v_{3+\frac{p}{2}}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
vi	$v(i-1))(\text{mod } p)$, $v(i+1))(\text{mod } p)$	$v(i-2))(\text{mod } p)$, $v(i+2))(\text{mod } p)$	. . .	$v(p-(k-i))\ (\text{mod } p)$, $v(i+k)\ (\text{mod } p)$	. . .	$v_{(i + \frac{p-1}{2})\ (\text{mod } p)}$ $v_{(i + \frac{p-1}{2} + 1)\ (\text{mod } p)}$	$v_{\left(1+\frac{p}{2}\right)(mod p)}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
vp	$vp\text{-}1, v1$	$vp\text{-}2, v2$		$vp\text{-}k, vk$		$v_{\frac{p-1}{2}}, v_{\frac{p-1}{2}+1}$	$v_{\frac{p}{2}}$

3.Circulant graph $C_p\ (\lfloor \frac{p}{2} \rfloor)$

The Circulant graph with jump size $\lfloor \frac{p}{2} \rfloor$; $p \geq 4$ vertices, is $C_p\ (\lfloor \frac{p}{2} \rfloor)$.

Proposition 3.1: [14] For any Circulant graph $C_p\ (\lfloor \frac{p}{2} \rfloor)$ with $p \geq 4$ vertices,
$\gamma_{ns}\left(C_p\ \left(\ \lfloor \frac{p}{2} \rfloor\right)\right) = p - 2.$

Theorem 4.1: The collection of all γ_{ns}-sets of a Circulant graph $C_p\ (\lfloor \frac{p}{2} \rfloor\ )$; $p \geq 4$ vertices form a PBIB-Designs with $\lfloor \frac{p}{2} \rfloor$ - association scheme and parameters are $v = p$, b = p,
$g = p - 2$, $r = p - 2$ and $\lambda_m = \begin{cases} p-4, & if\ 1 \leq m \leq \lfloor \frac{p-1}{2} \rfloor \\ p-3, & if\ m = \lfloor \frac{p}{2} \rfloor. \end{cases}$

Proof. Let $C_p\ (\lfloor \frac{p}{2} \rfloor)$ be a Circulant graph with $p \geq 4$ vertices and labelled as $v_1, v_2, \ldots, v_p$. By Proposition 3.1, we have $\gamma_{ns}\left(C_p\left(\lfloor \frac{p}{2} \rfloor\right)\right) = p - 2.$

Further, $C_p\ (\ \lfloor \frac{p}{2} \rfloor\ )$ with $p \geq 4$ have p blocks of γ_{ns}-set, it implies $b = \tau_{ns}\left(C_p\left(\lfloor \frac{p}{2} \rfloor\right)\right) = p.$

By Proposition 3.1, we have $g = \gamma_{ns}\left(C_p\left(\lfloor \frac{p}{2} \rfloor\right)\right) = p - 2$, where g is the number of elements contained exactly in a block.

From the above facts, we have $r = p - 2$.

To obtain the m – associates for the elements, where $1 \leq m \leq \lfloor \frac{p}{2} \rfloor$.

Two distinct elements are first associates, if they have jump size k and they are $\lfloor \frac{p}{2} \rfloor^{th}$ – associates $(1 \leq k \leq \lfloor \frac{p-2}{2} \rfloor)$, if they have $\lfloor \frac{p}{2} \rfloor$ jump sizes. These associates are as shown in Table 2 along with their matrix representations.

1. $p_{ij}^1 = 1$ for $1 \leq i \leq \frac{p}{2} - 1$, $j = i + 1$,
2. $p_{ij}^1 = 1$ for $i = 1 + j$, $1 \leq j \leq \frac{p}{2} - 1$.

If $2 \leq k \leq \frac{p}{2} - 1$, then

1. $p_{ij}^k = 1$ for $1 \leq j \leq \frac{p}{2} - 1$, $i + j = k$, $j = k + i$ and $i + j = p - k$.
2. $p_{ij}^k = 1$ for $1 \leq j \leq \frac{p}{2} - 1$, $i = k + j$ and $i + j = p - k$.

If $k = \frac{p}{2}$, then $p_{ij}^k = 2$ for $1 \leq i \leq \frac{p}{2} - 1$ and $j = k - i$ with other entries are all zero.

Hence the parameters of first kind are given by $v = p$, b = p, $g = p - 2$, $r = p - 2$, $\lambda_m = p - 4$; $1 \leq m \leq \lfloor \frac{p-1}{2} \rfloor$ and $\lambda_m = p - 3$; $m = \lfloor \frac{p}{2} \rfloor$.

4.Circulant graph with odd jump sizes

The Circulant graph with odd jump size $C_p\ (1, 3, \ldots, \lfloor \frac{p}{2} \rfloor)$; $p \geq 6$ vertices is known as a complete bipartite graph $K_{p_1 p_2}$ where $p_1 = p_2$; that is , $C_p\ (1, 3, \ldots, \lfloor \frac{p}{2} \rfloor) \cong K_{p_1 p_2}$.

Proposition 4.1: For any Circulant graph $C_p\ (\ 1, 3, \ldots, \lfloor \frac{p}{2} \rfloor\ )$ with $p \geq 6$ vertices,
$\gamma_{ns}\left(C_p\left(\ 1, 3, \ldots, \lfloor \frac{p}{2} \rfloor\right)\right) = 2.$

Proof : Let $C_p(\ 1, 3, \ldots, \lfloor \frac{p}{2} \rfloor\ )$ be a Circulant graph of odd jump sizes with $p \geq 6$ vertices and labelled as $v_1, v_2, \ldots, v_p$. Since $C_p\ (1, 3, \ldots, \lfloor \frac{p}{2} \rfloor) \cong K_{p_1 p_2}$ with $V\ (C_p\ (\ 1, 3, \ldots, \lfloor \frac{p}{2} \rfloor)) = V_1 \cup V_2$; $|\ V_1| = p_1$ and $|\ V_2| = p_2$, which is a bipartite graph. Hence the set $\{u, v\}$ form a γ_{ns}-set of $C_p\ (\ 1, 3, \ldots, \lfloor \frac{p}{2} \rfloor)$ for $u \in V_1$ and $v \in V_2$. Thus $\gamma_{ns}\left(C_p\left(\ 1, 3, \ldots, \lfloor \frac{p}{2} \rfloor\right)\right) = 2$ follows.

Table 2 Association schemes of circulants with even vertices.

Elements	Association scheme					
	First	Second	..	k	..	$\frac{p}{2}$
v_1	v_p, v_2	v_{p-1}, v_3	...	$v_{(p-(k-1))\ (\mathrm{mod}\ p)}$, $v_{(1+k)\ (\mathrm{mod}\ p)}$	...	$v_{1+\frac{p}{2}}$
v_2	v_1, v_3	v_p, v_4	...	$v_{(p-(k-2))\ (\mathrm{mod}\ p)}$, $v_{(2+k)\ (\mathrm{mod}\ p)}$	...	$v_{2+\frac{p}{2}}$
v_3	v_2, v_4	v_1, v_5	...	$v_{(p-(k-3))\ (\mathrm{mod}\ p)}$, $v_{(3+k)\ (\mathrm{mod}\ p)}$	...	$v_{3+\frac{p}{2}}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
v_i	$v_{(i-1))\ (\mathrm{mod}\ p)}$, $v_{(i+1)\ (\mathrm{mod}\ p)}$	$v_{(i-2))\ (\mathrm{mod}\ p)}$, $v_{(i+2))\ (\mathrm{mod}\ p)}$	...	$v_{(p-(k-i))\ (\mathrm{mod}\ p)}$, $v_{(i+k)\ (\mathrm{mod}\ p)}$	...	$v_{\left(i+\frac{p}{2}\right)(mod\ p)}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
v_p	v_{p-1}, v_1	v_{p-2}, v_2		v_{p-k}, v_k		$v_{\frac{p}{2}}$

With this association scheme, the parameters of second kind are given by $n_i = 2$ for $1 \leq j \leq \frac{p-1}{2} - 1$, $n_{\frac{p}{2}} = 1$ and

$$P^k = \begin{pmatrix} p_{11}^k & p_{12}^k & \cdots & p_{1\frac{p}{2}}^k \\ p_{21}^k & p_{22}^k & \cdots & p_{2\frac{p}{2}}^k \\ \vdots & \vdots & \cdots & \vdots \\ p_{\frac{p}{2}1}^k & p_{\frac{p}{2}2}^k & \cdots & p_{\frac{p}{2}\frac{p}{2}}^k \end{pmatrix}$$

The possible values of k in the matrix P^k are given below:
If $k = 1$, then

Theorem 4.1: The collection of all γ_{ns}-sets of a Circulant graph $C_p\ (1, 3, \ldots, \lfloor \frac{p}{2} \rfloor)$; $p \geq 6$ vertices form a PBIB-Designs with $\lfloor \frac{p}{2} \rfloor$ - association scheme and parameters are $v = p$, b = p,
$g = 2$, $r = 2$ and $\lambda_m = \begin{cases} 1, & if\ m = 1 \\ 0, & \text{otherwise}. \end{cases}$

Proof. Let $C_p\ (\ 1, 3, \ldots, \lfloor \frac{p}{2} \rfloor)$ be a Circulant graph with $p \geq 6$ vertices and labelled as $v_1, v_2, \ldots, v_p$. By Proposition 4.1, we have $\gamma_{ns}\left(C_p(\ 1, 3, \ldots, \lfloor \frac{p}{2} \rfloor\ \right) = 2$. Further, $C_p\ (\ 1, 3, \ldots, \lfloor \frac{p}{2} \rfloor)$ with $p \geq 6$ have p blocks of γ_{ns}-set, it implies

$b = \tau_{ns}\left(C_p\ (1, 3, \ldots, \lfloor \frac{p}{2} \rfloor\ )\right) = p.$

By Proposition 4.1, we have $g = \gamma_{ns}\left(C_p\ (1, 3, \ldots, \lfloor \frac{p}{2} \rfloor\ )\right) = 2$, where g is the number of elements contained exactly in a block.

From the above facts, we have $r = 2$.

To obtain the m – associates for the elements, where $1 \leq m \leq \lfloor \frac{p}{2} \rfloor$.

Two distinct elements are odd associates, if they have odd jump size and they are even associates ($2 \leq k \leq \lfloor \frac{p}{2} \rfloor$), if they have even jump sizes. These associates are as shown in Table 3 along with their matrix representations.

Table 3 Association schemes of circulants with odd vertices

Elements	**Association scheme**					
	First	Second	..	k	..	$\frac{p-1}{2}$
v_1	v_p, v_2	v_{p-1}, v_3	...	$v_{(p-(k-1)) \pmod p}$, $v_{(1+k) \pmod p}$	...	$v_{1+\frac{p-1}{2}}, v_{1+\frac{p-1}{2}+1}$
v_2	v_1, v_3	v_p, v_4	...	$v_{(p-(k-2)) \pmod p}$, $v_{(2+k) \pmod p}$	...	$v_{2+\frac{p-1}{2}}, v_{2+\frac{p-1}{2}+1}$
v_3	v_2, v_4	v_1, v_5	...	$v_{(p-(k-3)) \pmod p}$, $v_{(3+k) \pmod p}$	...	$v_{3+\frac{p-1}{2}}, v_{3+\frac{p-1}{2}+1}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
v_i	$v_{(i-1) \pmod p}$, $v_{(i+1) \pmod p}$	$v_{(i-2) \pmod p}$, $v_{(i+2) \pmod p}$	...	$v_{(p-(k-i)) \pmod p}$, $v_{(i+k) \pmod p}$	...	$v_{(i+\frac{p-1}{2}) \pmod p}$, $v_{(i+\frac{p-1}{2}+1) \pmod p}$
.	.	.	.	.	.	.
.	.	.	.	.	.	.
v_p	v_{p-1}, v_1	v_{p-2}, v_2		v_{p-k}, v_k		$v_{\frac{p-1}{2}}, v_{\frac{p-1}{2}+1}$

By Table 3, the parameters of second kind are given by $n_i = 2$ for $1 \leq i \leq \frac{p-1}{2}$ and $n_{\frac{p}{2}} = 1$.

With the association scheme for the Table 3, we have the matrix representation of the Circulant graph $C_{P(S_1, S_2, \ \ldots, \ S_t)}$ $p \geq 6$ is

$$P^k = \begin{pmatrix} p_{11}^k & p_{12}^k & \cdots & p_{1\frac{p-1}{2}}^k \\ p_{21}^k & p_{22}^k & \cdots & p_{2\frac{p-1}{2}}^k \\ \vdots & \vdots & \vdots & \vdots \\ p_{(\frac{p-1}{2})1}^k & p_{(\frac{p-1}{2})2}^k & \cdots & p_{(\frac{p-1}{2})(\frac{p-1}{2})}^k \end{pmatrix}$$

Therefore the possible values of k in the matrix P^k are given below:

If $k = 1$, then

1. $p_{ij}^1 = 1$ for $1 \leq i \leq \frac{p-1}{2} - 1$, $j = i + 1$,
2. $p_{ij}^1 = 1$ for $1 \leq j \leq \frac{p-1}{2} - 1$, $i = 1 + j$,
3. $p_{ij}^1 = 1$ for $i = \frac{p-1}{2}$, $j = \frac{p-1}{2}$.

If $2 \leq k \leq \frac{p-3}{2}$, then

1. $p_{ij}^k = 1$ for $1 \leq i \leq \frac{p-3}{2}$, $i + j = k$, $j = k + i$ and $i + j = p - k$,
2. $p_{ij}^k = 1$ for $1 \leq j \leq \frac{p-3}{2}$, $i = k + j$ and $i + j = p - k$.

If $k = \frac{p-1}{2}$, then

1. $p_{ij}^k = 1$ for $1 \leq i \leq \frac{p-3}{2}$, $j = \frac{p-1}{2} - i$,
2. $p_{ij}^k = 1$ for $1 \leq i \leq \frac{p-1}{2}$, $j = \frac{p+1}{2} - i$.

With other entries are all zero.

Hence the parameters of first kind are given by $v = p$, b = p, $g = 2$, $r = 2$, $\lambda_m = 1$ where $m = 1$ and otherwise, $\lambda_m = 0$.

5. Circulant graph with even jump sizes

The even jump sizes (2, 4, . . . , $\lfloor \frac{p}{2} \rfloor$) of Circulant graph is denoted by C_p (2, 4, . . . , $\lfloor \frac{p}{2} \rfloor$) with $p \geq 4$ vertices.

Proposition 5.1: For any Circulant graph C_p (2, 4, . . . , $\lfloor \frac{p}{2} \rfloor$) with $p \geq 4$ vertices,

$$\gamma_{ns}\left(C_p\left(2, 4, \ldots, \lfloor \frac{p}{2} \rfloor\right)\right) = \begin{cases} 3, & if\ p = 4n + 1; n \geq 1 \\ \frac{p}{2}, & if\ p = 4n + 4; n\ \geq 1. \end{cases}$$

Proof . Let C_p(2, 4, . . . , $\lfloor \frac{p}{2} \rfloor$) be a Circulant graph with $p = 4n + 1$ or $4(n + 1)$; $n \geq 1$ vertices. We have the following two cases.

Case 1: If $p = 4n + 1$, $n \geq 1$ vertices, then the Circulant graph C_p (2, 4, . . . , $\lfloor \frac{p}{2} \rfloor$) is a $(2n - 1)$-regular. Thus the result follows.

Case 2: If $p = 4(n + 1)$, $n \geq 1$ vertices and D is an γ_{ns}-set of C_p (2, 4, . . . , $\lfloor \frac{p}{2} \rfloor$), then it covers all the vertices of $V - D$ and it is also an nonsplit dominating set. This implies that $|D| = |V - D\,|$. Thus the result follows.

Theorem: The collection of all γ_{ns}-sets of a Circulant graph C_p (2, 4, . . . , $\lfloor \frac{p}{2} \rfloor$) ; $p \geq 4$ vertices form a PBIB-Designs with $\lfloor \frac{p}{2} \rfloor$ - association scheme and parameters are $v = p$, b = p,

$$g = 3,\ r = 3 \text{ and } \lambda_m = \begin{cases} 1, & if\ m = 1, \\ 0, & if\ 2 \leq m \leq \lfloor \frac{p-1}{2} \rfloor, \\ 2, & if\ m = \lfloor \frac{p}{2} \rfloor. \end{cases}$$

Proof. Let C_p (2, 4, . . . , $\lfloor \frac{p}{2} \rfloor$) be a Circulant graph with $p = 4n + 1$, $n \geq 1$ vertices and labelled as $v_1, v_2, \ldots, v_p$. By Proposition 5.1, we have $\gamma_{ns}\left(C_p\ (2, 4, \ldots, \lfloor \frac{p}{2} \rfloor\ )\right) = 3$.

Further, C_p(2, 4, . . . , $\lfloor \frac{p}{2} \rfloor$) with $p = 4n + 1$; $n \geq 1$ have p blocks of γ_{ns}-set, it implies

$b = \tau_{ns}\left(C_p(2, 4, \ldots, \lfloor \frac{p}{2} \rfloor\ )\right) = p.$

By Proposition 5.1, we have $g = \gamma_{ns}\left(C_p(2, 4, \ldots, \lfloor \frac{p}{2} \rfloor\ )\right) = 3$, where g is the number of elements contained exactly in a block.

From the above facts, we have $r = 3$.

To obtain the m – associates for the elements, where $1 \leq m \leq \lfloor \frac{p}{2} \rfloor$.

Two distinct elements are odd associates, if they have odd jump size and they are even associates ($2 \leq k \leq \lfloor \frac{p}{2} \rfloor$), if they have even jump sizes. These associates are as shown in the above Table 2 along with their matrix representations.

Hence the parameters of first kind are given by $v = p$, b = p, $g = 3$, $r = 3$, $\lambda_m = 1$ where $m = 1$; $\lambda_m = 0$ where $2 \leq m \leq \lfloor \frac{p-1}{2} \rfloor$ and $\lambda_m = 2$ where $m = \lfloor \frac{p}{2} \rfloor$.

Theorem 5.2: The collection of all γ_{ns}-sets of a Circulant graph C_p (2, 4, . . . , $\lfloor \frac{p}{2} \rfloor$) ; $p \geq 4$ vertices form a PBIB-Designs with $\lfloor \frac{p}{2} \rfloor$ - association scheme and parameters are $\nu = p$, b = 2, $g = \frac{p}{2}$, $r = 1$ and $\lambda_m = \begin{cases} 1, & if\ m\ is\ odd \\ 0, & if\ m\ is\ even. \end{cases}$

Proof. Let C_p (2, 4, . . . , $\lfloor \frac{p}{2} \rfloor$) be a Circulant graph with $p = 4n + 4$, $n \geq 1$ vertices and labelled as $v_1, v_2, \ldots, v_p$. By Proposition 5.1, we have $\gamma_{ns}\left(C_p(2, 4, \ldots, \lfloor \frac{p}{2} \rfloor)\right) = \frac{p}{2}$. Further, C_p (2, 4, . . . , $\lfloor \frac{p}{2} \rfloor$) with $p = 4n + 4$; $n \geq 1$ have two blocks of γ_{ns}-set, it implies

$b = \tau_{ns}\left(C_p\ (2, 4, \ldots, \lfloor \frac{p}{2} \rfloor)\right) = 2.$

By Proposition 5.1, we have $g = \left(C_p(2, 4, \ldots, \lfloor \frac{p}{2} \rfloor)\right) = \frac{p}{2}$, where g is the number of elements contained exactly in a block.

From the above facts, we have $r = 1$.
To obtain the m – associates for the elements, where $1 \leq m \leq \lfloor \frac{p}{2} \rfloor$.
Two distinct elements are odd associates, if they have odd jump size and they are even associates ($2 \leq k \leq \lfloor \frac{p}{2} \rfloor$), if they have even jump sizes. These associates are as shown in the above Table 2 along with their matrix representations.
Hence the parameters of first kind are given by $\nu = p$, b = 2, $g = \frac{p}{2}$, $r = 1$, $\lambda_m = 1$ where m is odd and $\lambda_m = 0$ where m is even.

6.Circulant graph C_p (1, 2, . . . , $\lfloor \frac{p}{2} \rfloor$)

The jump size of Circulant graph is 1, 2, . . . , $\lfloor \frac{p}{2} \rfloor$ are known as complete graph K_p with $p \geq 4$ vertices, that is C_p (1, 2, . . . , $\lfloor \frac{p}{2} \rfloor$) $\cong K_p$.

Proposition 6.1: For any Circulant graph C_p (1, 2, . . . , $\lfloor \frac{p}{2} \rfloor$) with $p \geq 4$ vertices,

$\gamma_{ns}\left(C_p\left(1, 2, \ldots, \lfloor \frac{p}{2} \rfloor\right)\right) = 1.$

Proof . Let C_p(1, 2, . . . , $\lfloor \frac{p}{2} \rfloor$) be a Circulant graph with $p \geq 4$ vertices and labelled as $v_1, v_2, \ldots, v_p$. If the Circulant graph C_p(1, 2, . . . , $\lfloor \frac{p}{2} \rfloor$) is a ($p$ - 1)- regular. Thus the result follows.

Theorem 6.1: The collection of all γ_{ns}-sets of a Circulant graph C_p (1, 2, . . . , $\lfloor \frac{p}{2} \rfloor$) ; $p \geq 4$ vertices form a PBIB-Designs with $\lfloor \frac{p}{2} \rfloor$ - association scheme and parameters are $\nu = p$, b = p, $g = 1$, $r = 1$ and $\lambda_m = 0$.

Proof. Let C_p (1, 2, . . . , $\lfloor \frac{p}{2} \rfloor$) be a Circulant graph with $p \geq 4$ vertices and labelled as $v_1, v_2, \ldots, v_p$. By Proposition 6.1, we have $\gamma_{ns}\left(C_p\ (1, 2, \ldots, \lfloor \frac{p}{2} \rfloor\right) = 1.$

Further, C_p (1, 2, . . . , $\lfloor \frac{p}{2} \rfloor$) with $p \geq 4$ have p blocks of γ_{ns}-set, it implies $b = \tau_{ns}\left(C_p(1, 2, \ldots, \lfloor \frac{p}{2} \rfloor)\right) = p.$

By Proposition 6.1, we have $g = \gamma_{ns}\left(C_p(1, 2, \ldots, \lfloor \frac{p}{2} \rfloor)\right) = 1$, where g is the number of elements contained exactly in a block.

From the above facts, we have $r = 1$.
Two distinct vertices $\{v_i\}$ and$\{v_j\}$ are said to be k^{th} associates where $1 \leq k \leq \lfloor \frac{p}{2} \rfloor$ if they are adjacent. These associates are as shown in the above Table 1 along with their matrix representations.

Hence the parameters of first kind are given by $\nu = p$, b = p, $g = 1$, $r = 1$ and $\lambda_m = 0$.

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