International Journal of Current Advanced Research
ISSN: O: 2319-6475, ISSN: P: 2319-6505, Impact Factor: SJIF: 5.995
Available Online at www.journalijcar.org
Volume 6; Issue 10; October 2017; Page No. 6453-6459
DOI: http://dx.doi.org/10.24327/ijcar.2017.6459.0946

Research Article

FRACTIONAL CUMULATIVE RISK MODEL

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A R T I C L E I N F O

Article History:

Received 15th July, 2017
Received in revised form 19th August, 2017 Accepted 25th September, 2017
Published online 28th October, 2017

Key words:

Cumulative risk model, Fractional Cumulative risk model, Weibull Triangular distribution, Weibull Trapezoidal distribution.

A B S T R A C T

This paper seeks about the fractional cumulative risk model based on weibull triangular and trapezoidal distribution.Weibull triangular distributions represents the life time distributions based on the continuous probability distribution which is shaped like a triangle. Weibull trapezoidal distribution is the life time distributions based on the shape of a quadrilateral trapezoid. The results are derived for finding the fractional cumulative risk model, which is helpful to find the life time in fractional order. Applications are also pointed out based on fractional cumulative risk model.

INTRODUCTION

The cumulative risk is a measure of the total risk that a certain event will happen during a given period of time. The cumulative risk model was introduced by N. Kannan introduced the models for experiments in which the stress levels are altered at intermediate stages during exposure. Cumulative risk model is already derived using exponential and weibull distribution.

In this paper the fractional cumulative risk model (FCRM) was derived using weibull triangular and weibull trapezoidal distributions. Fractional cumulative risk model gives the period of time in fractional order. The fractional cumulative risk model gives the accurate result compared to the cumulative risk model. The triangular and trapezoidal distribution is the shape parameter which becomes continuous. The weibull triangular distribution shaped like a triangle and weibull trapezoidal distribution is shaped like a quadrilaternal. This paper consists of three sections the first section is the introduction. Second section seeks about the cumulative risk model and fractional cumulative risk model based on weibull triangular and weibull trapezoidal distribution. The third section is the conclusion based on the applications of this model.

Cumulative Risk Model and Fractional Cumulative Risk Model based on Weibull Triangular Distribution

This section explains about the fractional cumulative risk model using weibull triangular distribution was derived this model will give better approximation and it helps to predict the survival time in fractional order. In this paper this model is helpful to find the predicting time of melanoma. Section 2.1 is the derivation fractional cumulative risk model for weibull distribution is derived. Section 2.2 is derivation based on the fractional cumulative risk model for weibull triangular distribution. Section 2.3 is derivations based on the fractional cumulative risk model for weibull trapezoidal distribution is derived these models are helpful to predict the time in fractional order.

Fractional Cumulative Risk Model: Weibull Distribution

In case of Weibull distribution the cumulative risk model takes the following form

The original form of Weibull distribution

$$h(x) = \begin{cases} \alpha_1 \lambda_1 t^{\alpha_1 - 1} & if\ 0 < t < \tau_1 \\ a + bt & if\ \tau_1 \le t < \tau_2 \\ \alpha_2 \lambda_2 t^{\alpha_2 - 1} & if\ t > \tau_2 \end{cases}$$

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The fractional cumulative risk model for the Weibull distribution

$$h^\alpha(x) = \begin{cases} \alpha_1 \lambda_1 \frac{\Gamma_{\alpha_1}}{\Gamma_{\alpha_1-\alpha}} t^{\alpha_1-1+\alpha} & if\ 0 < t < \tau_1 \\ a + b\frac{t^{\alpha+1}}{\Gamma_{\alpha+2}} & if\ \tau_1 \le t < \tau_2 \\ \alpha_2 \lambda_2 \frac{\Gamma_{\alpha_2}}{\Gamma_{\alpha_1-\alpha}} t^{\alpha_2-1+\alpha} & if\ t > \tau_2 \end{cases}$$

The parameters a and b are such that $h^\alpha(x)$ becomes continuous
Therefore

$a + b\tau_1 = \alpha_1 \lambda_1 \tau_1^{\alpha_1 - 1}$ and

$a + b\tau_2 = \alpha_2 \lambda_2 \tau_2^{\alpha_2 - 1}$

$a + b\tau_1 = \alpha_1 \lambda_1 \frac{\Gamma_{\alpha_1}}{\Gamma_{\alpha_1-1}} \tau_1^{\alpha_1-1+\alpha}$

$a + b\tau_2 = \alpha_2 \lambda_2 \frac{\Gamma_{\alpha_2}}{\Gamma_{\alpha_2-1}} \tau_2^{\alpha_2-1+\alpha}$

Based on the hazard function h(t) the corresponding fractional cumulative hazard function becomes

$$H(t) = \begin{cases} H_1(t) & if\ 0 < t < \tau_1 \\ H_2(t) & if\ \tau_1 < t < \tau_2 \\ H_3(t) & if\ t \ge \tau_2 \end{cases}$$

$$H^\alpha(t) = \begin{cases} H_1^\alpha(t) & if\ 0 < t < \tau_1 \\ H_2^\alpha(t) & if\ \tau_1 < t < \tau_2 \\ H_3^\alpha(t) & if\ t \ge \tau_2 \end{cases}$$

$$H_1^\alpha(t) = \lambda_1 \frac{\Gamma_{\alpha_1+1}}{\Gamma_{\alpha_1+\alpha+1}} t^{\alpha_1+\alpha}$$

$$H_2^\alpha(t) = \lambda_1 \tau_1^{\alpha_1} + \beta_0 t - \beta_0 \tau_1 + \frac{\beta_1}{2} t^2 - \frac{\beta_1}{2}\tau_1^2$$

$$= \lambda_1 \tau_1^{\alpha_1} + \beta_0 \frac{t^{\alpha+1}}{\Gamma_{\alpha+2}} - \beta_0 \tau_1 + \frac{\beta_1}{2}\frac{\Gamma_3 t^{\alpha+2}}{\Gamma_{\alpha+3}} - \frac{\beta_1}{2}\tau_1^2$$

$$H_3^\alpha(t) = \lambda_1 \tau_1^{\alpha_1} + \beta_0(\tau_2 - \tau_1) + \frac{\beta_1}{2}(\tau_2^2 - \tau_1^2) + \lambda_2(t^{\alpha_2} - \tau^{\alpha_2})$$

$$= \lambda_1 \tau_1^{\alpha_1} + \beta_0 \tau_2 - \beta_0 \tau_1 + \frac{\beta_1}{2}\tau_2^2 - \frac{\beta_1}{2}\tau_1^2 + \lambda_2 \frac{\Gamma_{\alpha_2+1}}{\Gamma_{\alpha_2+\alpha+1}} t^{\alpha_2+\alpha} - \lambda_2 \tau_2^{\alpha_2}$$

The fractional survival function $S^\alpha(t)$ is given by

$$S^\alpha(t) = e^{-H^\alpha(t)} = \begin{cases} S_1^\alpha(t) & if\ 0 < t < \tau_1 \\ S_2^\alpha(t) & if\ \tau_1 < t < \tau_2 \\ S_3^\alpha(t) & if\ t \ge \tau_2 \end{cases}$$

The fractional survival function $S^\alpha(t)$ is given by

$$S_1^\alpha(t) = e^{-\lambda_1 \frac{\Gamma_{\alpha_1+1}}{\Gamma_{\alpha_1+\alpha+1}} t^{\alpha_1+\alpha}}$$

$$S_2^\alpha(t) = e^{-\lambda_1 \tau_1^{\alpha_1}} - \beta_0 \frac{t^{\alpha+1}}{\Gamma_{\alpha+2}} + \beta_0 \tau_1 - \frac{1}{2}\beta_1 \frac{\Gamma_3}{\Gamma_{\alpha+3}} t^{\alpha+2} + \frac{1}{2}\beta_1 \tau_1^2$$

$$S_3^\alpha(t) = e^{-\lambda_1 \tau_1^{\alpha_1}} - \beta_0 \tau_2 + \beta_0 \tau_1 - \frac{1}{2}\beta_1 \tau_2^2 + \frac{1}{2}\beta_1 \tau_1^2 - \lambda_2 \left(\frac{\Gamma_{\alpha_2+1}}{\Gamma_{\alpha_2+\alpha+1}} t^{\alpha_2+\alpha}\right) + \lambda_2 \tau_2^{\alpha_2}$$

The corresponding probability density function becomes

$$f^\alpha(t) = \frac{-d^\alpha}{dt^\alpha} S^\alpha(t) = \begin{cases} f_1^\alpha(t) & if\ 0 < t < \tau_1 \\ f_2^\alpha(t) & if\ \tau_1 < t < \tau_2 \\ f_3^\alpha(t) & if\ t \ge \tau_2 \end{cases}$$

The corresponding probability density function becomes

$$f_1^\alpha(t) = \alpha_1 \lambda_1 \frac{\Gamma_{\alpha_1}}{\Gamma_{\alpha_1+\alpha}} t^{\alpha_1-1+\alpha} .\ e^{-\lambda_1 \frac{\Gamma_{\alpha_1+1}}{\Gamma_{\alpha_1+\alpha+1}} t^{\alpha+\alpha_1}}$$

$$f_2^\alpha(t) = \beta_0 e^{-\lambda_1 \tau_1^{\alpha_1}} + \beta_1 \frac{t^{\alpha+1}}{\Gamma_{\alpha+2}} e^{-\lambda_1 \tau_1^{\alpha_1}} - \beta_0 \frac{t^{\alpha+1}}{\Gamma_{\alpha+2}} + \beta_0 \tau_1 - \frac{1}{2}\beta_1 \frac{\Gamma_3 t^{\alpha+2}}{\Gamma_{\alpha+3}} + \frac{1}{2}\beta_1 \tau_1^2$$

$$f_3^\alpha(t) = \alpha_2 \lambda_2 \frac{\Gamma_{\alpha_2}}{\Gamma_{\alpha_2+\alpha}} t^{\alpha_2-1+\alpha}\ e^{-\lambda_2 \frac{\Gamma_{\alpha_2+1}}{\Gamma_{\alpha_2+\alpha+1}} t^{\alpha+\alpha_2}} e^{-\lambda_1 \tau_1^{\alpha_1}} - \beta_0 \tau_2 + \beta_0 \tau_1 - \frac{1}{2}\beta_1 \tau_2^2 + \frac{1}{2}\beta_1 \tau_1^2 + \lambda_2 \tau_2^{\alpha_2}$$

Fractional Cumulative Risk Model: Weibull Triangular Distribution

$$h^{\alpha}\left(t, X \middle/ a, b, c, d\right) = \begin{cases} 0 & if\ x<a_1 \\ \alpha_1 \lambda_1 \frac{\Gamma\alpha_1}{\Gamma\alpha_1-\alpha} t^{\alpha_1-1+\alpha} \frac{2(x-a_1)}{(b_1-a_1)(c_1-a_1)} & if\ {}^{0<t<a_1,}_{a_1 \le x \le b_1} \\ a + bt & if\ \tau_1 \le t \le \tau_2 \\ \alpha_2 \lambda_2 \frac{\Gamma\alpha_2}{\Gamma\alpha_2-\alpha} t^{\alpha_2-1+\alpha} \frac{2(b_1-x)}{(b_1-a_1)(b_1-c_1)} & if\ {}^{t>\tau_2,}_{c_1 \le x \le b_1} \\ 0 & if\ x>b_1 \end{cases}$$

The parameters a and b are such that $h^{\alpha}(t)$ becomes continuous

$$a + b\tau_1 = \alpha_1 \lambda_1 \tau_1^{\alpha_1-1} \frac{2(x-a_1)}{(b_1-a_1)(c_1-a_1)}$$

$$a + b\tau_2 = \alpha_2 \lambda_2 \tau_2^{\alpha_2-1} \frac{2(b_1-x)}{(b_1-a_1)(b_1-c_1)}$$

Based on the hazard function h(t) the corresponding Fractional cumulative hazard function becomes
The hazard function on the support of X is

$$h(x) = \frac{f(x)}{s(x)} = \begin{cases} \frac{2(a-x)}{ab-mb+ma+x^2-2ax} & if\ a < x < m \\ \frac{2}{b-x} & if\ m \le x < b \end{cases}$$

$$h(x) = \frac{f(x)}{s(x)} = \begin{cases} \frac{2(a-x)}{ab-cb+ca+x^2-2ax} & if\ a < x < c \\ \frac{2}{b-x} & if\ c \le x < b \end{cases}$$

Fractional hazard function on the support of X is

$$h^{\alpha}(x) = \frac{f^{\alpha}(x)}{s^{\alpha}(x)} = \begin{cases} \frac{2(a-x^{\alpha})}{ab-cb+ca+x^2-2ax} & if\ a < x < c \\ \frac{2}{b-x^{\alpha}} & if\ c \le x \le b \end{cases}$$

$$\frac{2(a-x^{\alpha})}{ab-cb+ca+x^2-2ax} = \alpha$$

$$2a - 2x^{\alpha} = \alpha(a_1b_1 - c_1b_1 + c_1a_1 + x^2 - 2a_1x)$$

$$-2x^{\alpha} = \alpha(a_1b_1 - c_1b_1 + c_1a_1 + x^2 - 2a_1x) - 2a_1$$

$$-2x^{\alpha} = (\alpha a_1b_1 - \alpha c_1b_1 + \alpha c_1a_1 + \alpha x^2 - 2a_1x\alpha) - 2a_1$$

$$-x^{\alpha} = \frac{(\alpha a_1b_1 - \alpha c_1b_1 + \alpha c_1a_1 + \alpha x^2 - 2a_1x\alpha) - 2a_1}{2}$$

$$x^{\alpha} = \frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\alpha) + 2a_1}{2}$$

$$\frac{2}{b_1 - x^{\alpha}} = \alpha$$

$$2 = \alpha b_1 - \alpha x^{\alpha}$$

$$2 - \alpha b_1 = -\alpha x^{\alpha}$$

$$\frac{2-\alpha b_1}{\alpha} = -x^{\alpha}$$

$$-\left[\frac{2-\alpha b_1}{\alpha}\right] = x^{\alpha}$$

$$\frac{-2+\alpha b_1}{-\alpha} = x^{\alpha}$$

$$H^{\alpha}(t) = \begin{cases} H_1^{\alpha}(t) & if\ 0 < t < \tau_1 \\ H_2^{\alpha}(t) & if\ \tau_1 < t < \tau_2 \\ H_3^{\alpha}(t) & if\ t \ge \tau_2 \end{cases}$$

$$H_1^{\alpha}(t) = \lambda_1 t^{\alpha_1} \frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}$$

$$H_2^{\alpha}(t) = \lambda_1 \tau_1^{\alpha_1} + \beta_0(t - \tau_1) + \frac{\beta_1}{2}(t^2 - \tau_1^2)$$

$$\begin{aligned} H_2^{\alpha}(t) = \lambda_1 \tau_1^{\alpha_1} & \frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2} + \beta_0 t \\ & - \beta_0 \tau_1 \frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2} + \frac{\beta_1}{2} t^2 \\ & - \frac{\beta_1}{2} \tau_1^2 \left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right]^2 \end{aligned}$$

$$H_3^\alpha(t) = \lambda_1\tau_1^{\alpha_1}\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2} + \beta_0\tau_2\left[\frac{-2+\alpha b_1}{-\alpha}\right]$$
$$-\beta_0\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2} + \frac{\beta_1}{2}\tau_2^2\left[\frac{-2+\alpha b_1}{-\alpha}\right]$$
$$-\frac{\beta_1}{2}\tau_1^2\left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right]^2 + \lambda_2(t^{\alpha_2}) - \lambda_2\tau_2^{\alpha_2}\left[\frac{-2+\alpha b_1}{-\alpha}\right]^{\alpha_2}$$

The Fractional survival function S(t) is given by

$$S^\alpha(t) = e^{-H^\alpha(t)} = \begin{cases} S_1^\alpha(t) & if\ 0 < t < \tau_1 \\ S_2^\alpha(t) & if\ \tau_1 < t < \tau_2 \\ S_3^\alpha(t) & if\ t \geq \tau_2 \end{cases}$$

The fractional survival function $S^\alpha(t)$ is given by

$$S_1^\alpha(t) = e^{-\lambda_1 t^{\alpha_1}}\frac{(-\alpha a_1b_1+\alpha c_1b_1-\alpha c_1a_1-\alpha x^2+2a_1x\boldsymbol{\alpha})+2\boldsymbol{a_1}}{2}$$
$$S_1^\alpha(t) = e^{-\lambda_1}$$
$$S_2^\alpha(t) = e^{-\lambda_1\tau_1^{\alpha_1}}\left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right] + \beta_0 t$$
$$-\beta_0\tau_1\left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right] + \frac{\beta_1}{2}t^2$$
$$-\frac{\beta_1}{2}\tau_1^2\left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right]$$

$$S_3^\alpha(t) = e^{-\lambda_1\tau_1^{\alpha_1}}\left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right] + \beta_0\tau_2\left[\frac{-2+\alpha b_1}{-\alpha}\right]$$
$$-\beta_0\left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right] + \frac{\beta_1}{2}\tau_2^2\left[\frac{-2+\alpha b_1}{-\alpha}\right]^2$$
$$-\frac{\beta_1}{2}\tau_1^2\left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right]^2 + \lambda_2(t^{\alpha_2}) - \lambda_2\tau_2^{\alpha_2}\left[\frac{-2+\alpha b_1}{-\alpha}\right]^{\alpha_2}$$

The corresponding PDF becomes

$$f(t) = -\frac{d}{dt}S^\alpha(t) = \begin{cases} f_1^\alpha(t) & if\ 0 < t < \tau_1 \\ f_2^\alpha(t) & if\ \tau_1 < t < \tau_2 \\ f_3^\alpha(t) & if\ t \geq \tau_2 \end{cases}$$

The corresponding PDF becomes

$$f_1(t) = \alpha_1\lambda_1\frac{\Gamma_{\alpha_1}}{\Gamma_{\alpha_1+\alpha}}t^{\alpha_1-1+\alpha}e^{-\lambda_1}\frac{\Gamma_{\alpha_1+1}}{\Gamma_{\alpha_1+\alpha+1}}t^{\alpha+\alpha_1}\left[\frac{(-\alpha a_1b_1+\alpha c_1b_1-\alpha c_1a_1-\alpha x^2+2a_1x\boldsymbol{\alpha})+2a_1}{2}\right]$$

$$f_2(t) = \beta_0 e^{-\lambda_1\tau_1\alpha_1}\left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right] + \beta 1\frac{t\alpha+1}{\Gamma(\alpha+2)}e^{-\lambda_1\tau_1{}^{\alpha_1}}$$
$$\left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right] - \beta 0\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}$$
$$\left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right] - \frac{1}{2}\beta 1\frac{\Gamma 3t^{\alpha+2}}{\Gamma\alpha+3}\left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right]$$
$$+\frac{1}{2}\beta_1\tau_1^2\left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right]$$

$$f_3^\alpha(t) = \alpha_2\lambda_2\frac{\Gamma_{\alpha_2}}{\Gamma_{\alpha_2+\alpha}}t^{\alpha_2-1+\alpha}.e^{-\lambda_2}\frac{\Gamma_{\alpha_2+1}}{\Gamma_{\alpha_2+\alpha+1}}t^{\alpha+\alpha_2}e^{-\lambda_1\tau_1{}^{\alpha_1}}\left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right]$$
$$-\beta 0\tau_2\left[\frac{-2+\alpha b_1}{-\alpha}\right] + \beta 0\tau_1\left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right]$$
$$-\frac{1}{2}\beta_1\tau_2^2\left[\frac{-2+\alpha b_1}{-\alpha}\right] + \frac{1}{2}\beta_1\tau_1^2\left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right]$$
$$+\lambda_2\tau_2^{\alpha_2}$$

Fractional Cumulative Risk Model: Weibull Trapezoidal Distribution

$$h^{\alpha}\left(t, X \middle/ a, b, c, d\right) = \begin{cases} 0 & if\ x<a_1 \\ \alpha_1 \lambda_1 \frac{\Gamma\alpha_1}{\Gamma\alpha_1-\alpha} t^{\alpha_1-1+\alpha} . u \frac{(x-a_1)}{(b_1-a_1)} & if \begin{matrix} 0<t<a_1, \\ a_1 \le x \le b_1 \end{matrix} \\ a + bt . u & if\ \tau_1 \le t \le \tau_2 \\ \alpha_2 \lambda_2 \frac{\Gamma\alpha_2}{\Gamma\alpha_2-\alpha} t^{\alpha_2-1+\alpha} . u \frac{(d_1-x)}{(d_1-c_1)} & if \begin{matrix} t>\tau_2 \\ c_1 \le x \le d_1 \end{matrix} \\ 0 & if\ x>d_1 \end{cases}$$

The parameters a and b are such that $h^{\alpha}(t)$ becomes continuous

$$a + b\tau_1 = \alpha_1 \lambda_1 \tau_1^{\alpha_1-1} . u \frac{(x-a_1)}{(b_1-a_1)}$$

$$a + b\tau_2 = \alpha_2 \lambda_2 \tau_2^{\alpha_2-1} . u \frac{(d_1-x)}{(d_1-c_1)}$$

Based on the hazard function h(t) the corresponding Fractional cumulative hazard function becomes

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{s(x)} = \begin{cases} \frac{2(x-a)}{ab-mb+ma+x^2-2ax} & if\ a < x < b < m \\ \frac{2}{d-x} & if\ m \le x < d \end{cases}$$

$$h(x) = \frac{f(x)}{s(x)} = \begin{cases} \frac{2(x-a)}{ab-cb+ca+x^2-2ax} & if\ a < x < b < c \\ \frac{2}{d-x} & if\ c \le x < d \end{cases}$$

Fractional hazard function on the support of X is

$$h^{\alpha}(x) = \frac{f^{\alpha}(x)}{s^{\alpha}(x)} = \begin{cases} \frac{2(a-x^{\alpha})}{ab-cb+ca+x^2-2ax} & if\ a < x < b < c \\ \frac{2}{d-x^{\alpha}} & if\ c \le x \le d \end{cases}$$

$$\frac{2(a-x^{\alpha})}{ab-cb+ca+x^2-2ax} = \alpha$$

$$2a - 2x^{\alpha} = \alpha(a_1b_1 - c_1b_1 + c_1a_1 + x^2 - 2a_1x)$$

$$-2x^{\alpha} = \alpha(a_1b_1 - c_1b_1 + c_1a_1 + x^2 - 2a_1x) - 2a_1$$

$$-2x^{\alpha} = (\alpha a_1b_1 - \alpha c_1b_1 + \alpha c_1a_1 + \alpha x^2 - 2a_1x\alpha) - 2a_1$$

$$-x^{\alpha} = \frac{(\alpha a_1b_1 - \alpha c_1b_1 + \alpha c_1a_1 + \alpha x^2 - 2a_1x\alpha) - 2a_1}{2}$$

$$x^{\alpha} = \frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\alpha) + 2a_1}{2}$$

$$\frac{2}{d_1-x^{\alpha}} = \alpha$$

$$2 = \alpha d_1 - \alpha x^{\alpha}$$

$$2 - \alpha d_1 = -\alpha x^{\alpha}$$

$$\frac{2-\alpha d_1}{\alpha} = -x^{\alpha}$$

$$-\left[\frac{2-\alpha d_1}{\alpha}\right] = x^{\alpha}$$

$$\frac{-2+\alpha d_1}{-\alpha} = x^{\alpha}$$

$$H^{\alpha}(t) = \begin{cases} H_1^{\alpha}(t) & if\ 0 < t < \tau_1 \\ H_2^{\alpha}(t) & if\ \tau_1 < t < \tau_2 \\ H_3^{\alpha}(t) & if\ t \ge \tau_2 \end{cases}$$

$$H_1^{\alpha}(t) = \lambda_1 t^{\alpha_1} \frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}$$

$$H_2^{\alpha}(t) = \lambda_1 \tau_1^{\alpha_1} + \beta_0(t - \tau_1) + \frac{\beta_1}{2}(t^2 - \tau_1^2)$$

$$\begin{aligned} H_2^{\alpha}(t) = \lambda_1 \tau_1^{\alpha_1} & \frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2} + \beta_0 t \\ & - \beta_0 \tau_1 \frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2} + \frac{\beta_1}{2} t^2 \\ & - \frac{\beta_1}{2} \tau_1^2 \left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right]^2 \end{aligned}$$

$$\begin{aligned} H_3^{\alpha}(t) = \lambda_1 \tau_1^{\alpha_1} & \frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2} + \beta_0 \tau_2 \left[\frac{-2 + \alpha d_1}{-\alpha}\right] \\ & - \beta_0 \frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2} + \frac{\beta_1}{2} \tau_2^2 \left[\frac{-2 + \alpha d_1}{-\alpha}\right] \\ & - \frac{\beta_1}{2} \tau_1^2 \left[\frac{(-\alpha a_1b_1 + \alpha c_1b_1 - \alpha c_1a_1 - \alpha x^2 + 2a_1x\boldsymbol{\alpha}) + 2a_1}{2}\right]^2 + \lambda_2(t^{\alpha_2}) - \lambda_2 \tau_2^{\alpha_2} \left[\frac{-2 + \alpha d_1}{-\alpha}\right]^{\alpha_2} \end{aligned}$$

The Fractional survival function S(t) is given by

$$S^{\alpha}(t) = e^{-H^{\alpha}(t)} = \begin{cases} S_1^{\alpha}(t) & if\ 0 < t < \tau_1 \\ S_2^{\alpha}(t) & if\ \tau_1 < t < \tau_2 \\ S_3^{\alpha}(t) & if\ t \geq \tau_2 \end{cases}$$

The fractional survival function $S^{\alpha}(t)$ is given by

$$S_1^{\alpha}(t) = e^{-\lambda_1 t^{\alpha_1}} \frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\alpha) + \mathbf{2a_1}}{2}$$

$$S_1^{\alpha}(t) = e^{-\lambda_1}$$

$$\begin{aligned} S_2^{\alpha}(t) = e^{-\lambda_1 \tau_1^{\alpha_1}} &\left[\frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\boldsymbol{\alpha}) + 2a_1}{2}\right] + \beta_0 t \\ &- \beta_0 \tau_1 \left[\frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\boldsymbol{\alpha}) + 2a_1}{2}\right] + \frac{\beta_1}{2} t^2 \\ &- \frac{\beta_1}{2} \tau_1^2 \left[\frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\boldsymbol{\alpha}) + 2a_1}{2}\right] \end{aligned}$$

$$\begin{aligned} S_3^{\alpha}(t) = e^{-\lambda_1 \tau_1^{\alpha_1}} &\left[\frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\boldsymbol{\alpha}) + 2a_1}{2}\right] + \beta_0 \tau_2 \left[\frac{-2 + \alpha d_1}{-\alpha}\right] \\ &- \beta_0 \left[\frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\boldsymbol{\alpha}) + 2a_1}{2}\right] + \frac{\beta_1}{2} \tau_2^2 \left[\frac{-2 + \alpha d_1}{-\alpha}\right]^2 \\ &- \frac{\beta_1}{2} \tau_1^2 \left[\frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\boldsymbol{\alpha}) + 2a_1}{2}\right]^2 + \lambda_2 (t^{\alpha_2}) - \lambda_2 \tau_2^{\alpha_2} \left[\frac{-2 + \alpha d_1}{-\alpha}\right]^{\alpha_2} \end{aligned}$$

The corresponding PDF becomes

$$f(t) = -\frac{d}{dt} S^{\alpha}(t) = \begin{cases} f_1^{\alpha}(t) & if\ 0 < t < \tau_1 \\ f_2^{\alpha}(t) & if\ \tau_1 < t < \tau_2 \\ f_3^{\alpha}(t) & if\ t \geq \tau_2 \end{cases}$$

The corresponding PDF becomes

$$f_1(t) = \alpha_1 \lambda_1 \frac{\Gamma_{\alpha_1}}{\Gamma_{\alpha_1 + \alpha}} t^{\alpha_1 - 1 + \alpha} e^{-\lambda_1} \frac{\Gamma_{\alpha_1 + 1}}{\Gamma_{\alpha_1 + \alpha + 1}} t^{\alpha + \alpha_1} \left[\frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\boldsymbol{\alpha}) + 2a_1}{2}\right]$$

$$\begin{aligned} f_2(t) = {} & \beta_0 e^{-\lambda_1 \tau_1 \alpha_1} \left[\frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\alpha) + 2a_1}{2}\right] + \beta 1 \frac{t\alpha + 1}{\Gamma(\alpha + 2)} e^{-\lambda_1 \tau_1 {}^{\alpha_1}} \\ & \left[\frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\boldsymbol{\alpha}) + 2a_1}{2}\right] - \beta 0 \frac{t^{\alpha + 1}}{\Gamma(\alpha + 2)} \\ & \left[\frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\alpha) + 2a_1}{2}\right] - \frac{1}{2} \beta 1 \frac{\Gamma 3 t^{\alpha + 2}}{\Gamma \alpha + 3} \left[\frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\alpha) + 2a_1}{2}\right] \\ & + \frac{1}{2} \beta_1 \tau_1^2 \left[\frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\boldsymbol{\alpha}) + 2a_1}{2}\right] \end{aligned}$$

$$\begin{aligned} f_3^{\alpha}(t) = {} & \alpha_2 \lambda_2 \frac{\Gamma_{\alpha_2}}{\Gamma_{\alpha_2 + \alpha}} t^{\alpha_2 - 1 + \alpha} . e^{-\lambda_2} \frac{\Gamma_{\alpha_2 + 1}}{\Gamma_{\alpha_2 + \alpha + 1}} t^{\alpha + \alpha_2} e^{-\lambda_1 \tau_1 {}^{\alpha_1}} \left[\frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\boldsymbol{\alpha}) + 2a_1}{2}\right] \\ & - \beta 0 \tau_2 \left[\frac{-2 + \alpha d_1}{-\alpha}\right] + \beta 0 \tau_1 \left[\frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\boldsymbol{\alpha}) + 2a_1}{2}\right] \\ & - \frac{1}{2} \beta_1 \tau_2^2 \left[\frac{-2 + \alpha d_1}{-\alpha}\right] + \frac{1}{2} \beta_1 \tau_1^2 \left[\frac{(-\alpha a_1 b_1 + \alpha c_1 b_1 - \alpha c_1 a_1 - \alpha x^2 + 2a_1 x\boldsymbol{\alpha}) + 2a_1}{2}\right] + \lambda_2 \tau_2^{\alpha_2} \end{aligned}$$

CONCLUSION

The fractional cumulative risk model for Weibull Triangular and Weibull Trapezoidal distribution is applied using the shape parameter, and is applied in the field of medicine, space research, engineering etc., for finding the period of time in fractional order. The cumulative risk of getting the disease like cancer, heart disease etc., and fractional cumulative risk model is helpful to predict the disease at the earlier stage. The Weibull Triangular and Weibull Trapezoidal distributions gives the more accurate and consistent result for cumulative risk model.

Acknowledgement

The author thanks the UGC commission for the financial support under the Rajiv Gandhi National fellowship programme. The author also thanks A. Divya, B.S Abdur Rahman University, vandalur, for her help in completing this paper.

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How to cite this article:

Jayakumar B and Elangovan R (2017) 'Fractional Cumulative Risk Model', *International Journal of Current Advanced Research*, 06(10), pp. 6453-6459. DOI: http://dx.doi.org/10.24327/ijcar.2017.6459.0946

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