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Research Article

A STOCHASTIC MODEL WITH ANTIGENIC DIVERSITY THRESHOLD OF HIV TRANSMISSION USING GEOMETRIC PROCESS

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A B S T R A C T

This study is based on a stochastic model for predicting the seroconversion time of HIV transmission using intercontact time between successive contacts to be a geometric process. In this study of HIV infection the extend of contribution to antigenic diversity is influence by several factors, such as different sources of infections, the frequent contact. In this paper a stochastic model to determine the expected time to seroconversion and its variances are derived under the assumption that the inter-arrival time between contact are distributed as a geometric process using Generalized Rayleigh distribution has a threshold. Numerical illustration is provided using simulated data.

INTRODUCTION

The study of HIV dynamics is one of the most important developments in recent AIDS research for understanding the pathogenesis of HIV infection. The HIV infection that leads to AIDS as become a important disease in the world. In the study of HIV infection and its progression, the antigenic diversity of the antigen namely the HIV plays an important role. The intensity of sexual contact is an important factor that adds to the antigenic diversity since more number of new antigens are acquired by the individual who is getting infected. The process of divergence of the antigenic properties of the invading antigens requires a constant monitoring so that the preventive strategies to arrest the growth of antigenic diversity can be adopted. The intercontact times do play a crucial role in the increase of antigenic diversity. If the antigenic diversity crosses a particular level which is known as antigenic diversity threshold, then there is a collapse of the immune system and seroconversion immediately takes place. The antigenic diversity threshold model has been discussed by Nowak and May (1991), Stiliankis *et al*. (1994), and Kirschner *et al*. (2000).

In the estimation of expected time to seroconversion there is an important role for interarrival times between successive contacts; and it has a significant influence. In the case of persons exposed to HIV infection through sexual contacts, the contribution to the antigenic diversity would depend upon the number of contacts in the interval (0, t]. If the number of contacts is increasing then interarrival times between contacts would decrease. If any person has contact with an unknown partner, there is a likelihood to have the fear of getting infected. However the person does not avoid the contacts. But there is a possibility that a person may postpone the event namely the contact due to the fear complex. This gives rise to the sequence of random variables in increasing order. Therefore it is considered here that the interarrival times between contacts may form a geometric process. For detailed study of geometric process and its property one can refer to Lam Yah (1988). A stochastic model for the estimation of statistical measures for time to seroconversion of HIV infected using geometric process has been derived under different threshold distribution (Gamma, Mixed Exponential distribution, Exponential -Geometric Distribution) by Kannan *et al*. (2008), Kannan *et al*. (2012), Kannan *et al*. (2017). In this paper, we propose a stochastic model to determine the expected time to seroconversion and its variance are discussed under the assumption that interarrival times between contacts are distributed as a geometric process using Generalized Rayleigh distribution as a threshold. In developing such a stochastic model and cumulative damage process discussed by Esary *et al*. (1973) is used. In this study the theoretical results are substantiated using numerical data simulated.

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Assumptions of the model

The following are the assumptions underlying in the model developed here

- The transmission of HIV is only through sexual contacts.
- An uninfected individual has sexual contacts with HIV infected partner, and a random number of HIV are getting transmitted, at each contact.
- An individual is exposed to a damage process acting on the immune system and the damage is assumed to be linear and cumulative.
- The interarrival times between successive contacts are random variables which forms a geometric process.
- The sequence of successive contacts and threshold level are independent.
- From the collection of large number of interarrival times between successive contacts of a person, a random sample of 'k' observations are taken.

Notations

The notations used in this model are as follows

X_i : a random variable denoting the amount of damage arising due to i^{th} contact $X_i^{'s}$ are identically and independently distributed with p.d.f. $g(.)$ and c.d.f. $G(.)$.

Y : a random variable representing the antigenic diversity threshold which follows Generalized Rayleigh distribution with parameter $'\mu'$ and $'\lambda'$ the p.d.f. $h(.)$ and c.d.f. $H(.)$.

U_i : a continuous random variable denoting the inter-arrival times between successive contacts with p.d.f. $f(.)$ and c.d.f. $F(.)$.

$g_k(.)$: the p.d.f. of the random variable $\sum_{i=1}^{k} X_i$

$F_k(.)$: the $'k'^{th}$ convolution of $F(.)$.

T : a continuous random variable denoting the time to seroconversion with p.d.f. $l(.)$ and c.d.f. $L(.)$.

$V_k(t)$: probability of exactly $'k'$ contacts in $(0,t]$.

$l^*(s)$: the Laplace Stieltjes transform of $l(t)$.

$f^*(s)$: the Laplace Stieltjes transform of $f(t)$.

RESULTS

It can be shown that

$$P\left[\sum_{i=1}^{k} X_i < Y\right] = \int_0^\infty g_k(x)\overline{H}(x)\,dx$$

Let Y ~ Generalized Rayleigh Distribution (α, λ, μ)

$$\overline{H}(x) = 1 - \left[1 - e^{-\lambda(x-\mu)^2}\right] = e^{-\lambda(x-\mu)^2}$$

Hence

$$P\left[\sum_{i=1}^{k} X_i < Y\right] = \int_0^\infty g_k(x)\left[e^{-\lambda(x-\mu)^2}\right] = \left[g^*\lambda(1-\mu)^2\right]^k$$

The survival function S(t) is

S(t) = P[T > t]

$$= \sum_{k=0}^{\infty} V_k(t) P\left[\sum_{i=1}^{k} X_i < Y\right] = \sum_{k=0}^{\infty}\left[F_k(t) - F_{k+1}(t)\right]\left[g^*\lambda(1-\mu)^2\right]^k \qquad ......(1)$$

$$= \sum_{k=0}^{\infty} \Pr\{\text{there are exactly k contacts in } (0,t]\}$$

$$* \Pr\{\text{the cumulative total of antigenic diversity} < Y\}$$

L(t) = 1 – S(t)

$$L(t) = \left[1 - g^*\left(\lambda + \lambda\mu^2 - 2\lambda\mu\right)\right] \sum_{k=1}^{\infty} F_k(t) \left[g^*\left(\lambda + \lambda\mu^2 - 2\lambda\mu\right)\right]^{k-1}$$

On simplification

$$l^*(s) = \left[1 - g^*\left(\lambda + \lambda\mu^2 - 2\lambda\mu\right)\right] \sum_{k=1}^{\infty} \left[f^*(s)\right]^k \left[g^*\left(\lambda + \lambda\mu^2 - 2\lambda\mu\right)\right]^{k-1}$$

Taking Laplace Stieltjes transform of l(t),we get

$$l^*(s) = \left[1 - g^*\left(\lambda + \lambda\mu^2 - 2\lambda\mu\right)\right] \sum_{k=1}^{\infty} f_k^*(s) \left[g^*\left(\lambda + \lambda\mu^2 - 2\lambda\mu\right)\right]^{k-1} \quad \ldots$$

(2)

Since $U_i^t s \sim$ Geometric process $f_k^*(s)$ can be written as,

$$f_k^*(s) = \prod_{n=1}^{k} f^*\left(\frac{s}{a^{n-1}}\right) \quad \ldots (3)$$

Substituting equation (3) in (2), we get

$$l^*(s) = \left[1 - g^*\left(\lambda + \lambda\mu^2 - 2\lambda\mu\right)\right] \sum_{k=1}^{\infty} \left[g^*\left(\lambda + \lambda\mu^2 - 2\lambda\mu\right)\right]^{k-1} \prod_{n=1}^{k} f^*\left(\frac{s}{a^{n-1}}\right)$$

$$E(T) = -\left.\frac{d l^* s}{ds}\right|_{s=0}$$

$$= -\left[1 - g^*\left(\lambda + \lambda\mu^2 - 2\lambda\mu\right)\right] \sum_{k=1}^{\infty} \left[g^*\left(\lambda + \lambda\mu^2 - 2\lambda\mu\right)\right]^{k-1} \left.\left(\frac{d}{ds} \prod_{n=1}^{k} f^*\left(\frac{s}{a^{n-1}}\right)\right)\right|_{s=0} \quad \ldots (4)$$

Consider

$$\left.\frac{d}{ds}\left(\prod_{n=1}^{k} f^*\left(\frac{s}{a^{n-1}}\right)\right)\right|_{s=0}$$

$$= \left.\frac{d}{ds}\left\{f^*(s) f^*\left(\frac{s}{a}\right) f^*\left(\frac{s}{a^2}\right) \cdots f^*\left(\frac{s}{a^{k-1}}\right)\right\}\right|_{s=0}$$

$$= f^{*\prime}(0)\left[\frac{a^k - 1}{(a-1)a^{(k-1)}}\right] \quad \ldots (5)$$

Substituting equation (5) in (4), we get

$$E(T) = -\left[1 - g^*\left(\lambda + \lambda\mu^2 - 2\lambda\mu\right)\right] \sum_{k=1}^{\infty} \left[g^*\left(\lambda + \lambda\mu^2 - 2\lambda\mu\right)\right]^{k-1} f^{*\prime}(0)\left[\frac{a^k - 1}{(a-1)a^{(k-1)}}\right]$$

$$= -\frac{\left[1 - g^*\left(\lambda + \lambda\mu^2 - 2\lambda\mu\right)\right]}{(a - 1)} f^{*\prime}(0) \sum_{k=1}^{\infty} \left\{ a\left[g^*\left(\lambda + \lambda\mu^2 - 2\lambda\mu\right)\right]^{k-1} - \left[\frac{g^*\left(\lambda + \lambda\mu^2 - 2\lambda\mu\right)}{a}\right]^{k-1} \right\} \quad \ldots (6)$$

Consider

$$\sum_{k=1}^{\infty}\left\{a\left[g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]^{k-1}-\left[\frac{g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)}{a}\right]^{k-1}\right\}$$

$$=\frac{a}{\left[1-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]}-\frac{a}{\left[a-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]}$$

$$=\frac{a(a-1)}{\left[1-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]\left[a-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]} \quad \ldots (7)$$

Substituting equations (7) in (6), we get

$$E(T)=-\frac{\left[1-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]}{a-1}f^{*\prime}(0)\frac{a(a-1)}{\left[1-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]\left[a-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]} \quad \ldots (8)$$

Let f(.) ~ Exponential distribution with parameter c then

$$f^*(s)=\frac{c}{c+s}, \qquad f^{*\prime}(0)=-\frac{1}{c} \text{ and } f^{*\prime\prime}(0)=\frac{2}{c^2}$$

$$g^*(\lambda)=\frac{\alpha}{\alpha+\lambda}, g^*2(\lambda\mu)=\frac{\alpha}{\alpha+2\lambda\mu}, g^*\left(\lambda\mu^2\right)=\frac{\alpha}{\alpha+\lambda\mu^2}$$

From equation (8)

$$=\frac{-\left[1-\left\{\frac{\alpha}{\alpha+\lambda}+\frac{\alpha}{\alpha+\lambda\mu^2}-\frac{\alpha}{\alpha+2\lambda\mu}\right\}\right]}{(a-1)}\left(\frac{-1}{c}\right)$$

$$\frac{a(a-1)}{\left[1-\left\{\frac{\alpha}{\alpha+\lambda}+\frac{\alpha}{\alpha+\lambda\mu^2}-\frac{\alpha}{\alpha+2\lambda\mu}\right\}\right]\left[a-\left\{\frac{\alpha}{\alpha+\lambda}+\frac{\alpha}{\alpha+\lambda\mu^2}-\frac{\alpha}{\alpha+2\lambda\mu}\right\}\right]}$$

$$=\frac{a(\alpha+\lambda)(\alpha+2\lambda\mu)(\alpha+\lambda\mu^2)}{c\begin{pmatrix}a\alpha^3-\alpha^3+a\alpha^2\lambda\mu^2+2a\alpha^2\lambda\mu+a\alpha^2\lambda-4\alpha^2\lambda\mu\\ +2a\alpha\lambda^2\mu^3+a\alpha\lambda^2\mu^2+2a\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\\ -2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+2a\lambda^3\mu^3\end{pmatrix}}$$

$$E(T)=\frac{a\begin{pmatrix}\alpha^3+\alpha^2\lambda\mu^2+2\alpha^2\lambda\mu+\alpha\lambda^2\mu^3\\ +\alpha^2\lambda+\alpha\lambda^2\mu^2+2\alpha\lambda^2\mu+2\lambda^3\mu^3\end{pmatrix}}{c\begin{pmatrix}a\alpha^3-\alpha^3+a\alpha^2\lambda\mu^2+2a\alpha^2\lambda\mu+a\alpha^2\lambda-4\alpha^2\lambda\mu\\ +2a\alpha\lambda^2\mu^3+a\alpha\lambda^2\mu^2+2a\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\\ -2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+2a\lambda^3\mu^3\end{pmatrix}}$$

On simplification … (9)

$$E\left(T^2\right)=\left.\frac{d^2 l*s}{ds^2}\right|_{s=0}$$

$$E\left(T^2\right)=\left[1-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]\sum_{k=1}^{\infty}g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)^{k-1}\frac{d^2}{ds^2}\left.\left(\prod_{n=1}^{k}f^*\left(\frac{s}{a^{n-1}}\right)\right)\right|_{s=0} \quad \ldots (10)$$

Consider

$$\frac{d^2}{ds^2}\left(\prod_{n=1}^{k} f^*\left(\frac{s}{a^{n-1}}\right)\right)\Bigg|_{s=0}$$

$$=\frac{d^2}{ds^2}\left\{f^*(s)f^*\left(\frac{s}{a}\right)f^*\left(\frac{s}{a^2}\right)\cdots f^*\left(\frac{s}{a^{k-1}}\right)\right\}\Bigg|_{s=0}$$

$$= f^{*\prime\prime}(0)\frac{a^{2k}-1}{\left(a^2-1\right)\left(a^{2(k-1)}\right)}-\left[f^{*\prime}(0)\right]^2\frac{a^{2k}-1}{\left(a^2-1\right)\left(a^{2(k-1)}\right)}+\left[f^{*}(0)\right]^2\frac{\left(a^{2k}-2a^k+1\right)}{(a-1)^2\left(a^{2(k-1)}\right)}$$

On simplification

We get

$$=\left[f^{*\prime\prime}(0)-\left[f^{*\prime}(0)\right]^2\right]\frac{\left(a^{2k}-1\right)}{\left(a^2-1\right)\left(a^{2(k-1)}\right)}+\left[f^{*\prime}(0)\right]^2\left(\frac{a^{2k}-2a^k+1}{(a-1)^2\left(a^{2(k-1)}\right)}\right)$$

Using

$$V(U_1)=\left[f^{*\prime\prime}(0)-\left[f^{*\prime}(0)\right]^2\right] \text{ and } \left[E(U_1)\right]^2=\left[f^{*\prime}(0)\right]^2$$

From equation (10)

$$E\left(T^2\right)=\left[1-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]V(U_1)\sum_{k=1}^{\infty} g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)^{k-1}\left[\frac{a^{2k}-1}{\left(a^2-1\right)\left(a^{2(k-1)}\right)}\right]$$

$$+\left[1-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]\left[E(U_1)\right]^2\sum_{k=1}^{\infty} g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)^{k-1}\left[\frac{a^{2k}-2a^k+1}{\left(a^2-1\right)\left(a^{2(k-1)}\right)}\right]$$

… (11)

We note,

$$V(U_1)=\frac{1}{c^2} \quad \text{and} \quad \left[E(U_1)\right]^2=\frac{1}{c^2}$$

$$E\left(T^2\right)=\frac{\left[1-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]}{\left(a^2-1\right)}V(U_i)\left[\frac{a^2}{\left[1-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]}-\frac{a^2}{\left[a^2-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]}\right]$$

$$\frac{\left[1-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]}{(a-1)^2}\left[E(U_i)\right]^2\left[\frac{a^2}{\left[1-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]}-\frac{2a^2}{\left[a-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]}+\frac{a^2}{\left[a^2-g^*\left(\lambda+\lambda\mu^2-2\lambda\mu\right)\right]}\right]$$

$$\frac{\left[1-\left(\frac{\alpha}{\alpha+\lambda}+\frac{\alpha}{\alpha+\lambda\mu^2}-\frac{\alpha}{\alpha+2\lambda\mu}\right)\right]}{(a-1)^2}\left(\frac{a^2}{c^2}\right)\left[\frac{1}{\left[1-\left(\frac{\alpha}{\alpha+\lambda}+\frac{\alpha}{\alpha+\lambda\mu^2}-\frac{\alpha}{\alpha+2\lambda\mu}\right)\right]}\frac{2}{\left[a-\left(\frac{\alpha}{\alpha+\lambda}+\frac{\alpha}{\alpha+\lambda\mu^2}-\frac{\alpha}{\alpha+2\lambda\mu}\right)\right]}+\frac{1}{\left[a^2-\left(\frac{\alpha}{\alpha+\lambda}+\frac{\alpha}{\alpha+\lambda\mu^2}-\frac{\alpha}{\alpha+2\lambda\mu}\right)\right]}\right]$$

----- (12)

Let

$$A_1=\frac{\left[1-\left(\frac{\alpha}{\alpha+\lambda}+\frac{\alpha}{\alpha+\lambda\mu^2}-\frac{\alpha}{\alpha+2\lambda\mu}\right)\right]}{\left(a^2-1\right)}\left(\frac{a^2}{c^2}\right)$$

$$A_1=\frac{a^2\left(\alpha^2\lambda\mu^2-2\alpha^2\lambda\mu+\alpha^2\lambda+2\lambda^2\mu^2+2\lambda^3\mu^3\right)}{c^2\left(a^2-1\right)\left(\begin{array}{c}\alpha^3+\alpha^2\lambda\mu^2+2\alpha^2\lambda\mu+\alpha^2\lambda+2\alpha\lambda^2\mu^3\\+\alpha\lambda^2\mu^2+2\alpha\lambda^2\mu+2\lambda^3\mu^3\end{array}\right)}$$

On simplification ---- (13)

$$A_2 = \left\{ \frac{1}{1-\left(\frac{\alpha}{\alpha+\lambda}+\frac{\alpha}{\alpha+\lambda\mu^2}-\frac{\alpha}{\alpha+2\lambda\mu}\right)} - \frac{1}{\mathrm{a}^2-\left(\frac{\alpha}{\alpha+\lambda}+\frac{\alpha}{\alpha+\lambda\mu^2}-\frac{\alpha}{\alpha+2\lambda\mu}\right)} \right\}$$

$$A_2 = \left\{ \begin{array}{l} \dfrac{\alpha^3+\alpha^2\lambda\mu^2+2\alpha^2\lambda\mu+\alpha^2\lambda+2\alpha\lambda^2\mu^3+\alpha\lambda^2\mu^2+2\alpha\lambda^2\mu+2\lambda^3\mu^3}{\alpha^2\lambda\mu^2-2\alpha^2\lambda\mu+\alpha^2\lambda+2\lambda^2\mu^2+2\lambda^3\mu^3} \\ \quad -\dfrac{\alpha^3+\alpha^2\lambda\mu^2+2\alpha^2\lambda\mu+\alpha^2\lambda+2\alpha\lambda^2\mu^3+\alpha\lambda^2\mu^2+2\alpha\lambda^2\mu+2\lambda^3\mu^3}{\begin{array}{l}\mathrm{a}^2\alpha^3+\mathrm{a}^2\alpha^2\lambda\mu^2+2\mathrm{a}^2\alpha^2\lambda\mu+2\mathrm{a}^2\alpha\lambda^2\mu^3+\mathrm{a}^2\alpha^2\lambda \\ +\mathrm{a}^2\alpha\lambda^2\mu^2+2\mathrm{a}^2\alpha\lambda^2\mu+2\mathrm{a}^2\lambda^3\mu^3-\alpha^3-4\alpha^2\lambda\mu-2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\end{array}} \end{array} \right\}$$

On simplification ----- (14)

$$A_3 = \frac{\left[1-\left(\frac{\alpha}{\alpha+\lambda}+\frac{\alpha}{\alpha+\lambda\mu^2}-\frac{\alpha}{\alpha+2\lambda\mu}\right)\right]}{(\mathrm{a}-1)^2}\left(\frac{\mathrm{a}^2}{\mathrm{c}^2}\right)$$

$$A_3 = \frac{\mathrm{a}^2\left(\alpha^2\lambda\mu^2-2\alpha^2\lambda\mu+\alpha^2\lambda+2\lambda^2\mu^2+2\lambda^3\mu^3\right)}{\mathrm{c}^2(\mathrm{a}-1)^2\left(\alpha^3+\alpha^2\lambda\mu^2+2\alpha^2\lambda\mu+\alpha^2\lambda+2\alpha\lambda^2\mu^3+\alpha\lambda^2\mu^2+2\alpha\lambda^2\mu+2\lambda^3\mu^3\right)}$$

On simplification -------- (15)

$$A_4 = \left\{ \begin{array}{l} \dfrac{1}{1-\left(\frac{\alpha}{\alpha+\lambda}+\frac{\alpha}{\alpha+\lambda\mu^2}-\frac{\alpha}{\alpha+2\lambda\mu}\right)} - \dfrac{2}{\mathrm{a}-\left(\frac{\alpha}{\alpha+\lambda}+\frac{\alpha}{\alpha+\lambda\mu^2}-\frac{\alpha}{\alpha+2\lambda\mu}\right)} \\ \qquad + \dfrac{1}{\mathrm{a}^2-\left(\frac{\alpha}{\alpha+\lambda}+\frac{\alpha}{\alpha+\lambda\mu^2}-\frac{\alpha}{\alpha+2\lambda\mu}\right)} \end{array} \right\}$$

$$A_4 = \left\{ \begin{array}{l} \dfrac{\alpha^3+\alpha^2\lambda\mu^2+2\alpha^2\lambda\mu+\alpha^2\lambda+2\alpha\lambda^2\mu^3+\alpha\lambda^2\mu^2+2\alpha\lambda^2\mu+2\lambda^3\mu^3}{\alpha^2\lambda\mu^2-2\alpha^2\lambda\mu+\alpha^2\lambda+2\lambda^2\mu^2+2\lambda^3\mu^3} \\ \quad -\dfrac{2}{\dfrac{\begin{array}{l}\mathrm{a}\alpha^3+\mathrm{a}\alpha^2\lambda\mu^2+2\mathrm{a}\alpha^2\lambda\mu+\mathrm{a}\alpha^2\lambda+2\mathrm{a}\alpha\lambda^2\mu^3 \\ +\mathrm{a}\alpha\lambda^2\mu^2+2\mathrm{a}\alpha\lambda^2\mu+2\mathrm{a}\lambda^3\mu^3-\alpha^3-4\alpha^2\lambda\mu-2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\end{array}}{\alpha^3+\alpha^2\lambda\mu^2+2\alpha^2\lambda\mu+\alpha^2\lambda+2\alpha\lambda^2\mu^3+\alpha\lambda^2\mu^2+2\alpha\lambda^2\mu+2\lambda^3\mu^3}} \\ \quad -\dfrac{\alpha^3+\alpha^2\lambda\mu^2+2\alpha^2\lambda\mu+\alpha^2\lambda+2\alpha\lambda^2\mu^3+\alpha\lambda^2\mu^2+2\alpha\lambda^2\mu+2\lambda^3\mu^3}{\begin{array}{l}\mathrm{a}^2\alpha^3+\mathrm{a}^2\alpha^2\lambda\mu^2+2\mathrm{a}^2\alpha^2\lambda\mu+2\mathrm{a}^2\alpha\lambda^2\mu^3+\mathrm{a}^2\alpha^2\lambda \\ +\mathrm{a}^2\alpha\lambda^2\mu^2+2\mathrm{a}^2\alpha\lambda^2\mu+2\mathrm{a}^2\lambda^3\mu^3-\alpha^3-4\alpha^2\lambda\mu-2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\end{array}} \end{array} \right\}$$

On simplification -------- (16)

Substituting equations (13),(14),(15) and (16) in (12), we get

$$E\left(T^2\right) = A_1 A_2 + A_3 A_4$$

The Variance of time to seroconversion is

$$V[T] = E\left[T^2\right] - \left[E(T)\right]^2$$

$$V[T]=\frac{\begin{aligned}&2a^2\left\{\begin{aligned}&a\left(\alpha^2\lambda\mu^2-2\alpha^2\lambda\mu+\alpha^2\lambda+2\lambda^2\mu^2+2\lambda^3\mu^3\right)\\&\quad\left(\begin{aligned}&a\alpha^3+a\alpha^2\lambda\mu^2+2a\alpha^2\lambda\mu+a\alpha^2\lambda+2a\alpha\lambda^2\mu^3+a\alpha\lambda^2\mu^2+2a\alpha\lambda^2\mu\\&+2a\lambda^3\mu^3-\alpha^3-4\alpha^2\lambda\mu-2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\end{aligned}\right)\\&-\left(\alpha^2\lambda\mu^2-2\alpha^2\lambda\mu+\alpha^2\lambda+2\lambda^2\mu^2+2\lambda^3\mu^3\right)\\&\quad\left(\begin{aligned}&a\alpha^3+a\alpha^2\lambda\mu^2+2a\alpha^2\lambda\mu+a\alpha^2\lambda+2a\alpha\lambda^2\mu^3+a\alpha\lambda^2\mu^2+2a\alpha\lambda^2\mu\\&+2a\lambda^3\mu^3-\alpha^3-4\alpha^2\lambda\mu-2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\end{aligned}\right)\\&+a^2\left(\begin{aligned}&a^2\alpha^3+a^2\alpha^2\lambda\mu^2+2a^2\alpha^2\lambda\mu+2a^2\alpha\lambda^2\mu^3+a^2\alpha^2\lambda+a^2\alpha\lambda^2\mu^2+2a^2\alpha\lambda^2\mu\\&+2a^2\lambda^3\mu^3-\alpha^3-4\alpha^2\lambda\mu-2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\end{aligned}\right)\\&\quad\left(\begin{aligned}&a\alpha^3+a\alpha^2\lambda\mu^2+2a\alpha^2\lambda\mu+a\alpha^2\lambda+2a\alpha\lambda^2\mu^3+a\alpha\lambda^2\mu^2+2a\alpha\lambda^2\mu\\&+2a\lambda^3\mu^3-\alpha^3-4\alpha^2\lambda\mu-2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\end{aligned}\right)\end{aligned}\right.\\&-a\left(\begin{aligned}&a^2\alpha^3+a^2\alpha^2\lambda\mu^2+2a^2\alpha^2\lambda\mu+2a^2\alpha\lambda^2\mu^3+a^2\alpha^2\lambda+a^2\alpha\lambda^2\mu^2+2a^2\alpha\lambda^2\mu\\&+2a^2\lambda^3\mu^3-\alpha^3-4\alpha^2\lambda\mu-2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\end{aligned}\right)\\&\quad\left(\begin{aligned}&a\alpha^3+a\alpha^2\lambda\mu^2+2a\alpha^2\lambda\mu+a\alpha^2\lambda+2a\alpha\lambda^2\mu^3+a\alpha\lambda^2\mu^2+2a\alpha\lambda^2\mu\\&+2a\lambda^3\mu^3-\alpha^3-4\alpha^2\lambda\mu-2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\end{aligned}\right)\\&-a^2\left(\begin{aligned}&a^2\alpha^3+a^2\alpha^2\lambda\mu^2+2a^2\alpha^2\lambda\mu+2a^2\alpha\lambda^2\mu^3+a^2\alpha^2\lambda+a^2\alpha\lambda^2\mu^2+2a^2\alpha\lambda^2\mu\\&+2a^2\lambda^3\mu^3-\alpha^3-4\alpha^2\lambda\mu-2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\end{aligned}\right)\\&\quad\left(\begin{aligned}&a\alpha^3+a\alpha^2\lambda\mu^2+2a\alpha^2\lambda\mu+a\alpha^2\lambda+2a\alpha\lambda^2\mu^3+a\alpha\lambda^2\mu^2+2a\alpha\lambda^2\mu\\&+2a\lambda^3\mu^3-\alpha^3-4\alpha^2\lambda\mu-2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\end{aligned}\right)\\&\quad\left(\alpha^2\lambda\mu^2-2\alpha^2\lambda\mu+\alpha^2\lambda+2\lambda^2\mu^2+2\lambda^3\mu^3\right)\\&+\left(\alpha^2\lambda\mu^2-2\alpha^2\lambda\mu+\alpha^2\lambda+2\lambda^2\mu^2+2\lambda^3\mu^3\right)\\&\quad\left(\begin{aligned}&a^2\alpha^3+a^2\alpha^2\lambda\mu^2+2a^2\alpha^2\lambda\mu+2a^2\alpha\lambda^2\mu^3+a^2\alpha^2\lambda+a^2\alpha\lambda^2\mu^2+2a^2\alpha\lambda^2\mu\\&+2a^2\lambda^3\mu^3-\alpha^3-4\alpha^2\lambda\mu-2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\end{aligned}\right)\end{aligned}}{\begin{aligned}&c^2\left(a^2-1\right)\left(a-1\right)^2\\&\left(\begin{aligned}&a^2\alpha^3+a^2\alpha^2\lambda\mu^2+2a^2\alpha^2\lambda\mu+2a^2\alpha\lambda^2\mu^3+a^2\alpha^2\lambda+a^2\alpha\lambda^2\mu^2+2a^2\alpha\lambda^2\mu\\&+2a^2\lambda^3\mu^3-\alpha^3-4\alpha^2\lambda\mu-2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\end{aligned}\right)\\&\left(\begin{aligned}&a\alpha^3+a\alpha^2\lambda\mu^2+2a\alpha^2\lambda\mu+a\alpha^2\lambda+2a\alpha\lambda^2\mu^3+a\alpha\lambda^2\mu^2+2a\alpha\lambda^2\mu\\&+2a\lambda^3\mu^3-\alpha^3-4\alpha^2\lambda\mu-2\alpha\lambda^2\mu^3-2\alpha\lambda^2\mu+\alpha\lambda^2\mu^2\end{aligned}\right)\end{aligned}}$$

Numerical Illustrations

Table 1

a	$\mu=0.5,\alpha=0.2,\lambda=0.3$ $c=0.5$	
	Mean	**Variance**
2	1.222222	13.87125
3	0.725275	10.96233
4	0.523810	9.96901
5	0.411985	9.472471
6	0.340206	9.175408
7	0.290019	8.977927
8	0.252874	8.837204
9	0.224236	8.731867
10	0.201465	8.650068
11	0.182918	8.584713

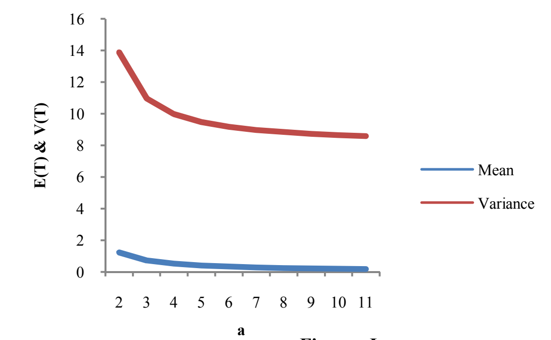

Figure - I

Table 2

μ	$\alpha = 0.2, \lambda = 0.3, a = 2$, $c = 0.5$ Mean	Variance
0.5	1.222222	13.87125
1	1.159420	11.44935
1.5	1.125731	10.32889
2	1.111111	9.876543
2.5	1.104587	9.680903
3	1.101614	9.59298
3.5	1.100309	9.554605
4	1.099831	9.540598
4.5	1.099779	9.539093
5	1.099951	9.544117

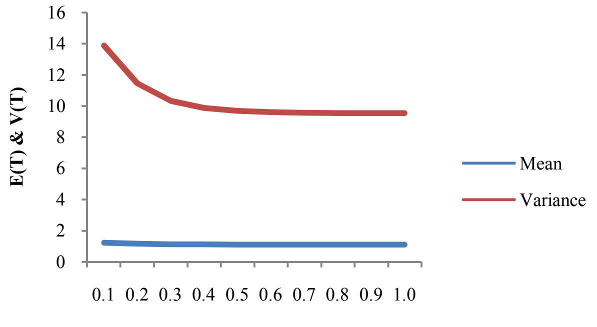

Figure - 2

Table 3

λ	$\mu = 0.5, \alpha = 0.2, a = 2$, $c = 0.5$ Mean	Variance
0.5	1.181818	12.25987
1	1.125000	10.3058
1.5	1.095238	9.40697
2	1.076923	8.891519
2.5	1.064516	8.557573
3	1.055556	8.323711
3.5	1.048780	8.150841
4	1.043478	8.017872
4.5	1.039216	7.912428
5	1.035714	7.826767

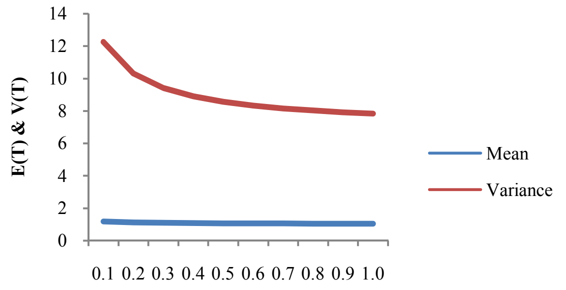

Figure - 3

Table 4

α	$\mu = 0.5, c = 0.5, \lambda = 0.3$, $a = 2$ Mean	Variance
0.1	1.166667	11.70556
0.2	1.222222	13.87125
0.3	1.250000	15.10417
0.4	1.266667	15.89859
0.5	1.277778	16.45275
0.6	1.285714	16.86122
0.7	1.291667	17.17473
0.8	1.296296	17.42293
0.9	1.300000	17.62429
1	1.303030	17.79091

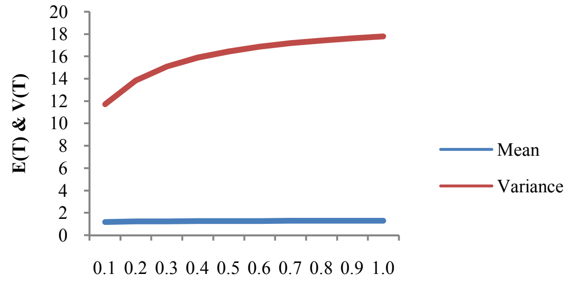

Figure - 4

Table -5

c	$\alpha = 0.2, \lambda = 0.3, a = 2$, $\mu = 0.5$ Mean	Variance
1	0.6111	3.4678
2	0.3056	0.8670
3	0.2037	0.3853
4	0.1528	0.2167
5	0.1222	0.1387
6	0.1019	0.0963
7	0.0873	0.0708
8	0.0764	0.0542
9	0.0679	0.0428
10	0.0611	0.0347

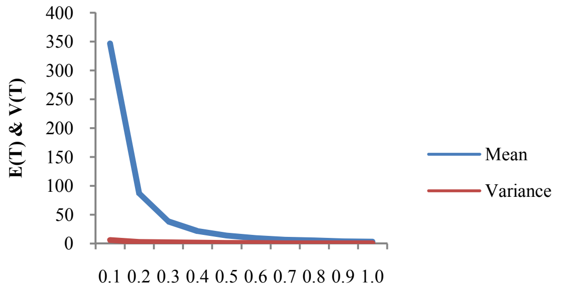

Figure - 5

CONCLUSIONS

It can be seen from Table-1 that as $'a'$ increases the mean time to seroconversion decreases. This is due to the fact that in a geometric process if $a > 1$ then the sequence of random variables would be decreasing and so the inter arrival times form a decreasing sequence. Hence $E(T)$ decreases by the fact that the contacts would be more frequent. Similarly the variance of the seroconversion time also decreases. It is easily seen from Table (1) and Fig.(1).

If μ which is parameter of threshold which follows Generalized Rayleigh distribution, increases then the expected time to seroconversion decreases. This due to the fact that $E(T)$ decreases if μ increases. Hence the average threshold level is less and hence it takes less time to cross the same. Hence the variance of seroconversion also decreases as indicated in Table-2 and Fig.2.

The behavior of $E(T)$ for fixed k,α,c and μ but with variation in λ. An increase in λ which is the parameter of mixed exponential of threshold increases then the expected time to seroconversion and its variance decrease as indicated in Table-3 and Fig.3.

The value of $E(T)$ corresponding to the variation in α with k,μ,c and λ are fixed. α happens to be the parameter of the random variable depicting the amount of antigenic diversity contribution in successive contacts. If α increases both expected time to seroconversion and its variance also increase as indicated in Table-4 and Fig.4.

If $'c'$ the parameter of the distribution of the random variable, U_i denoting the inter contact time is increasing then expected time to seroconversion decreases. It is due to fact that since U_i follows Exp(c) and so $E(U_i)=\frac{1}{c}$ Hence if c increases, then the inter-arrival time on the average decreases. Then there would be more number of contacts. Hence the expected time to seroconversion decreases and its variance also decrease as indicated in Table-5 and Fig.5.

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