International Journal of Current Advanced Research
ISSN: O: 2319-6475, ISSN: P: 2319-6505, Impact Factor: SJIF: 5.995
Available Online at www.journalijcar.org
Volume 6; Issue 11; November 2017; Page No. 7154-7160
DOI: http://dx.doi.org/10.24327/ijcar.2017.7160.1091

Research Article

DEGREE BASED TOPOLOGICAL INDICES OF JAHANGIR GRAPHS

Gayathri P[1]* and Priyanka U[2]

Department of Mathematics, A.V.C. College (Autonomous), Mannampandal-609305

A R T I C L E I N F O

Article History:
Received 8th August, 2017
Received in revised form 29th September, 2017
Accepted 4th October, 2017
Published online 28th November, 2017

Key words:

M-polynomial; Topological index; Jahangir graph

A B S T R A C T

In this Paper, we make progress to many degree based indices like that Randic, Geometric - Arithmetic, Sum-Connectivity, Harmonic, First Zagreb and Second Zagreb, Second Modified Zagreb, Inverse sum, Alberston, Atom - Bond Connectivity, Symmetric Division Index and Augmented Zagreb and also obtained closed forms all the above said indices using M-polynomial of Jahangir graphs $J_{n,m}$ for all values of $n\geq2$,$m\geq3$.Our results are easier to evaluate and simpler than existing results in literature.

INTRODUCTION

In mathematical chemistry, mathematical tools like polynomials and topological-based numbers predict properties of compounds without using quantum mechanics. These tools, in combination, capture information hidden in the symmetry of molecular graphs. A topological index is a functionthat characterizes the topology of the graph. Most commonly known invariants of such kinds aredegree-based topological indices. These are actually the numerical values that correlate the structure with various physical properties, chemical reactivity, and biological activities [1-5]. It is an established fact that many properties such as heat of formation, boiling point, strain energy, rigidity and fracture toughness of a molecule are strongly connected to its graphical structure and this fact plays a synergic role in chemical graph theory. Algebraic polynomials play a significant part in chemistry. Hosoya polynomial [6] is one such well-known example which determines distance-based topological indices. M-polynomial [7], introduced in 2015, plays the same role in determining closed forms of many degree-based topological indices [8-11]. The main advantage of M-polynomial [36-40] is the wealth of information that it contains about degree-based graph invariants. The Jahangir graph $J_{n,m}$ is a graph on nm + 1 vertices and m(n + 1) edges for all n ≥ 2 and m $\geq$ 3. Jn,m consists of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance to each other. Figure [1] shows some particular cases of $J_{n,m}$. The Figure $J_{2,8}$ is carved on Jahangir's tomb. It is situated 5 km northwest of Lahore, Pakistan.

Corresponding author: **Gayathri P**
Department of Mathematics, A.V.C. College (Autonomous), Mannampandal-609305

In [12], Laurdusamy *et al.* computed the pebbling number of Jahangir graph $J_{2,m}$ for $m \geq 8$. Mojdeh *et al.* in [13] computed domination number in $J_{2,m}$ and Ramsey number for $J_{3,m}$ in [14] by Ali *et al*. Weiner index and Hosoya Polynomial of $J_{2,m}$ $J_{3,m}$ and $J_{4,m}$ are computed in [15-17]. All these results are partial and need to be generalized for all values of m and n.

Computational Procedure of M-Polynomial

M-Polynomial of graph G is defined as if $G = (V, E)$ is a graph and $v \in V$, then $d_v(G)$ (or d_v for short if G is clear from the context) denotes the degree of v. Let G be a graph and let $m_{ij}(G), i, j \geq 1,$ be the number of edges e = uv of G such that $\{d_u(G), d_v(G)\} = \{i, j\}$. The M-polynomial of G as $M(G; x, y) = \sum_{i\leq j} m_{ij}(G)x^i y^j$ .For a graph $G = (V, E)$, a degree-based topological index is a graph invariant of the form $I(G) = \sum_{e=uv\in E} f(d_u, d_v)$ where $f = f(x, y)$ is a function appropriately selected for possible chemical applications.In this article, we compute closed form of some degree-based topological indices of the Jahangir graph by using the *M*-polynomial. Let "v" be the number of vertices and "e" be the number of edges for $J_{n,m}$ graph represents (nm+1) vertices and m(n+1) edges respectively for all $n\geq2$, $m\geq3$. i.e., a graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm}. Here we considered the Jahangir graphs for $n = 2, m \geq 3$, the results are given in case 1 to case 3 and for $n = 3, m \geq 3$, the results are given in case 4 to case 6 and for

$n = 4, m \geq 3$, the results are given in case 7 to case 9 , for $n = 5, m \geq 3$, the results are given in case 10 to case 12, finally, the results are generalized for $n \geq 2, m \geq 3$.

Case 1: For $J_{2,3}$, the number of edges with end degrees (2,2) is equal to 0, the number of edges with end degrees (2,3) is equal to 6, the number of edges with end degrees (3,3) is equal to 3, therefore the total number of edges isequal to 9.

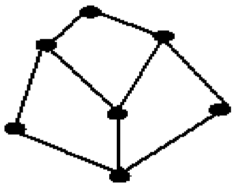

Fig 1 $J_{(2,3)}$

Case 2: For $J_{2,4}$, the number of edges with end degrees (2,2) is equal to 0, the number of edges with end degrees (2,3) is equal to 8, the number of edges with end degrees (3,3) isequal to 0,the number of edges with end degrees (3,4) isequal to 4, therefore the total number of edges isequal to 12.

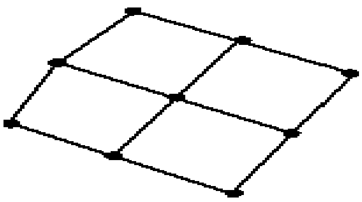

Fig 2 J(2,4)

Case (3): For $J_{2,5}$, the number of edges with end degrees (2,2) is equal to 0, the number of edges with end degrees (2,3) is equal to10, the number of edges with end degrees (3,3) is equal to 0,the number of edges with end degrees (3,4) is equal to 0, the number of edges with end degrees (3,5) isequal to 5, therefore the total number of edges is equal to 15.

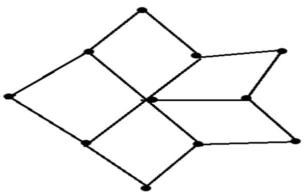

Fig 3 J(2,5)

Case 4: For $J_{3,3}$, the number of edges with end degrees (2,2) is equal to 3, the number of edges with end degrees (2,3) is equal to 6, the number of edges with end degrees (3,3) is equal to 3, the therefore total number of edges is equal to12.

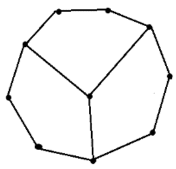

Fig 4 J(3,3)

Case 5: For $J_{3,4}$, the number of edges with end degrees (2,2) isequal to4, the number of edges with end degrees (2,3) is equal to 8, the number of edges with end degrees (3,3) is equal to 4, therefore the total number of edges is equal to16.

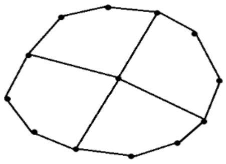

Fig 5 B(3,4)

Case 6: For $J_{3,5}$, the number of edges with end degrees (2,2) is equal to 5, the number of edges with end degrees (2,3) is equal to 10, the number of edges with end degrees (3,3) is equal to 5, therefore the total number of edges is equal to 20.

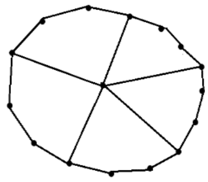

Fig 6 B(3,5)

Case 7: For $J_{4,3}$, the number of edges with end degrees (2,2) is equal to 6, the number of edges with end degrees (2,3) is equal to 6, the number of edges with end degrees (3,3) is equal to 3, therefore the total number of edges is equal to 15.

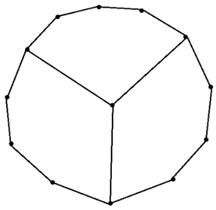

Fig 7 B(4,3)

Case 8: For $J_{4,4}$, the number of edges with end degrees (2,2) is equal to 8, the number of edges with end degrees (2,3) is equal to 8, the number of edges with end degrees (3,3) is equal to 4, therefore the total number of edges is equal to 20.

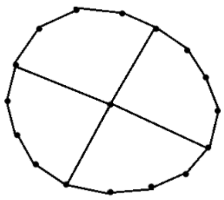

Fig 8 B(4,4)

Case (9): For $J_{4,5}$, the number of edges with end degrees (2,2) is equal to 10, the number of edges with end degrees (2,3) is equal to 10, the number of edges with end degrees (3,3) is equal to 5, therefore the total number of edges is equal to 25.

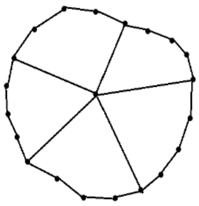

Fig 9 B(4,5)

Case 10:For $J_{5,3}$, the number of edges with end degrees (2,2) is equal to 10, the number of edges with end degrees (2,3) is equal to 10, the number of edges with end degrees (3,3) is equal to 5, therefore the total number of edges is equal to 25.

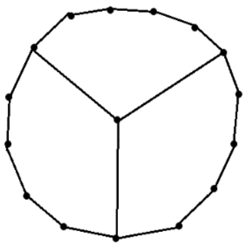

Fig 10 B(5,3)

Case 11: For $J_{5,4}$, the number of edges with end degrees (2,2) is equal to 12, the number of edges with end degrees (2,3) is equal to 8, the number of edges with end degrees (3,3) is equal to 4, therefore the total number of edges is equal to 24.

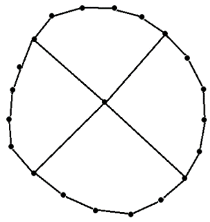

Fig 11 B(5,4)

Case 12: For $J_{5,5}$, the number of edges with end degrees (2,2) is equal to 15, the number of edges with end degrees (2,3) is equal to 10, the number of edges with end degrees (3,3) isequal to 5, therefore the total number of edges is equal to 30.

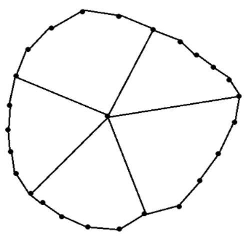

Fig 12 B(5,5)

M-Polynomial of chain of Jahangir graph is developed and it is given by

$$J_{m,n} = \{(n-2)m\}x^2y^2 + \{2m\}x^2y^3$$
$$+ \{m\}x^3y^3 \ for\ all\ n \geq 2, m \geq 3$$

Theorem 1

Let $J_{n,m}$ is a Jahangir graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm}, then the Randic index is given by $\chi(J_{nm}) = m\left\{\frac{n}{2}+\sqrt{\frac{2}{3}}-1\right\}+\sqrt{\frac{m}{3}}$

Proof

Randic Index is denoted by $\chi(J_{nm}) = \frac{1}{\sqrt{ij}}$

$$=\frac{1}{\sqrt{ij}}\{(n-2)m\}+\frac{1}{\sqrt{ij}}\{2m\}+\frac{1}{\sqrt{ij}}\{m\}$$

$$=\frac{1}{\sqrt{2*2}}\{(n-2)m\}+\frac{1}{\sqrt{2*3}}\{2m\}+\frac{1}{\sqrt{3*m}}\{m\}$$

$$=\frac{1}{\sqrt{4}}\{(n-2)m\}+\frac{1}{\sqrt{6}}\{2m\}+\frac{1}{\sqrt{3*m}}\{m\}$$

$$=\frac{1}{2}\{(n-2)m\}+\frac{1}{\sqrt{2}\sqrt{3}}\{2m\}+\frac{1}{\sqrt{3*m}}\{m\}$$

$$=\frac{1}{2}\{(n-2)m\}+\frac{\sqrt{2}\sqrt{2}m}{\sqrt{2}\sqrt{3}}+\frac{1}{\sqrt{3*m}}\{m\}$$

$$=\frac{1}{2}\{(nm-2m)\}+\frac{\sqrt{2}\sqrt{2}m}{\sqrt{2}\sqrt{3}}+\frac{\sqrt{m}\sqrt{m}}{\sqrt{3}\sqrt{m}}$$

$$=\frac{1}{2}\{(nm-2m)\}+\frac{\sqrt{2}m}{\sqrt{3}}+\frac{\sqrt{m}}{\sqrt{3}}$$

$$=\frac{nm-2m}{2}+\sqrt{\frac{2}{3}}\ m+\sqrt{\frac{m}{3}}$$

$$=m\left\{\frac{n}{2}+\sqrt{\frac{2}{3}}-1\right\}+\sqrt{\frac{m}{3}}$$

Theorem 2

Let $J_{n,m}$ is a Jahangir graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm}, then the Arithmetic index is given by

$$GA(J_{n,m}) = m\left\{n-2+\frac{4\sqrt{6}}{5}+\frac{2\sqrt{3m}}{3+m}\right\}$$

Proof

Geometric - Arithmetic Index is denoted by $GA(J_{nm}) = \frac{2\sqrt{ij}}{i+j}$

$$=\frac{2\sqrt{ij}}{i+j}\{(n-2)m\}+\frac{2\sqrt{ij}}{i+j}\{2m\}+\frac{2\sqrt{ij}}{i+j}\{m\}$$

$$=\frac{2\sqrt{2*2}}{2+2}\{(n-2)m\}+\frac{2\sqrt{2*3}}{2+3}\{2m\}+\frac{2\sqrt{3*m}}{3+m}\{m\}$$

$$=\frac{2\sqrt{4}}{4}\{(n-2)m\}+\frac{2\sqrt{6}}{5}\{2m\}+\frac{2\sqrt{3m}}{3+m}\{m\}$$

$$=\frac{4}{4}\{(n-2)m\}+\frac{4\sqrt{6}}{5}\{m\}+\frac{2\sqrt{3m}}{3+m}\{m\}$$

$$=\{(n-2)m\}+\frac{4\sqrt{6}}{5}\{m\}+\frac{2\sqrt{3m}}{3+m}\{m\}$$

$$=nm-2m+\frac{4\sqrt{6}m}{5}+\frac{2\sqrt{3}\sqrt{m}}{3+m}\{m\}$$

$$=nm-2m+\frac{4\sqrt{6}m}{5}+\frac{2\sqrt{3}(m)^{1/2}(m)}{3+m}$$

$$=m\left\{n-2+\frac{4\sqrt{6}}{5}+\frac{2\sqrt{3m}}{3+m}\right\}$$

Theorem 3

Let $J_{n,m}$ is a Jahangir graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm}, then the Sum-Connectivity index is given by $SCI(J_{nm}) = m\left\{\frac{1}{\sqrt{m+3}}+\frac{2}{\sqrt{5}}+\frac{n}{2}-1\right\}$

Proof

Sum - Connectivity is denoted by $SCI(J_{nm}) = \frac{1}{\sqrt{i+j}}$

$$=\frac{1}{\sqrt{i+j}}\{(n-2)m\}+\frac{1}{\sqrt{i+j}}\{2m\}+\frac{1}{\sqrt{i+j}}\{m\}$$
$$=\frac{1}{\sqrt{2+2}}\{(n-2)m\}+\frac{1}{\sqrt{2+3}}\{2m\}+\frac{1}{\sqrt{3+m}}\{m\}$$
$$=\frac{1}{\sqrt{4}}\{(n-2)m\}+\frac{1}{\sqrt{5}}\{2m\}+\frac{1}{\sqrt{3+m}}\{m\}$$
$$=\frac{1}{2}\{(n-2)m\}+\frac{1}{\sqrt{5}}\{2m\}+\frac{1}{\sqrt{3+m}}\{m\}$$
$$=\frac{nm}{2}-\frac{2m}{2}+\frac{2m}{\sqrt{5}}+\frac{m}{\sqrt{3+m}}$$
$$=m\left\{\frac{1}{\sqrt{m+3}}+\frac{2}{\sqrt{5}}+\frac{n}{2}-1\right\}$$

Theorem 4

Let $J_{n,m}$ is a Jahangir graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm}, then the Harmonic index is given by

$$HI(J_{nm})=\frac{m}{10(3+m)}\{15n+5nm-2m+14\}$$

Proof

Harmonic index is denoted by $HI(J_{nm}) = \frac{2}{i+j}$

$$=\frac{2}{i+j}\{(n-2)m\}+\frac{2}{i+j}\{2m\}+\frac{2}{i+j}\{m\}$$
$$=\frac{2}{2+2}\{(n-2)m\}+\frac{2}{2+3}\{2m\}+\frac{2}{3+m}\{m\}$$
$$=\frac{2}{4}\{(n-2)m\}+\frac{2}{5}\{2m\}+\frac{2}{3+m}\{m\}$$
$$=\frac{1}{2}\{(n-2)m\}+\frac{2}{5}\{2m\}+\frac{2}{3+m}\{m\}$$
$$=\frac{nm}{2}-\frac{2m}{2}+\frac{4m}{5}+\frac{2m}{3+m}=\frac{nm}{2}-m+\frac{4m}{5}+\frac{2m}{3+m}$$
$$=\frac{nm}{2}-\frac{m}{5}+\frac{2m}{3+m}$$
$$=\frac{5nm-2m}{10}+\frac{2m}{3+m}=\frac{(3+m)(5nm-2m)+20m}{10(3+m)}$$
$$=\frac{15nm-6m+5m^2n-2m^2+20m}{10(3+m)}$$
$$=\frac{15nm+5m^2n-2m^2+14m}{10(3+m)}$$
$$=\frac{m}{10(3+m)}\{15n+5nm-2m+14\}$$

Theorem 5

Let $J_{n,m}$ is a Jahangir graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm}, then the First Zagreb index is given by $M_1(J_{nm}) = m\{4n+m+5\}$

Proof

First Zagreb Index is denoted by

$M_1(J_{nm}) = i+j$

$$=(i+j)\{(n-2)m\}+(i+j)\{2m\}+(i+j)\{m\}$$
$$=(2+2)\{(n-2)m\}+(2+3)\{2m\}+(3+m)\{m\}$$
$$=(4)\{(n-2)m\}+(5)\{2m\}+(3+m)\{m\}$$
$$=(4)\{(n-2)m\}+\{10m\}+(3m+m^2)$$
$$=4nm-8m+\{10m\}+(3m+m^2)$$
$$=4nm-8m+10m+3m+m^2$$
$$=4nm+2m+3m+m^2=4nm+5m+m^2$$
$$=m\{4n+m+5\}$$

Theorem 6

Let $J_{n,m}$ is a Jahangir graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm}, then the Second Zagreb index is given by $M_2(J_{nm}) = m\{4n+3m+5\}$

Proof

Second Zagreb index is denoted by $M_2(J_{nm}) = ij$

$$=(ij)\{(n-2)m\}+(ij)\{2m\}+(ij)\{m\}$$
$$=(2*2)\{(n-2)m\}+(2*3)\{2m\}+(3*m)\{m\}$$
$$=(4)\{(n-2)m\}+(6)\{2m\}+(3*m)\{m\}$$
$$=(4)\{(n-2)m\}+\{12m\}+(3m^2)$$
$$=4nm-8m+\{12m\}+(3m^2)=4nm-8m+12m+3m^2$$
$$=4nm+4m+3m^2=m\{3m+4n+4\}$$

Theorem 7

Let $J_{n,m}$ is a Jahangir graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm}, then the Second Modified Zagreb index is given by $M_3(J_{nm}) = \frac{1}{12}\{3nm-2m+4\}$

Proof

Second Modified Zagreb index is denoted by $M_3(J_{nm}) = \frac{1}{ij}$

$$=\frac{1}{ij}\{(n-2)m\}+\frac{1}{ij}\{2m\}+\frac{1}{ij}\{m\}$$

$$=\frac{1}{2*2}\{(n-2)m\}+\frac{1}{2*3}\{2m\}+\frac{1}{3*m}\{m\}$$

$$=\frac{1}{4}\{(n-2)m\}+\frac{1}{6}\{2m\}+\frac{1}{3*m}\{m\}$$

$$=\frac{nm}{4}-\frac{2m}{4}+\frac{2m}{6}+\frac{m}{3*m}$$

$$=\frac{nm}{4}-\frac{m}{2}+\frac{m}{3}+\frac{1}{3}=\frac{nm}{4}-\frac{m}{2}+\frac{(m+1)}{3}$$

$$=\frac{nm}{4}-\frac{m}{2}+\frac{(m+1)}{3}=\frac{3nm-6m+4(m+1)}{12}$$

$$=\frac{3nm-6m+4m+4}{12}=\frac{3nm-2m+4}{12}$$

$$=\frac{1}{12}\{3nm-2m+4\}$$

Theorem 8

Let $J_{n,m}$ is a Jahangir graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm}, then the Inverse sum index is given by $IS(J_{nm})=\frac{m(5mn+15n+17m+6)}{5(3+m)}$

Proof

Inverse Sum Index is denoted by

$$IS(J_{nm}) = \frac{ij}{i+j}$$

$$=\frac{ij}{i+j}\{(n-2)m\}+\frac{ij}{i+j}\{2m\}+\frac{ij}{i+j}\{m\}$$

$$=\frac{2*2}{2+2}\{(n-2)m\}+\frac{2*3}{2+3}\{2m\}+\frac{3*m}{3+m}\{m\}$$

$$=\frac{4}{4}\{(n-2)m\}+\frac{6}{5}\{2m\}+\frac{3*m}{3+m}\{m\}$$

$$=\{(n-2)m\}+\frac{12m}{5}+\frac{3m^2}{3+m}$$

$$=nm-2m+\frac{12m}{5}+\frac{3m^2}{3+m}=nm+\frac{2m}{5}+\frac{3m^2}{3+m}$$

$$=\frac{5nm+2m}{5}+\frac{3m^2}{3+m}=\frac{(3+m)5nm+2m+15m^2}{5(3+m)}$$

$$=\frac{15nm+5m^2n+(3+m)2m+15m^2}{5(3+m)}$$

$$=\frac{15nm+5m^2n+6m+2m^2+15m^2}{5(3+m)}$$

$$=\frac{15nm+5m^2n+6m+17m^2}{5(3+m)}$$

$$=\frac{m(5mn+15n+17m+6)}{5(3+m)}$$

Theorem 9

Let $J_{n,m}$ is a Jahangir graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm}, then the Albertson index is given by $Alb(J_{nm}) = 2m+|3-m|\{m\}$

Proof

Albertson index is denoted by

$$\text{Alb}(J_{nm}) = |i-j|$$

$$=|i-j|\{(n-2)m\}+|i-j|\{2m\}+|i-j|\{m\}$$

$$=|2-2|\{(n-2)m\}+|2-3|\{2m\}+|3-m|\{m\}$$

$$=|0|\{(n-2)m\}+|-1|\{2m\}+|3-m|\{m\}$$

$$=2m+|3-m|\{m\}$$

Theorem 10

Let $J_{n,m}$ is a Jahangir graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm}, then the Atom-Bond connectivity index is given by

$$ABC(J_{nm})=(m)\left\{\frac{n}{\sqrt{2}}+\frac{\sqrt{\left(\frac{m+1}{m}\right)}}{\sqrt{3}}\right\}$$

Proof

Atom Bomb Connectivity is denoted by

$$ABC(j_{nm})=\sqrt{\frac{i+j-2}{ij}}$$

$$=\sqrt{\frac{i+j-2}{ij}}\{(n-2)m\}+\sqrt{\frac{i+j-2}{ij}}\{2m\}$$

$$+\sqrt{\frac{i+j-2}{ij}}\{m\}$$

$$=\sqrt{\frac{2+2-2}{2*2}}\{(n-2)m\}+\sqrt{\frac{2+3-2}{2*3}}\{2m\}$$

$$+\sqrt{\frac{3+m-2}{3*m}}\{m\}$$

$$=\sqrt{\frac{2}{4}}\{(n-2)m\}+\sqrt{\frac{3}{6}}\{2m\}+\sqrt{\frac{1+m}{3m}}\{m\}$$

$$=\sqrt{\frac{1}{2}}\{(n-2)m\}+\sqrt{\frac{1}{2}}\{2m\}+\sqrt{\frac{1+m}{3m}}\sqrt{m}\sqrt{m}$$

$$=\sqrt{\frac{1}{2}}\{(n-2)m\}+\sqrt{\frac{1}{2}}\{2m\}+\frac{\sqrt{1+m}}{\sqrt{3}\sqrt{m}}\sqrt{m}\sqrt{m}$$

$$=\sqrt{\frac{1}{2}}\{(n-2)m\}+\sqrt{\frac{1}{2}}\{2m\}+\frac{\sqrt{1+m\ (m)}}{\sqrt{3}}$$

$$=\frac{1}{\sqrt{2}}\{(n-2)m\}+\frac{2m}{\sqrt{2}}+\frac{\sqrt{(m)(1+m)}}{\sqrt{3}}$$

$$=\frac{nm-2m}{\sqrt{2}}+\frac{2m}{\sqrt{2}}+\frac{\sqrt{(m)(m)\left(\frac{1}{m}+1\right)}}{\sqrt{3}}$$

$$=\frac{nm-2m}{\sqrt{2}}+\frac{2m}{\sqrt{2}}+(m)\frac{\sqrt{\left(\frac{1}{m}+1\right)}}{\sqrt{3}}$$

$$=\frac{nm}{\sqrt{2}}+(m)\frac{\sqrt{\left(\frac{1}{m}+1\right)}}{\sqrt{3}}=(m)\left\{\frac{n}{\sqrt{2}}+\frac{\sqrt{\left(\frac{m+1}{m}\right)}}{\sqrt{3}}\right\}$$

Theorem 11

Let $J_{n,m}$ is a Jahangir graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm}, then the Symmetric Division Index is given by $SD(J_{nm})=\frac{m}{3}\{m+6n+1\}+\{9\}$.

Proof

Symmetric Division Index is denoted by

SD (J_{nm}) = $\frac{i^2+j^2}{ij}$

$$=\frac{i^2+j^2}{ij}\{(n-2)m\}+\frac{i^2+j^2}{ij}\{2m\}+\frac{i^2+j^2}{ij}\{m\}$$

$$=\frac{(2)^2+(2)^2}{2*2}\{(n-2)m\}+\frac{(2)^2+(3)^2}{2*3}\{2m\}+\frac{(3)^2+(m)^2}{3m}\{m\}$$

$$=\frac{4+4}{4}\{(n-2)m\}+\frac{4+9}{6}\{2m\}+\frac{9+(m)^2}{3m}\{m\}$$

$$=\frac{8}{4}\{(nm-2m)\}+\frac{13m}{3}+\frac{9+(m)^2}{3}$$

$$=2\{(nm-2m)\}+\frac{13m+9+(m)^2}{3}$$

$$=\frac{6nm-12m+13m+9+(m)^2}{3}$$

$$=\frac{6nm+m+9+(m)^2}{3}=\frac{m(6n+1+m)+9}{3}$$

$$=\frac{m}{3}\{m+6n+1\}+\{9\}.$$

Theorem 12

Let $J_{n,m}$ is a Jahangir graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance n to each other on C_{nm}, then the Augmented Zagreb index is given by $AZI(J_{nm})=8nm+\frac{27m^4}{(1+m)^3}$

Proof

Augmented Zagreb index is denoted by AZI(J_{nm}) = $\left(\frac{ij}{i+j-2}\right)^3$

$$=\left(\frac{ij}{i+j-2}\right)^3\{(n-2)m\}+\left(\frac{ij}{i+j-2}\right)^3\{2m\}$$

$$+\left(\frac{ij}{i+j-2}\right)^3\{m\}$$

$$=\left(\frac{2*2}{2+2-2}\right)^3\{(n-2)m\}+\left(\frac{2*3}{2+3-2}\right)^3\{2m\}$$

$$+\left(\frac{3*m}{3+m-2}\right)^3\{m\}$$

$$=\left(\frac{4}{2}\right)^3\{(n-2)m\}+\left(\frac{6}{3}\right)^3\{2m\}+\left(\frac{3*m}{1+m}\right)^3\{m\}$$

$$=(2)^3\{(n-2)m\}+(2)^3\{2m\}+\left(\frac{3*m}{1+m}\right)^3\{m\}$$

$$=(8)\{(n-2)m\}+(8)\{2m\}+\left(\frac{3*m}{1+m}\right)^3\{m\}$$

$$=(8)\{nm-2m\}+16m+\left(\frac{3*m}{1+m}\right)^3\{m\}$$

$$=8nm-16m+16m+\left(\frac{3*m}{1+m}\right)^3\{m\}$$

$$=8nm+\left(\frac{3m}{1+m}\right)^3\{m\}=8nm+\frac{27m^4}{(1+m)^3}$$

CONCLUSION

In this article, we computed closed forms of degree based topological indices and M-polynomial of $J_{n,m}$ for all n≥2 ,m≥ 3. These facts are invariants of graphs and remain preserved under isomorphism.These results can also play a vital role in industry and pharmacy in the realm of that molecular graph which contains $J_{n,m}$ as its sub-graphs.

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