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Triple layered complete fuzzy graph

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A B S T R A C T **RESEARCH ARTICLE**

In this Paper, a new fuzzy graph named Triple layered complete fuzzy graph is proposed. The Triple layered complete fuzzy graph gives a 3-dimension structure in its nature. We have also discussed about order, size, degree and □-complement of Triple layered complete fuzzy graph.

2010AMS Subject Classification: 03E72

Keywords:

Order, Size, Vertex Degree, □-complement, Strong Fuzzy Graph, Triple layered complete fuzzy graph.

1. Introduction

Azriel Rosenfeld introduced fuzzy graph in 1975 [5]. Though introduced recently, it has been growing fast and has numerous applications in various fields. During the same time Yeh and Bang have also introduced various concepts in connectedness with fuzzy graphs [7]. Mordeson and Peng introduced the concept of operations on fuzzy graphs, Sunitha and Vijayakumar discussed about the operations of union, join, Cartesian product and composition on two fuzzy graphs [4]. The degree of a vertex in some fuzzy graphs was discussed by Nagoorgani and Radha [6]. Nagoorgani and Malarvizhi have defined different types of fuzzy graphs and discussed its relationships with isomerism in fuzzy graphs [3].

In this paper we define Triple layered complete fuzzy graph (TLCFG) or 3 – D Fuzzy graph which gives a 3-D structure in fuzzy graph theory and some of its properties were discussed. Section two contains the basic definitions in fuzzy graphs, in section three we introduce a new fuzzy graph called a Triple layered complete fuzzy graph, section four presents the theoretical concepts of TLCFG and finally we give conclusion on (TLCFG).

2. Preliminaries

2.1 Definition: A fuzzy graph G is a pair of functions G: (σ, μ) where a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of G: (σ, μ) is denoted by G* :(σ* , μ*) [5].

2.2 Definition: Let G: (σ, μ) be a fuzzy graph, the order of G is defined as $O(G) = \sum \sigma(u) / u \in V$ [8].

2.3 Definition: Let G: (σ, μ) be a fuzzy graph, the size of G is defined as $S(G) = \sum \mu(u, v) / u, v \in V$ [8].

2.4 Definition: Let G: (σ, μ) be a fuzzy graph, the degree of a vertex u in G is defined as

$dG(u) = \sum \mu(u, v) / v \neq u, v \in V$ and is denoted as dG (u) [10].

2.5 Definition: A fuzzy graph G: (σ , μ) is said to be strong fuzzy graph if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ for all (u , v) in μ* [9].

2.6 Definition: Let G be a fuzzy graph, the μ –compliment of G is denoted as $G\mu'$: (σμ,μμ) where σ* ∪ μ*and μμ (u, v) ={ $\sigma(u) \wedge \sigma(v) - \mu(u, v)$ *if* $\mu(u, v) > 0$ *if* $\mu(u, v) = 0$ [4].

3. Triple Layered Complete Fuzzy Graph(Tlcfg)

3.1 Definition: Let σ_{TL}: V→ [0, 1] be a subset of V and μ_{TL}: V× V→ [0, 1] be a symmetric fuzzy relation on σ_{TL}. Any two vertex of the Triple Layered Complete Fuzzy graph are adjacent. The vertex set of complete triple layered fuzzy graph be σ ∪ μ∪ μ and it's denoted by K σ ∪ μ ∪μ.

Or

Let σ_{TL}: V→ [0, 1] be a fuzzy subset of V then the complete triple layered fuzzy graph on σ_{TL} is defined on K σ ∪ μ∪ μ= (σ_{TL}, μ_{TL}). Any two vertices of the TLCFG are adjacent.

Example: 3.1.1. Consider the complete fuzzy graph with 3 vertices (K_3)

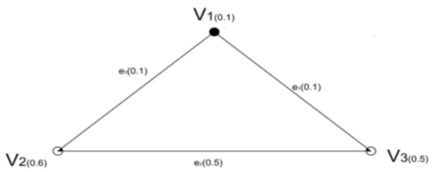

Figure 1. A complete fuzzy graph (K_3)

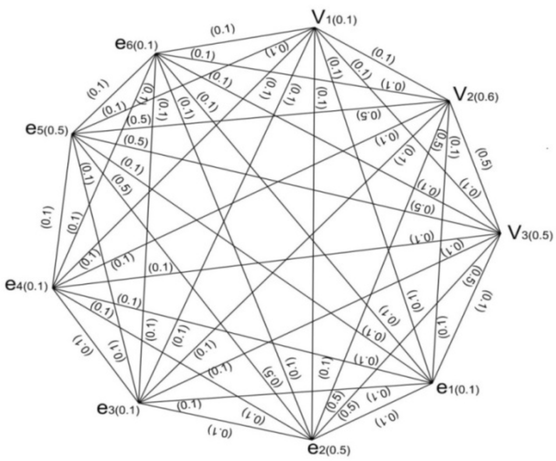

Figure 2. TLCFG of K_3

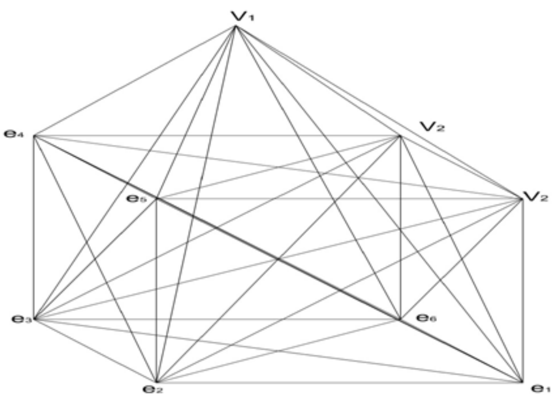

Figure 3. Image of TL (K_3)

Example 3.1.2: Consider the complete fuzzy graph with 4 vertices (K_4).

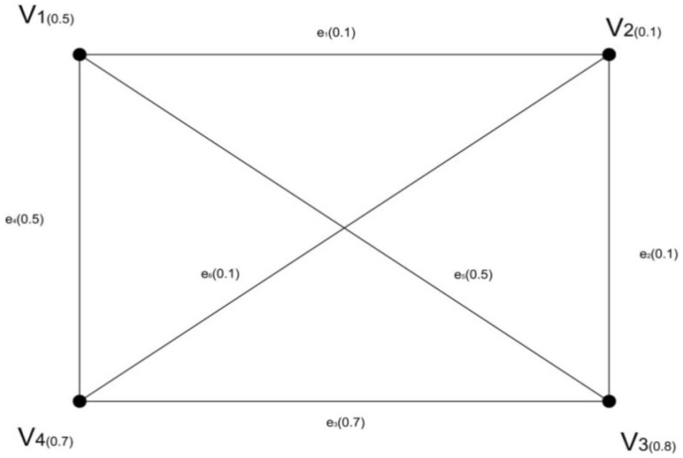

Figure 4. A complete fuzzy graph (K_4)

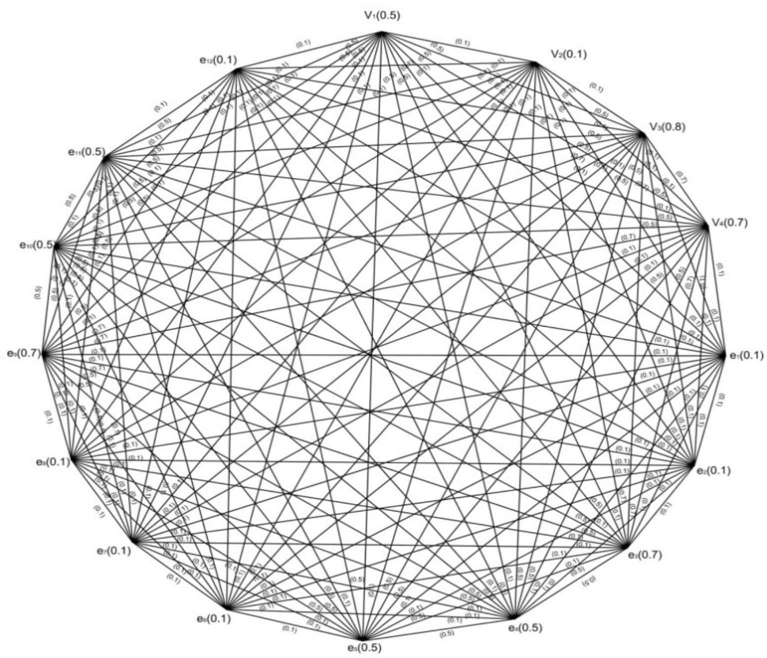

Figure 5. TLCFG of K_4

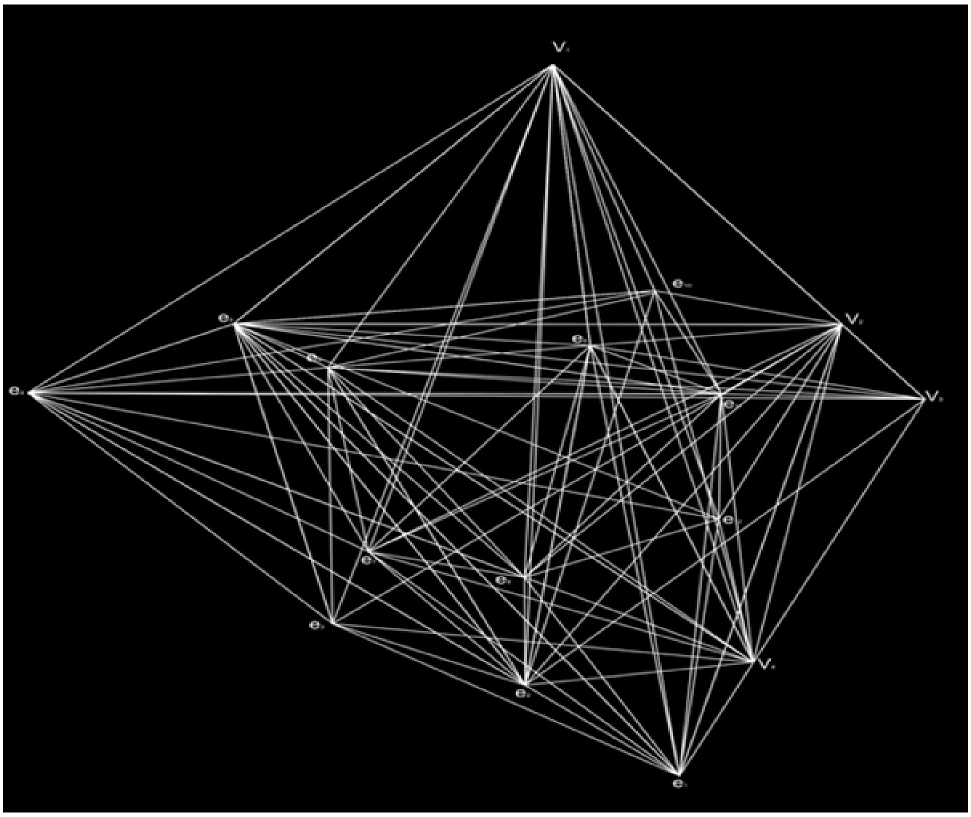

Figure 6. Image of TL(K_4)

Example 3.1.3: Consider the complete fuzzy graph with 5 vertices (K_5).

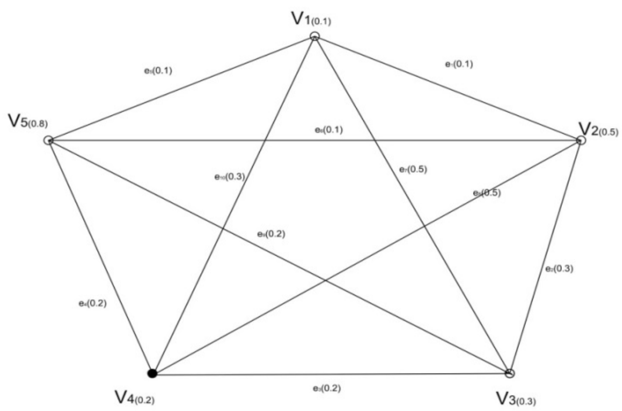

Figure 7. A complete fuzzy graph (K_5)

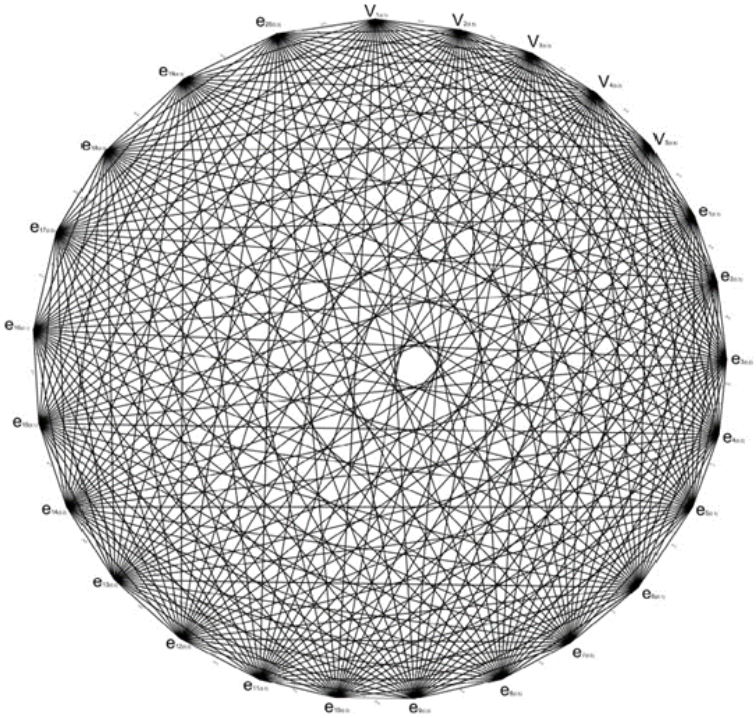

Figure 8. TLCFG of K_5

Similarly we can convert complete fuzzy graph into Triple layered complete fuzzy graph.

4. Theoritical Concepts

4.1 Theorem: The order of Triple layered complete fuzzy graph $K\sigma\cup\mu\cup\mu$ is equal to the sum of the order and twice size of the complete graph

Proof: As the node set of complete Triple layered fuzzy graph and the fuzzy subset σ_{TL} on

$\sigma *\cup \mu* \cup \mu *$ is defined as,

σ_{TL} ={ $\sigma(u)$ if $u \in \sigma *$
$2\ \mu\ (uv)$ $uv \in \mu *$

By the definition, order of the Triple layered fuzzy graph is,

O (TL (G)) = $\sum_{u\in V\cup E\cup E} \sigma TL(u)$ (by definition 2.2)

= $\sum_{u\in V} \sigma TL(u)$ + $\sum_{u\in E} \sigma TL(u)$

=$\sum_{u\in V} \sigma(u)$ + 2 $\sum_{u\in E} \mu(u)$ (by definition of $\sigma_{TL}(u)$)

O (TL (G))=Order (G) + 2 size (G)

4.2 Theorem: Every Triple layered complete fuzzy graph is a strong fuzzy graph

Proof: As the node set of TL (G) is $\sigma *\cup \mu* \cup \mu *$ and the fuzzy subset σ_{TL} on $\sigma *\cup \mu* \cup \mu *$ is defined as,

σ_{TL} ={ $\sigma(u)$ if $u\ \epsilon\sigma *$
$2\mu(uv)$ if uv $\epsilon\mu *$

By the definition of Triple layered complete fuzzy graph

μ (u, v) =σ (u) ∧ σ (v) -----------------------①

And also by the definition of strong fuzzy graph

μ (u, v) =min (σ (u), σ (v)) -----------------②

From equation① &②; we get

Every Triple layered complete fuzzy graph is a strong fuzzy graph

Example 4.2.1: We choose TL (G) of K_3 graph,

v_1=0.1; v_2=0.6; v_3=0.5 and e_1=0.1; e_2=0.5; e_3=0.1; e_4=0.1; e_5=0.5; e_6=0.1

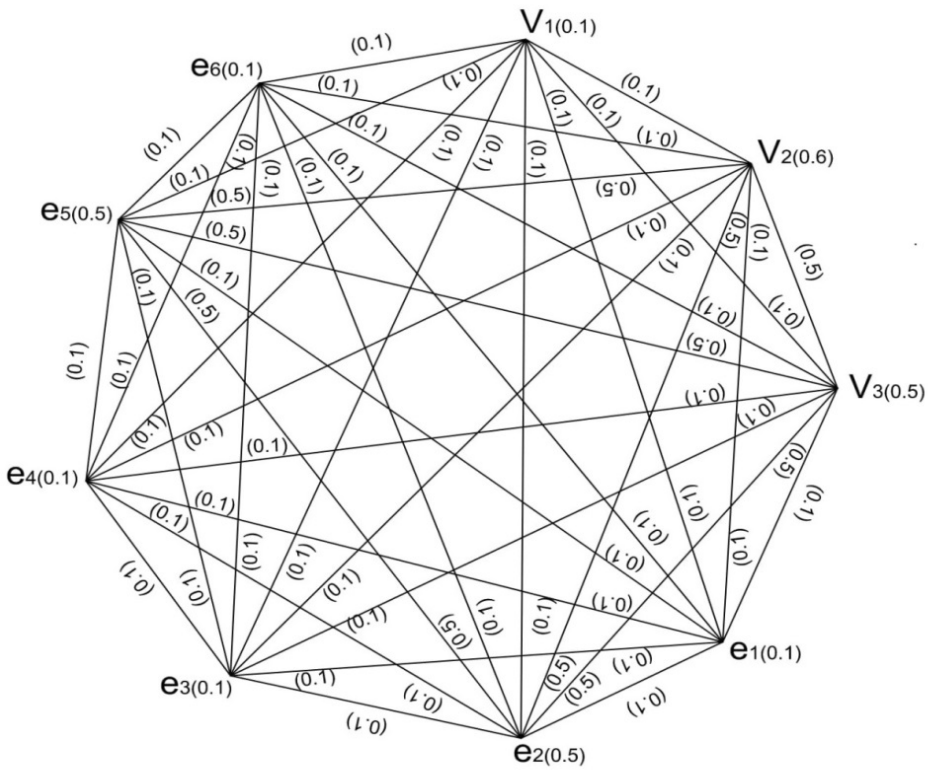

Figure 9. TLCFG of K_3

(i) $\mu(v_1, v_2) = \sigma(v_1) \wedge \sigma(v_2)$
$= 0.1 \wedge 0.6$
$= 0.1$

(ii) $\mu(e_1, e_2) = \sigma(e_1) \wedge \sigma(e_2)$
$= 0.1 \wedge 0.5$
$= 0.1$

(iii) $\mu(v_1, e_1) = \sigma(v_1) \wedge \sigma(e_1)$
$= 0.1 \wedge 0.1$
$= 0.1$

(iv) $\mu(e_2, e_3) = \sigma(e_2) \wedge \sigma(e_3)$
$= 0.5 \wedge 0.1$
$= 0.1$

(v) $\mu(e_4, e_5) = \sigma(e_4) \wedge \sigma(e_5)$
$= 0.1 \wedge 0.5$
$= 0.1$

(vi) $\mu(e_4, e_6) = \sigma(e_4) \wedge \sigma(e_6)$
$= 0.1 \wedge 0.1$
$= 0.1$

(vii) $\mu(v_2, e_4) = \sigma(v_2) \wedge \sigma(e_4)$
$= 0.6 \wedge 0.1$
$= 0.1$

every triple layered fuzzy graph is a strong fuzzy graph

4.3 Theorem

If G is a strong fuzzy graph then the μ-complement of TL (G) is isolated vertices

Proof

Let G be a strong fuzzy graph by the previous theorem, Every Triple layered complete graph is strong fuzzy graph

$\mu(u, v) = \sigma(u) \wedge \sigma(v) \rightarrow ①$

And by the definition of μ-complement,

$\mu^{\mu}(u, v) = \sigma(u) \wedge \sigma(v) - \mu(u, v)$

$= \mu(u, v) - \mu(u, v)$

$= 0$

$\mu^{\mu}(u, v) = 0$ for all u, v in $\sigma^* \cup \mu^* \cup \mu^*$

$d_{TL}(u) = 0$ for all u in $\sigma^* \cup \mu^* \cup \mu^*$

Every vertices of complement of TL (G) have isolated vertices.

xample 4.3.1

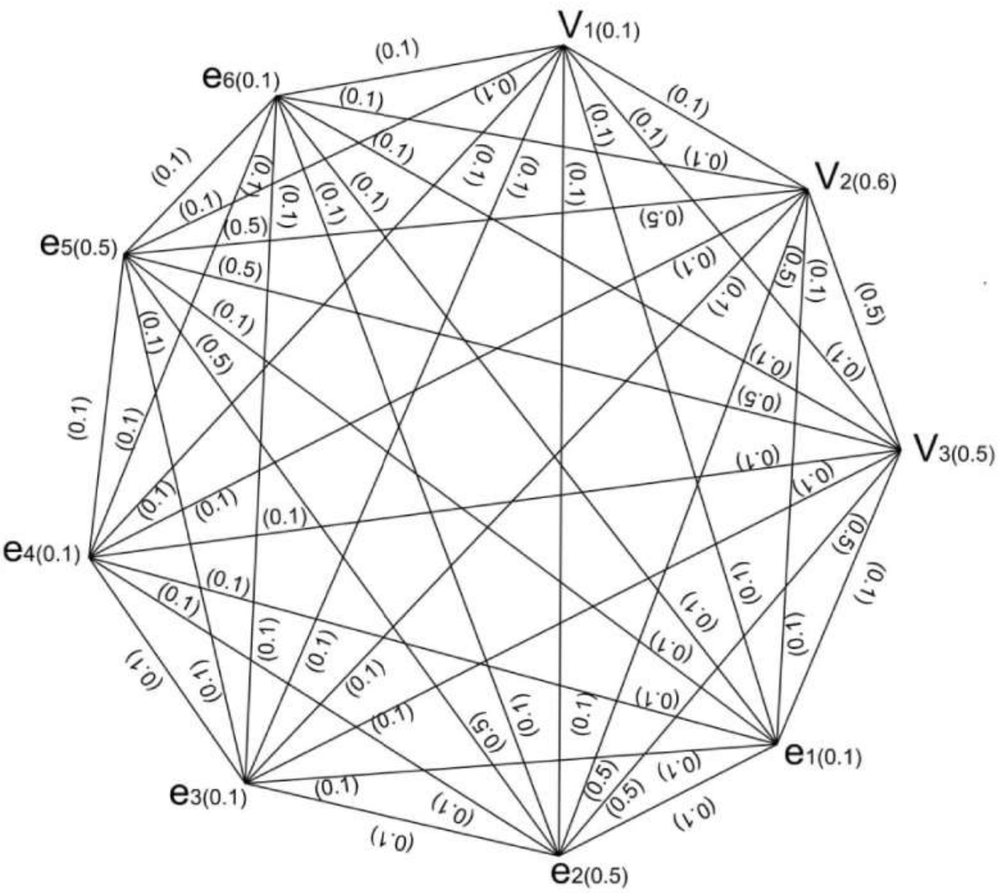

Figure 10. TLCFG of K_3

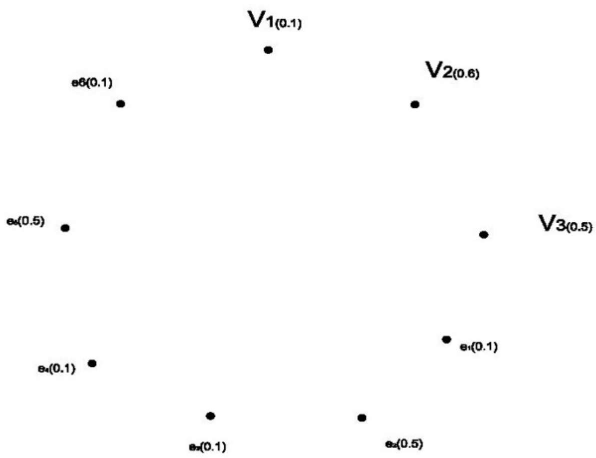

Figure 11. μcomplement TLCFG of K_3 .

TLCFG $(K_n) = K_{2n-1} +$ TLCFG (K_{n-1})

Example 4.3.1

(i) $TLCFG(K_4) = K_{2(4)-1} + TLCFG(K_3)$

$= K_7 + K_9$

$= K_{16}$

$TLCFG\ (K_4) = CFG\ (K_{16})$

(ii) $TLCFG(K_5) = K_{2(5)-1} + TLCFG(K_4)$

$= K_9 + K_{16}$

$= K_{25}$

$TLCFG\ (K_5) = CFG\ (K_{25})$

Table 1: Relation between complete fuzzy graph and Triple layered complete fuzzy

COMPLETE FUZZY GRAPH	TRIPLE LAYERED COMPLETE FUZZY GRAPH
K_3	$TLCFG(K_3)=K_9$
K_4	$TLCFG(K_4)=K_{16}$
K_5	$TLCFG(K_5)=K_{25}$
K_6	$TLCFG(K_6)=K_{36}$
K_7	$TLCFG(K_7)=K_{49}$
K_8	$TLCFG(K_8)=K_{64}$
K_9	$TLCFG(K_9)=K_{81}$
K_{10}	$TLCFG(K_{10})=K_{100}$
K_{11}	$TLCFG(K_{11})=K_{121}$
K_{12}	$TLCFG(K_{12})=K_{144}$
K_{13}	$TLCFG(K_{13})=K_{169}$
K_{14}	$TLCFG(K_{14})=K_{196}$
K_{15}	$TLCFG(K_{15})=K_{225}$
K_{16}	$TLCFG(K_{16})=K_{256}$
K_{17}	$TLCFG(K_{17})=K_{289}$
K_{18}	$TLCFG(K_{18})=K_{324}$
K_{19}	$TLCFG(K_{19})=K_{361}$
K_{20}	$TLCFG(K_{20})=K_{400}$
K_{21}	$TLCFG(K_{21})=K_{441}$
K_{22}	$TLCFG(K_{22})=K_{484}$

Remark:

The edge relation between complete fuzzy graph and Triple layered complete fuzzy graph is, $TLCFG\ (K_n) = K_{2n-1} + TLCFG\ (K_{n-1})$ Number of edges $(E_{TL}) = 2n_{TL}(n_{TL} - 1)\ /2$ n_{TL} represents number of vertices in TLCFG.

Conclusion

In this paper we laid a concept triple layered complete fuzzy graph(TLCFG) and illustrated with some examples. Further structures can be developed by increasing number of cycles. These structural patterns with the cycles gives further different patterns in networking models.

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