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Research Article

APPROXIMATE SOLUTION OF NONLINEAR DIFFUSION EQUATION USING POWER SERIES METHOD (PSM)

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A B S T R A C T

In this paper, Power Series Method (PSM) is discussed which is useful to find the solution of nonlinear partial differential equations (PDEs). The approximate solution in the form of infinite series is obtained for the nonlinear diffusion equation. Power Series Method (PSM) is easy to apply on linear as well as nonlinear problems and is capable of reducing the labour of computational work.

INTRODUCTION

It is well known that nonlinear differential equations are much more difficult to solve than linear differential equations, especially by means of analytic methods. Perturbation techniques are widely applied to obtain analytic approximations of nonlinear problems in science and engineering.

For many years, nonlinear differential equations have been an important topic of study and in many branches of knowledge. This interest has led to the development of many techniques through the last few years in order to obtain exact solutions without requiring further properties of the differential equation. In [4], it has been suggested that the power series method is a general technique which can be used to solve any kind of nonlinear differential equations and verified that the solution obtained by PSM exactly match with the analytic solution [12]. Also the solution of Burger's equation, Korteweg-de vries equation and coupled of Korteweg-de vries equations obtained in the form of infinite series by Power Series Method [7].

The methodology to solve the nonlinear partial differential equation is to obtain a solution by using the approximate analytical method such as Homotopy Analysis Method (HAM) [15], the Adomian Decomposition Method (ADM) [10] and Generalised Separable Method [02]. In some cases, a partial differential equation can be solved via perturbation analysis in which the solution is considered to be a correction to an equation with known solution [1]. Alternatively, there are numerical techniques that solve nonlinear partial differential equations such as the Finite Element Methods [8, 13] and the Finite Difference Method [9, 14]. Furthermore, other direct methods were developed to find closed form solution for nonlinear partial differential equations such as the Exp-function method [11], Tanh method [16] and Extended Tanh method [3].

In this paper, we propose to apply in order to find particular solution of typical PDE widely use in mathematical physics, namely, Nonlinear Diffusion Equation. In this problem, we are able to evaluate the coefficientof truncated power series.

Power Series Method (PSM)

In mathematics, the power series method is used to seek a power series solution to certain differential equations [5]. In general, such a solution assumes a power series with unknown coefficients, then substitutes that solution into the differential equation to find a recurrence relation for the coefficients. The power series method is the standard strategy to solve linear differential equation with variable coefficients. We study an extension of it is called the frobenius method [6] for the general method problem which do not admit power series solutions about a particular point.

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The general algebra for solving differential equations is explained by considering an analytical function $u(x)$ defined in the interval $[0,1]$. Assuming its expansion in power series as

$$u(x) = \sum_{n=0}^{\infty} c_{1n}\, x^n \tag{1}$$

Taking k^{th} power, we get

$$u(x)^k = \sum_{n=0}^{\infty} c_{kn}\, x^n \tag{2}$$

The following is the necessary condition to be satisfied in order to get the required recurrence relation

$$u(x)^k = u(x)^{k-1}\, u(x) \tag{3}$$

On replacing the series expressions from (1) and (2) in each term of equation (3), we get the recurrence relation

$$c_{kn} = \sum_{i=0}^{n} c_{(k-1)i}\, c_{1(n-i)} = \sum_{i=0}^{n} c_{1i}\, c_{(k-1)(n-i)} \tag{4}$$

To obtain the product of two series expansion of $f(x)$and $g(x)$. Let us assume that $f(x)$ and $g(x)$ be analytic at $x = 0$ and defined in $[0,1]$. Let

$$f(x) = \sum_{n=0}^{\infty} c_n\, x^n, \qquad g(x) = \sum_{n=0}^{\infty} d_n\, x^n \tag{5}$$

And

$$h(x) = f(x)g(x) \tag{6}$$

then function of $h$is also analytic at $x = 0$ and defined in$[0,1]$, therefore the series expansion of $h(x)$ is

$$h(x) = \sum_{n=0}^{\infty} b_n\, x^n \tag{7}$$

where,

$$b_n = \sum_{s=0}^{n} c_s\, d_{n-s} = \sum_{s=0}^{n} c_{n-s}\, d_s, n = 0,1,2,\ldots \tag{8}$$

Taking k^{th} derivative of $f(x)$, we get

$$f^{(k)}(x) = \sum_{n=0}^{\infty} (n+1)(n+2)\ldots(n+k)\, c_{n+k} x^n = \sum_{n=0}^{\infty} \phi_{kn}\, c_{n+k} x^n \tag{9}$$

where, $\phi_{kn} = (n+1)(n+2)\ldots(n+k)$, where k and n are positive integers.

To generalize this method, let $z(x,y)$be an analytic function in the domain $G \subseteq R^2$. The function $z(x,y)$is of theform

$$z(x,y) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} c_{ij}\, (x-x_0)^i (y - y_0)^j, where\ (x_0,y_0) \in G \tag{10}$$

For instance, the terms up to order 2 can be written as,

$$z(x,y) = c_{00} + c_{10}(x-x_0) + c_{01}(y-y_0) + c_{20}(x-x_0)^2 + c_{11}(x-x_0)(y-y_0) + c_{02}(y-y_0)^2$$

To represent the higher power of $z(x,y)$ in series, the necessary condition, as in (3) will be

$$z^k(x,y) = z^{k-1}(x,y)\, z(x,y) \tag{11}$$

The series expansion of $z^k(x,y)$ can be written as

$$z^k(x,y) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} c_{ij}^k\, (x-x_0)^i (y-y_0)^j \tag{12}$$

By using relation (11), the coefficient of z^k can be expressed as

$$c_{ij}^k = \sum_{s=0}^{j}\sum_{t=0}^{i} c_{ts}^{k-1}\, c_{(i-t)(j-s)}^1 \tag{13}$$

Substituting$(x_0,y_0) = (0,0)$, $k = 2$ in equation (12) and (13), $z^2(x,y)$can be expressed in the series form as

$$z^2(x,y) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left(\sum_{s=0}^{j}\sum_{t=0}^{i} c_{ts}\, c_{(i-t)(j-s)}\right) x^i y^j \tag{14}$$

Series expansion for any derivative of z with respect to x and y, for any order, and for any power, can be obtained by generalization of equivalent relation forordinary differential equations. The series expansion of

$\frac{\partial z}{\partial y}, \frac{\partial z}{\partial x}, \left(\frac{\partial z}{\partial x}\right)^2, \frac{\partial^2 z}{\partial x^2}, z\left(\frac{\partial z}{\partial x}\right)^2, z\,\frac{\partial^2 z}{\partial x^2}$ and $z^2\,\frac{\partial^2 z}{\partial x^2}$ are as follows:

$$\frac{\partial z}{\partial y} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}(j+1)\, c_{i(j+1)} x^i y^j \tag{15}$$

$$\frac{\partial z}{\partial x} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}(i+1)\, c_{(i+1)j} x^i y^j \tag{16}$$

$$\left(\frac{\partial z}{\partial x}\right)^2 = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\sum_{s=0}^{j}\sum_{t=0}^{i}(t+1)(i-t+1)\, c_{(t+1)s} c_{(i-t+1)(j-s)}\right] x^i y^j \tag{17}$$

$$\frac{\partial^2 z}{\partial x^2} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}(i+1)(i+2)\, c_{(i+2)j} x^i y^j \tag{18}$$

$$z\left(\frac{\partial z}{\partial x}\right)^2 = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\sum_{q=0}^{j}\sum_{p=0}^{i} c_{(i-p)(j-q)}\left(\sum_{s=0}^{q}\sum_{t=0}^{p}(t+1)(p-t+1)\, c_{(t+1)s} c_{(p-t+1)(q-s)}\right)\right] x^i y^j \tag{19}$$

$$z\,\frac{\partial^2 z}{\partial x^2} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\sum_{s=0}^{j}\sum_{t=0}^{i}(i-t+1)(i-t+2)\, c_{ts} c_{(i-t+2)(j-s)}\right] x^i y^j \tag{20}$$

$$z^2\,\frac{\partial^2 z}{\partial x^2} = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\sum_{q=0}^{j}\sum_{p=0}^{i}(i-p+1)(i-p+2)\, c_{(i-p+1)(j-q)}\left(\sum_{s=0}^{q}\sum_{t=0}^{p} c_{ts}\, c_{(p-t)(q-s)}\right)\right] x^i y^j \tag{21}$$

Consider the nonlinear diffusion equation is

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial x}\left(z^k \frac{\partial z}{\partial x}\right) \tag{22}$$

where$k$ is positive integer.

Case I: Let $k = 1$, then equation (22) gives

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial x}\left(z \frac{\partial z}{\partial x}\right) = z\,\frac{\partial^2 z}{\partial x^2} + \left(\frac{\partial z}{\partial x}\right)^2 \tag{23}$$

with initial condition

$$z(x,0) = e^{-x} \tag{24}$$

Let the series solution of (23) be of the form

$$z(x,y) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} c_{ij}\, x^i y^j \tag{25}$$

Substituting equations (15), (17) and (20) into (23), to obtain the recurrence relation:

$$c_{i(j+1)} = \frac{1}{j+1}\left[\sum_{s=0}^{j}\sum_{t=0}^{i}\Big((i-t+1)(i-t+2)c_{ts}c_{(i-t+2)(j-s)} + (t+1)(i-t+1)c_{(t+1)s}c_{(i-t+1)(j-s)}\Big)\right], \forall\, i,j \geq 0 \tag{26}$$

where

$$c_{i0} = \frac{(-1)^n}{i!}, \forall\, i \geq 0 \tag{27}$$

By applying the recurrence relation (26) for several values of i and j the approximate series solution obtained is as follows,

$$z(x,y) = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + 2y - 4xy + 4x^2y - \frac{8}{3}x^3y + \cdots \tag{28}$$

Case II: Let $k = 2$, then equation (22) gives

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial x}\left(z^2 \frac{\partial z}{\partial x}\right) = z^2\frac{\partial^2 z}{\partial x^2} + 2z\left(\frac{\partial z}{\partial x}\right)^2 \tag{29}$$

with initial condition

$$z(x,0) = e^{-x} \tag{30}$$

Let the series solution of (29) be of the form

$$z(x,y) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} c_{ij}\, x^i y^j \tag{31}$$

Substituting equations (15), (19) and (21) into (29), to obtain the recurrence relation:

$$c_{i(j+1)} = \frac{1}{j+1}\left[\sum_{q=0}^{j}\sum_{p=0}^{i}(i-p+1)(i-p+2)\, c_{(i-p+2)(j-p)}\left(\sum_{s=0}^{q}\sum_{t=0}^{p} c_{ts}\, c_{(p-t)(q-s)}\right) + 2\sum_{q=0}^{j}\sum_{p=0}^{i} c_{(i-p)(q-j)}\left(\sum_{s=0}^{q}\sum_{t=0}^{p}(t+1)(p-t+1)\, c_{(t+1)s}c_{(p-t+1)(q-s)}\right)\right], \forall\, i,j \geq 0 \tag{32}$$

where

$$c_{i0} = \frac{(-1)^n}{i!}, \forall\, i \geq 0 \tag{33}$$

By applying the recurrence relation (32) for several values of i and j the approximate series solution obtained is as follows,

$$z(x,y) = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + 3y - 9xy + \frac{27}{2}x^2y - \frac{27}{2}x^3y + \cdots \tag{34}$$

RESULTS AND DISCUSSION

In present study, we have discussed power series method to solve nonlinear PDEs. To generalized power series method, we have considered analytic function $z(x,y)$ as two dimension power series which is expressed in equation (10). Then the required derivatives have been obtained in the form of series expansion in equations (15) to (21). Then substituting these relations in given equation one can obtain recurrence relation. In this paper, power series method is appliedon second order general nonlinear diffusion equation (22). Then to discuss particular case, we have studied two cases for $k = 1$ and $k = 2$. In both the cases using approximate initial condition, the approximate solutions have been obtained for $k = 1$ in equation (28) and for $k = 2$ in equation (34) in the form of infinite series.

CONCLUSION

In this paper, we have studied PSM for solving nonlinear PDEs which gives the approximate solution in the form of infinite series. Here we have applied PSM to solve second order nonlinear PDEs. The PSM can be extend to solve higher order nonlinear PDEs.

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