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Research Article

SELF-FORCE IN A CPT ODD LORENTZ VIOLATING ELECTRODYNAMICS

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ABSTRACT

Abraham-Lorentz's equation of radiation reaction suffers from both runaway solutions and pre-acceleration. In the point charge limit, we cannot eliminate both these difficulties simultaneously. In this study, the self-force due to a classical charged particle is explored in a CPT odd Lorentz violating electrodynamics. It is shown that the electromagnetic mass behaves like a tensor but the radiation reaction force is not modified.

INTRODUCTION

Lorentz symmetry is the fundamental symmetry of two important theories, the standard model of particle physics and Einstein's general relativity. Lorentz symmetry is a global symmetry in flat space in the standard model, whereas in general relativity, this symmetry is the local one. Quantum versions of general relativity like string theory, loop quantum gravity, etc, are shown lead to Spontaneous Lorentz violation [1, 2]. Recent cosmological observations that establish the presence of dark matter and dark energy strongly suggest that the standard model needs to be extended. The minimal standard model extension (mSME), which is a subset of Standard model extension, deals with Lorentz violation in electrodynamics in flat space [3]. This mSME framework preserves Lorentz symmetry under observer Lorentz transformations but violates it under particle transformations. The mSME Lagrangian density contains the usual standard model term plus Lorentz violating terms with CPT even and odd coefficients which are very small. In an inertial frame, if we fix these coefficients, space-space rotational symmetry breaks and hence Lorentz symmetry also gets broken since the Lorentz symmetry group includes Lorentz transformations and rotations in ordinary three-dimensional space. In this type of Lorentz symmetry violation, the mass behaves like a tensor[4]. Accelerated classical charged particle emits radiation. Self-electromagnetic fields produced by an accelerated electron produce a force on it, which is called self-force. The self-force has a finite part which is called as radiation reaction, and the infinite part is the electromagnetic mass term [5,6]. In the point charge limit, the radiation reaction force suffers from both runaway solutions and casualty violation. We cannot eliminate these difficulties simultaneously [7]. Radiation reaction in the usual Maxwell theory has been studied in the subject matter of several recent studies[8,9]. It is possible that when one considers the inputs from various versions of mSME, these problems in classical electrodynamics might find a resolution. This is the motivation for the work done in this article, which is to capture the effects due to mSME on these problems.

Birefringence in vacuum is discussed in Lorentz violating electrodynamics with the nonzero CPT even coefficient κ_{0123}[10]. For the same nonzero CPT even coefficient self-force is derived, due to a charged particle in which the radiation reaction force is not altered but electromagnetic mass behaves like a tensor[11]. In the present work, a spherically symmetric charge distributed particle's radiation reaction and electromagnetic mass, both are calculated in the CPT odd Lorentz violating electrodynamics. In Section 2 retarded potentials are calculated in the presence of CPT odd coefficients. In section 3 self-force due to a charged particle is calculated using these retarded potentials.

Electromagnetic potentials in CPT odd Lorentz violating electrodynamics

In the minimal Standard model extension, Lagrangian density for the pure photon sector can be written as [12]

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}(\kappa_F)_{k\lambda\mu\nu}F^{k\lambda}F^{\mu\nu} + \frac{1}{2}(\kappa_{AF})^k\epsilon_{k\lambda\mu\nu}A^\lambda F^{\mu\nu} - j^\mu A_\mu \quad (1)$$

Here the symmetric four-current density vector $j^\mu(\rho,\vec{J})$ is coupled with the four vector potential $A^\mu(\phi,\vec{A})$ and $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$ is the electromagnetic field strength tensor. We are using natural units (c = 1) and metric $(1,-1,-1,-1)$ through out the paper. $(\kappa_F)_{k\lambda\mu\nu}$ and $(\kappa_{AF})^k$ are CPT even and

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odd Lorentz violating coefficients respectively. This $U(1)$ gauge invariant Lagrangian density, (1) preserves Lorentz symmetry under observer Lorentz transformation but violates under particle transformations.

Lagrangian density with CPT odd coefficients in the pure photon sector can be written as

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\kappa_{AF})^{k}\epsilon_{k\lambda\mu\nu}A^{\lambda}F^{\mu\nu} - j^{\mu}A_{\mu} \qquad (2)$$

and the equations of motion

$$\partial_\alpha F^{\alpha}_{\mu} + (\kappa_{AF})^{\alpha}\epsilon_{\mu\alpha\beta\gamma}F^{\beta\gamma} + j_\mu = 0 \qquad (3)$$

Under the Lorentz gauge ($\partial_\alpha A^\alpha = 0$),

$$\Box A_\mu + (\kappa_{AF})^{\alpha}\epsilon_{\mu\alpha\beta\gamma}F^{\beta\gamma} + j_\mu = 0 \qquad (4)$$

In a particular frame, if we fix the CPT odd coefficients, explicit Lorentz symmetry breaking occurs. Based on the symmetry properties of Levi-Civita tensor, $\epsilon_{\mu\alpha\beta\gamma}$, equation (4) gives the following coupled equations.

$$\Box\phi - 2(\kappa_{AF})^{1}\left(\frac{\partial A_y}{\partial z} - \frac{\partial A_z}{\partial y}\right) + 2(\kappa_{AF})^{2}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)$$
$$-2(\kappa_{AF})^{3}\left(\frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x}\right) + \rho = 0 \qquad (5)$$

$$\Box A_{\mathbf{x}} - 2(\kappa_{AF})^{0}\left(\frac{\partial A_y}{\partial z} - \frac{\partial A_z}{\partial y}\right) + 2(\kappa_{AF})^{2}\left(\frac{\partial A_z}{\partial t} - \frac{\partial \phi}{\partial z}\right)$$
$$-2(\kappa_{AF})^{3}\left(\frac{\partial A_y}{\partial t} + \frac{\partial \phi}{\partial y}\right) + J_{\mathbf{x}} = 0 \qquad (6)$$

$$\Box A_{\mathbf{y}} + 2(\kappa_{AF})^{0}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) - 2(\kappa_{AF})^{1}\left(\frac{\partial A_z}{\partial t} + \frac{\partial \phi}{\partial z}\right)$$
$$+2(\kappa_{AF})^{3}\left(\frac{\partial A_x}{\partial t} + \frac{\partial \phi}{\partial x}\right) + J_{\mathbf{y}} = 0 \qquad (7)$$

$$\Box A_{\mathbf{z}} - 2(\kappa_{AF})^{0}\left(\frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x}\right) + 2(\kappa_{AF})^{1}\left(\frac{\partial A_y}{\partial t} + \frac{\partial \phi}{\partial y}\right)$$
$$-2(\kappa_{AF})^{2}\left(\frac{\partial A_x}{\partial t} + \frac{\partial \phi}{\partial x}\right) + J_{\mathbf{z}} = 0 \qquad (8)$$

The solutions of these equations yield the potentials in CPT odd Lorentz violating electrodynamics in the leading order in (κ_{AF}) (see Appendix A).

$$\phi(\vec{\mathbf{x}},t) = -\frac{1}{4\pi}\int \frac{[\rho(\vec{\mathbf{x}}',t')]_{ret}}{|\vec{\mathbf{x}}-\vec{\mathbf{x}}'|}d\vec{\mathbf{x}}' \qquad (9)$$

$$A_p(\vec{\mathbf{x}},t) = -\frac{1}{4\pi}\int \frac{J_p(\vec{\mathbf{x}}',t')_{ret}}{|\vec{\mathbf{x}}-\vec{\mathbf{x}}'|}d\vec{\mathbf{x}}' - \frac{(\kappa_{AF})^{r}}{4\pi}\epsilon_{pqr}\int J_q(\vec{\mathbf{x}}',t')_{ret}d\vec{\mathbf{x}}' \qquad (10)$$

Here $p = x, q = y$ and $r = z$ and ϵ_{pqr} is Levi-Civita symbol in three dimensions ($p,q,r = 1,2,3$). Vector potential expression (10) indicates that the rotational symmetry breaking and hence Lorentz symmetry. An estimate of the bounds on the parameters described in the work can be found in [13].

Self-force Due to a Classical Charged Parti- Cle in CPT odd Lorentz violating electro- dynamics

Self-force is calculated by assuming the electron as a uniformly charged sphere [7]. In this model, Lorentz force expression is used to calculate Self-force.

$$\vec{F}^{\text{self}} = -\int \rho(\vec{\mathbf{x}},t)\left[\vec{\nabla}\phi + \frac{\partial\vec{A}}{\partial t}\right]d\vec{\mathbf{x}} \qquad (11)$$

Here, the retarded potentials are expanded by using the following Taylor series.

$$[---]_{ret} = \sum_{n=0}^{\infty}\frac{(-1)^n}{n!}R^n\frac{\partial^n}{\partial t^n}[----]_{t'=t} \qquad (12)$$

This usual Lorentz force expression can also hold even in the case of Lorentz violating theory, but now the potentials are modified potentials due to Lorentz violation [12]. By substituting the equations (9) and (10) in (11) we can get self-force components as

$$\begin{aligned} F_p^{\text{self}} = {} & \frac{4}{3}U\dot{v}_p - \frac{2}{3}e^2\ddot{v}_p + \\ & \frac{(\kappa_{AF})^r}{4\pi}\epsilon_{pqr}\int \rho(\vec{\mathbf{x}},t)d\vec{\mathbf{x}}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}R^n\frac{\partial^{n+1}}{\partial t^{n+1}}\int \rho(\vec{\mathbf{x}}')v_q(t)d\vec{\mathbf{x}}' \\ = {} & \frac{4}{3}U\dot{v}_p + \frac{(\kappa_{AF})^r e^2}{4\pi}\epsilon_{pqr}\dot{v}_q - \\ & \frac{2}{3}e^2\ddot{v}_p - \frac{(\kappa_{AF})^r e^2}{4\pi}\epsilon_{pqr}\ddot{v}_q\iint R\rho(\vec{\mathbf{x}})\rho(\vec{\mathbf{x}}')d\vec{\mathbf{x}}d\vec{\mathbf{x}}' + O(R^n, n \geq 2) \\ = {} & \frac{4}{3}U\dot{v}_p + \frac{(\kappa_{AF})^r e^2}{4\pi}\epsilon_{pqr}\dot{v}_q - \frac{2}{3}e^2\ddot{v}_p + O(R^n, n \geq 1) \qquad (13) \end{aligned}$$

where the electrostatic energy is given by

$$U = \frac{1}{2}\iint \frac{\rho(\vec{\mathbf{x}},t)\rho(\vec{\mathbf{x}}',t')}{|\mathbf{x}-\mathbf{x}'|}d\vec{\mathbf{x}}d\vec{\mathbf{x}}' \qquad (14)$$

In the equation (13) $\dot{v}, \ddot{v}$ are first and second order time derivatives of the velocity, respectively. We can write the self-force as

$$F_i^{\text{self}} = M_{ij}^{em}\dot{v}_j - \frac{2}{3}e^2\ddot{v}_i + O(R^n, n \geq 1)\ i,j = x,y,z \qquad (15)$$

where the electromagnetic mass tensor is given by

$$M_{ij}^{em} = \begin{bmatrix} \frac{4}{3}U & \frac{(\kappa_{AF})^3 e^2}{4\pi} & -\frac{(\kappa_{AF})^2 e^2}{4\pi} \\ -\frac{(\kappa_{AF})^3 e^2}{4\pi} & \frac{4}{3}U & \frac{(\kappa_{AF})^1 e^2}{4\pi} \\ \frac{(\kappa_{AF})^2 e^2}{4\pi} & -\frac{(\kappa_{AF})^1 e^2}{4\pi} & \frac{4}{3}U \end{bmatrix} \qquad (16)$$

Therefore, the electromagnetic mass behaves like a tensor of rank two. If we fix the CPT coefficients, the bare mass also behaves like tensor[4]. Hence the equation of motion can be written as

$$\begin{aligned} F_i^{ex} &= \left(M_{ij}^{\text{bare}} + M_{ij}^{em}\right)\dot{v}_j - \frac{2}{3}e^2\ddot{v}_i + O(R^n, n \geq 1) \\ &= M_{ij}\dot{v}_j - \frac{2}{3}e^2\ddot{v}_i + O(R^n, n \geq 1) \qquad (17) \end{aligned}$$

where F^{ex} is external force and M_{ij} is the renormalized mass. In the point charge limit $R \to 0$, we can neglect the term $O(R^n, n \geq 1)$ in the above equation. Then, the equation of motion is given by

$$F_i^{ex} = M_{ij}\dot{v}_j - \frac{2}{3}e^2\ddot{v}_i \qquad (18)$$

Therefore, in the point charge limit, the radiation reaction force is equal to Abraham-Lorentz's equation. The radiation reaction force is given by

$$F_i^{rad} = -\frac{2}{3}e^2\ddot{v}_i\ i = x,y,z \qquad (19)$$

In mSME, the laws of physics are Lorentz invariant under observer Lorentz transformations but not under particle transformations. In this work, although the self-force is calculated in the instantaneous rest frame of the particle, we can transform it to any inertial frame by using suitable observer Lorentz transformations.

CONCLUSIONS

In crystals, due to anisotropy, the electron effective mass behaves like a tensor. In the minimal standard model extension, Lorentz violation induces vacuum anisotropy. Therefore like in crystals, the electromagnetic mass of the electron also behaves like a tensor in an anisotropy-induced vacuum. However, in the point charge limit, the nonrelativistic radiation reaction is not modified. Therefore, still it suffers from both runaway solutions and casualty violations. Although the corrections to the standard results are small, the results are nevertheless insightful since the study develops a classical framework or a subsequent study using Quantum Field theory techniques. The current crisis is cosmology in terms of finding candidates for dark matter and dark energy and the new observations and inputs coming from gravity wave astronomy, which promises to probe the properties of space and time, making these studies worthwhile. The problems that plague radiation reaction can also be studied in the context of non-commutativity or nonlocal theories. Such studies are already being pursued [15].

Appendix: Calculation of the modified potentials

By using Fourier transformation, we can express the potentials in the following way.

$$\phi(\mathbf{\vec{x}},t)=\frac{1}{(2\pi)^4}\iint\ \tilde{\phi}(\vec{K},\omega)e^{-i(\vec{K}\cdot\mathbf{\vec{x}}-\omega t)}d\vec{K}d\omega \qquad (20)$$

$$\vec{A}(\mathbf{\vec{x}},t)=\frac{1}{(2\pi)^4}\iint\ \tilde{\vec{A}}(\vec{K},\omega)\exp-i(\vec{K}\cdot\mathbf{\vec{x}}-\omega t)d\vec{K}d\omega \qquad (21)$$

$$\vec{j}(\mathbf{\vec{x}},t)=\frac{1}{(2\pi)^4}\iint\ \tilde{\vec{j}}(\vec{K},\omega)\exp-i(\vec{K}\cdot\mathbf{\vec{x}}-\omega t)d\vec{K}d\omega \qquad (22)$$

$$\rho(\mathbf{\vec{x}},t)=\frac{1}{(2\pi)^4}\iint\ \tilde{\rho}(\vec{K},\omega)\exp-i(\vec{K}\cdot\mathbf{\vec{x}}-\omega t)d\vec{K}d\omega \qquad (23)$$

$$(M_{4\times4})\times\begin{bmatrix}\tilde{\phi}\\ \tilde{A}_x\\ \tilde{A}_y\\ \tilde{A}_z\end{bmatrix}=-\begin{bmatrix}\tilde{\rho}\\ \tilde{j}_x\\ \tilde{j}_y\\ \tilde{j}_z\end{bmatrix} \qquad (24)$$

where $M_{4\times4}=$

$$\begin{bmatrix}(\vec{K}^2-\omega^2) & 2i\left((\kappa_{AF})^3K_y-(\kappa_{AF})^2K_z\right) & 2i((\kappa_{AF})^1K_z-(\kappa_{AF})^3K_x) & 2i\left((\kappa_{AF})^2K_x-(\kappa_{AF})^1K_y\right)\\ 2i\left(((\kappa_{AF})^3K_y-(\kappa_{AF})^2K_z\right) & (\vec{K}^2-\omega^2) & 2i((\kappa_{AF})^0K_z-(\kappa_{AF})^3\omega) & 2i\left((\kappa_{AF})^2\omega-(\kappa_{AF})^0K_y\right)\\ 2i((\kappa_{AF})^1K_z-(\kappa_{AF})^3K_x) & 2i((\kappa_{AF})^3\omega-(\kappa_{AF})^0K_z) & (\vec{K}^2-\omega^2) & 2i((\kappa_{AF})^0K_x-(\kappa_{AF})^1\omega)\\ 2i\left((\kappa_{AF})^2K_x-(\kappa_{AF})^1K_y\right) & 2i\left((\kappa_{AF})^0K_y-(\kappa_{AF})^2\omega\right) & 2i((\kappa_{AF})^1\omega-(\kappa_{AF})^0K_x) & (\vec{K}^2-\omega^2)\end{bmatrix}$$

$$\begin{bmatrix}\tilde{\phi}\\ \tilde{A}_x\\ \tilde{A}_y\\ \tilde{A}_z\end{bmatrix}=-M_{4\times4}^{-1}\begin{bmatrix}\tilde{\rho}_{\tilde{j}}\\ \tilde{j}_x\\ \tilde{j}_y\\ \tilde{j}_z\end{bmatrix} \qquad (25)$$

$$M_{4\times4}^{-1}=\frac{1}{(\vec{K}^2-\omega^2)}(I_{4\times4}+A_{4\times4})^{-1} \qquad (26)$$

After using the binomial expansion for the above equation and restricting the series up to the first order in (K_{AF}) gives

$$M_{4\times4}^{-1}=\frac{1}{(\vec{K}^2-\omega^2)}(I_{4\times4}-A_{4\times4})$$

where $A_{4\times4}=$

$$\frac{1}{(\vec{K}^2-\omega^2)}\begin{bmatrix}(0 & 2i((\kappa_{AF})^3K_y-(\kappa_{AF})^2K_z) & 2i\Lambda((\kappa_{AF})^1K_z-(\kappa_{AF})^3K_x) & 2i((\kappa_{AF})^2K_x-(\kappa_{AF})^1K_y)\\ 2i((\kappa_{AF})^3K_y-(\kappa_{AF})^2K_z) & 0 & 2i((\kappa_{AF})^0K_z-(\kappa_{AF})^3\omega) & 2i((\kappa_{AF})^2\omega-(\kappa_{AF})^0K_y)\\ 2i((\kappa_{AF})^1K_z-(\kappa_{AF})^3K_x) & 2i((\kappa_{AF})^3\omega-(\kappa_{AF})^0K_z) & 0 & 2i((\kappa_{AF})^0K_x-(\kappa_{AF})^1\omega)\\ 2i((\kappa_{AF})^2K_x-(\kappa_{AF})^1K_y) & 2i((\kappa_{AF})^0K_y-(\kappa_{AF})^2\omega) & 2i((\kappa_{AF})^1\omega-(\kappa_{AF})^0K_x) & 0\end{bmatrix}$$

$$\tilde{\phi}=\frac{-\tilde{\rho}}{\left(\vec{K^2}-\omega^2\right)}+\frac{2i\left((\kappa_{AF})^3K_y-(\kappa_{AF})^2K_z\right)}{\left(\vec{K}^2-\omega^2\right)^2}\tilde{j}_x+\frac{2i\left((\kappa_{AF})^2K_x-(\kappa_{AF})^1K_y\right)}{\left(\vec{K}^2-\omega^2\right)^2}\tilde{j}_y+\frac{2i\left((\kappa_{AF})^2K_x-(\kappa_{AF})^1K_y\right)}{\left(\vec{K}^2-\omega^2\right)^2}\tilde{j}_z \qquad (28)$$

$$\tilde{A}_x=\frac{-\tilde{j}_x}{\left(\vec{K}^2-\omega^2\right)}+\frac{2i\left((\kappa_{AF})^3K_y-(\kappa_{AF})^2K_z\right)}{\left(\vec{K}^2-\omega^2\right)^2}\tilde{\rho}+\frac{2i((\kappa_{AF})^0K_z-(\kappa_{AF})^3\omega)}{\left(\vec{K}^2-\omega^2\right)^2}\tilde{j}_y+\frac{2i\left((\kappa_{AF})^2\omega-(\kappa_{AF})^0K_y\right)}{\left(\vec{K}^2-\omega^2\right)^2}\tilde{j}_z \qquad (29)$$

$$\tilde{A}_y=\frac{-\tilde{j}_y}{\left(\vec{K}^2-\omega^2\right)}+\frac{2i((\kappa_{AF})^1K_z-(\kappa_{AF})^3K_x)}{\left(\vec{K}^2-\omega^2\right)^2}\tilde{\rho}+\frac{2i((\kappa_{AF})^3\omega-(\kappa_{AF})^0K_z)}{\left(\vec{K}^2-\omega^2\right)^2}\tilde{j}_x+\frac{2i((\kappa_{AF})^0K_x-(\kappa_{AF})^1\omega)}{\left(\vec{K}^2-\omega^2\right)^2}\tilde{j}_z \qquad (30)$$

$$\tilde{A}_z=\frac{-\tilde{j}_z}{\left(\vec{K}^2-\omega^2\right)}+\frac{2i\left((\kappa_{AF})^2K_x-(\kappa_{AF})^1K_y\right)}{\left(\vec{K}^2-\omega^2\right)^2}\tilde{\rho}+\frac{2i\left((\kappa_{AF})^0K_y-(\kappa_{AF})^2\omega\right)}{\left(\vec{K}^2-\omega^2\right)^2}\tilde{j}_x+\frac{2i((\kappa_{AF})^1\omega-(\kappa_{AF})^0K_x)}{\left(\vec{K}^2-\omega^2\right)^2}\tilde{j}_y \qquad (31)$$

By substituting the above equations in the equations $(20)-(24)$, we can get the retarded potentials. During the integration, the terms which have only ω in the numerator are non-zero and the remaining terms become zero. Therefore, in the retarded potentials, the term $(\kappa_{AF})^0$ is absent, even if it is in $\tilde{A}_i(k)$.

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