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Research Article

SQUARE SUM DIFFERENCE PRODUCT PRIME LABELING OF SOME CYCLE RELATED GRAPHS

Sunoj B S[1]* and Mathew Varkey T K[2]

[1]Department of Mathematics, Government Polytechnic College, Attingal
[2]Department of Mathematics, T K M College of Engineering, Kollam 5

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A B S T R A C T

Square sum difference product prime labeling of a graph is the labeling of the vertices with {0,1,2---------,p-1} and the edges with absolute difference of the sum of the squares of the labels of the incident vertices and product of the labels of the incident vertices. The greatest common incidence number of a vertex (***gcin***) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the ***gcin*** of each vertex of degree greater than one is one, then the graph admits square sum difference product prime labeling. In this paper we investigate some cycle related graphs for square sum difference product prime labeling.

INTRODUCTION

All graphs in this paper are simple, finite, connected and undirected. The symbol V(G) and E(G) denote the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4] . Some basic concepts are taken from [1] and [2]. In [5], we introduced the concept of square sum difference product prime labeling and proved that some path related graphs admit this kind of labeling In this paper we investigated square sum difference product prime labeling of some cycle related graphs.

Definition: Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

Main Results

Definition: Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection

f : V(G) → {0,1,2,3,----------------,p-1} by $f(v_i)$ = i−1 , for every i from 1 to p and define a 1-1 mapping $f^*_{sqsdppl}$: E(G) → set of natural numbers N by $f^*_{sqsdppl}(uv)$ = $|\{f(u)\}^2 + \{f(v)\}^2 - f(u)f(v)|$.The induced function $f^*_{sqsdppl}$ is said to be square sum difference product prime labeling, if for each vertex of degree at least 2, the greatest common incidence number is 1.

Definition: A graph which admits square sum difference product prime labeling is called a square sum difference product prime graph.

Theorem: Cycle C_n admits square sum difference product prime labeling.

Proof: Let G = C_n and let v_1,v_2,----------------,v_n are the vertices of G

Here |V(G)| = n and |E(G)| = n

Define a function f : V → {0,1,2,3,----------------,n-1 } by

$f(v_i)$ = i-1 , i = 1,2,------,n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i\, v_{i+1})$ = i^2-i+1, i = 1,2,------------,n-1

$f^*_{sqsdppl}(v_1\, v_n)$ = n^2-2n+1,

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_1) = gcd of $\{f^*_{sqsdppl}(v_1\, v_2)$, $f^*_{sqsdppl}(v_1\, v_n)\}$
= gcd of { 1, $(n-1)^2$} = 1.

gcin of (v_{i+1}) = gcd of $\{f^*_{sqsdppl}(v_i\, v_{i+1})$, $f^*_{sqsdppl}(v_{i+1}\, v_{i+2})\}$
= gcd of { i^2-i+1, i^2+i+1}
= gcd of {2i , i^2-i+1}
= gcd of {i , i^2-i+1}

Corresponding author:* **Sunoj B S
Department of Mathematics, Government Polytechnic College, Attingal

= 1, i = 1,2,------------,n-2

gcin of (v_n) = gcd of {$f^*_{sqsdppl}(v_1\ v_n)$, $f^*_{sqsdppl}(v_{n-1}\ v_n)$ }
= gcd of { $(n-1)^2$, n^2-3n+3}
= gcd of {n-1 , (n-1)(n-2)+1}
= 1

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence C_n , admits square sum difference product prime labeling.

Theorem: Middle graph of cycle C_n admits square sum difference product prime labeling.

Proof: Let G =C(P_n) and let v_1,v_2,----------------,v_{2n} are the vertices of G
Here |V(G)| = 2n and |E(G)| = 3n
Define a function f : V → {0,1,2,3,----------------,2n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,2n
Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i\ v_{i+1})$ = i^2-i+1, i = 1,2,------,2n-1
$f^*_{sqsdppl}(v_{2i-1}\ v_{2i+1})$ = $4i^2$-4i+4, i = 1,2,-------,n-1
$f^*_{sqsdppl}(v_1\ v_{2n})$ = $(2n-1)^2$
$f^*_{sqsdppl}(v_1\ v_{2n-1})$ = $(2n-2)^2$
Clearly $f^*_{sqsdppl}$ is an injection.
gcin of (v_1) = gcd of {$f^*_{sqsdppl}(v_1\ v_2)$, $f^*_{sqsdppl}(v_1\ v_{2n})$ }
= gcd of { 1, $(2n-1)^2$} = 1.
gcin of (v_{i+1}) =1, i = 1,2,------,2n-2
gcin of (v_{2n}) = gcd of {$f^*_{sqsdppl}(v_1\ v_{2n})$, $f^*_{sqsdppl}(v_{2n-1}\ v_{2n})$ }
= gcd of { $(2n-1)^2$, (2n-1)(2n-2)+1}
=1
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence M(C_n), admits square sum difference product prime labeling.

Theorem: Duplicate graph of cycle C_n admits square sum difference product prime labeling, when n is odd.

Proof: When n is odd, duplicate graph of cycle C_n becomes cycle C_{2n}. By Theorem -1, C_{2n} admits square sum difference product prime labeling.

Definition: A sunflower graph SF(n) is defined as a graph obtained by starting with cycle C_n with consecutive vertices v_1,v_2,-----------,v_n and creating new vertices w_1,w_2,----------,w_n with w_i connected to vi and v_{i+1}.

Theorem: Sunflower graph admits square sum difference product prime labeling.

Proof: Let G = SF(n) and let v_1,v_2,----------------,v_{2n} are the vertices of G
Here |V(G)| = 2n and |E(G)| = 3n
Define a function f : V → {0,1,2,3,----------------,2n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,2n
Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows
$f^*_{sqsdppl}(v_i\ v_{i+1})$ = i^2-i+1, i = 1,2,------,2n-1
$f^*_{sqsdppl}(v_{2i-1}\ v_{2i+1})$ = $4i^2$-4i+4, i = 1,2,-------,n-1
$f^*_{sqsdppl}(v_1\ v_{2n})$ = $(2n-1)^2$
$f^*_{sqsdppl}(v_1\ v_{2n-1})$ = $(2n-2)^2$
Clearly $f^*_{sqsdppl}$ is an injection.
gcin of (v_1) = 1
gcin of (v_{i+1}) = 1, i = 1,2,------------,2n-2
gcin of (v_{2n}) = 1
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence SF(n), admits square sum difference product prime labeling.

Theorem: Corona of cycle C_n admits square sum difference product prime labeling.

Proof: Let G = C_n ʘ K_1 and let v_1,v_2,----------------,v_{2n} are the vertices of G
Here |V(G)| = 2n and |E(G)| = 2n
Define a function f : V → {0,1,2,3,----------------,2n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,2n
Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows
$f^*_{sqsdppl}(v_{2i-1}\ v_{2i+1})$ = $4i^2$-4i+4, i = 1,2,------------,n-1
$f^*_{sqsdppl}(v_{2i-1}\ v_{2i})$ = $4i^2$-6i+3, i = 1,2,------------,n
$f^*_{sqsdppl}(v_1\ v_{2n-1})$ = $(2n-2)^2$
Clearly $f^*_{sqsdppl}$ is an injection.
gcin of (v_1) = 1
gcin of (v_{2i+1}) = gcd of {$f^*_{sqsdppl}(v_{2i-1}\ v_{2i+1})$, $f^*_{sqsdppl}(v_{2i+1}\ v_{2i+2})$, $f^*_{sqsdppl}(v_{2i+1}\ v_{2i+3})$ }
= gcd of { $4i^2$-4i+4, $4i^2$+2i+1, $4i^2$+4i+4}, i= 1,2,------------,n-2
gcin of (v_{2n-1}) = gcd of {$f^*_{sqsdppl}(v_1\ v_{2n-1})$, $f^*_{sqsdppl}(v_{2n-1}\ v_{2n})$ }
= gcd of {$(2n-2)^2$, $4n^2$-6n+3}
= gcd of {2n-2 , $4n^2$-6n+3}
= gcd of {2n-2, 2n-1}
= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence C_n ʘ K_1, admits square sum difference product prime labeling.

Theorem: Split graph of cycle C_n admits square sum difference product prime labeling, when n is odd.
Proof: Let G = S'(C_n) and let v_1,v_2,----------------,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| = 3n
Define a function f : V → {0,1,2,3,----------------,2n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,2n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i\ v_{i+1})$ = i^2-i+1, i = 1,2,------,2n-1
$f^*_{sqsdppl}(v_{2i-1}\ v_{n+2i})$ = $(n+1)^2$+(n+2i-1)(2i-2), i = 1,2,-------,$\frac{n-1}{2}$
$f^*_{sqsdppl}(v_{2i+1}\ v_{n+2i})$ = $(n-1)^2$+(n+2i-1)(2i), i = 1,2,-------,$\frac{n-1}{2}$
$f^*_{sqsdppl}(v_1\ v_{2n})$ = $(2n-1)^2$
$f^*_{sqsdppl}(v_1\ v_n)$ = $(n-1)^2$

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_1) = 1

gcin of (v_{i+1}) = 1, i = 1,2,------,2n-2

gcin of (v_{2n}) = 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence S(C_n), admits square sum difference product prime labeling.

Definition: Let G be the graph obtained by joining a path P_m to a cycle C_n. G is called tadpole graph and is denoted by $C_n(P_m)$.

Theorem: The graph $C_n(P_m)$ admits square sum difference product prime labeling, if n is odd.

Proof: Let G = $C_n(P_m)$ and let v_1,v_2,----------------,v_{n+m-1} are the vertices of G

Here |V(G)| = n+m-1 and |E(G)| = n+m-1

Define a function f : V → {0,1,2,3,----------------,n+m-2 } by

f(v_i) = i-1 , i = 1,2,------,n+m-1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i\, v_{i+1})$ = i^2-i+1, i = 1,2,---------,n+m-2

$f^*_{sqsdppl}(v_1\, v_n)$ = $(n-1)^2$

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_1) = 1

gcin of (v_{i+1}) = 1, i = 1,2,---------,n+m-2

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $C_n(P_m)$, admits square sum difference product prime labeling.

Definition: Let G be the graph obtained by joining two copies of path P_m to two consecutive vertices of cycle C_n. G is denoted by $C_n(2P_m)$.

Theorem: The graph $C_n(2P_m)$ admits square sum difference product prime labeling, when n and m are odd.

Proof: Let G = $C_n(P_m)$ and let v_1,v_2,----------------,v_{n+2m-2} are the vertices of G

Here |V(G)| = n+2m-2 and |E(G)| = n+2m-2

Define a function f : V → {0,1,2,3,----------------,n+2m-3 } by

f(v_i) = i-1 , i = 1,2,------,n+2m-2

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i\, v_{i+1})$ = i^2-i+1, i = 1,2,--------,n+2m-3

$f^*_{sqsdppl}(v_m\, v_{n+m-1})$ = $(n-1)^2$+(n+m-2)(m-1)

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,--------,n+2m-4

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $C_n(2P_m)$, admits square sum difference product prime labeling.

Definition: Let u_1, u_2,----------u_n be the vertices of the first copy of cycle C_n and v_1, v_2,-------------,v_n are the vertices of second copy of cycle C_n. Let G be the graph obtained by joining v_1 to u_2, v_2 to u_3 ------------,v_n to u_1. G is called Z graph of cycle C_n and is denoted by Z(C_n).

Theorem: Z graph of cycle C_n admits square sum difference product prime labeling, if n is odd.

Proof: Let G = Z(C_n) and let v_1,v_2,----------------,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| = 3n

Define a function f : V → {0,1,2,3,----------------,2n-1 } by

f(v_i) = i-1 , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i\, v_{i+1})$ = i^2-i+1, i = 1,2,------------,n-1

$f^*_{sqsdppl}(v_{n+i}\, v_{n+i+1})$ = $(n+i)^2$-(n+i)+1, i = 1,2,------------,n-1

$f^*_{sqsdppl}(v_{i+1}\, v_{n+i})$ = $(n-1)^2$ + (n+i-1)i, i = 1,2,------------,n-1

$f^*_{sqsdppl}(v_1\, v_{2n})$ = $(2n-1)^2$

$f^*_{sqsdppl}(v_1\, v_n)$ = $(n-1)^2$

$f^*_{sqsdppl}(v_{n+1}\, v_{2n})$ = $3n^2$-3n+1

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_1) = 1

gcin of (v_{i+1}) = 1, i= 1,2,------------,n-2

gcin of (v_{n+i+1}) = 1, i= 1,2,------------,n-2

gcin of (v_{2n}) = gcd of {$f^*_{sqsdppl}(v_1\, v_{2n})$, $f^*_{sqsdppl}(v_{2n-1}\, v_{2n})$ }
= gcd of { $(2n-1)^2$, (2n-1)(2n-2)+1}
= 1.

gcin of (v_n) = gcd of {$f^*_{sqsdppl}(v_1\, v_n)$, $f^*_{sqsdppl}(v_{n-1}\, v_n)$ }
= gcd of { $(n-1)^2$, (n-1)(n-2)+1}
= 1.

gcin of (v_{n+1}) = gcd of {$f^*_{sqsdppl}(v_2\, v_{n+1})$, $f^*_{sqsdppl}(v_{n+1}\, v_{n+2})$ }
= gcd of {n^2-n+1, n^2+n+1}
= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence Z(C_n), admits square sum difference product prime labeling.

References

1. Apostol. Tom M, Introduction to Analytic Number Theory, Narosa, (1998).
2. F Harary, Graph Theory, Addison-Wesley, Reading, Mass, (1972)
3. Joseph A Gallian, A Dynamic Survey of Graph Labeling, *The Electronic Journal of Combinatorics* (2016), #DS6, pp 1-408.
4. T K Mathew Varkey, Some Graph Theoretic Generations Associated with Graph Labeling, PhD Thesis, University of Kerala 2000.
5. Sunoj B S, Mathew Varkey T K, Square Sum Difference Product Prime Labeling of Some Path Related Graphs, *International Journal for Research in Applied Science and Engineering Technology*, Vol.5, Issue 10, pp 1404-1408 (October 2017).

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