



## SG $\hat{G}$ -HOMEOMORPHISMS AND - $\hat{G}$ SG HOMEOMORPHISMS IN TOPOLOGICAL SPACES

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### ARTICLE INFO

#### Article History:

Received 18<sup>th</sup> December, 2016

Received in revised form 19<sup>th</sup> January, 2017

Accepted 4<sup>th</sup> February, 2017

Published online 28<sup>th</sup> March, 2017

### ABSTRACT

In the present paper we introduce two new types of mappings called  $sg\hat{G}$ -homeomorphism and  $\hat{G}sg$ -homeomorphism and then shown that one of these mapping has a group structure. Further we investigate some properties of these two homeomorphisms.

#### Key words:

Homeomorphism;  $sg\hat{G}$ -homeomorphism;

$\hat{G}sg$ -homeomorphism.

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### INTRODUCTION

Levine<sup>(3)</sup> in 1970 generalized the concept of closed sets to generalized closed sets. Bhattacharya and Lahiri<sup>(7)</sup> in 1987 generalized the concept of closed sets to semi-generalized closed sets with the helps of semi-open sets and obtained various topological properties. Arya and Nour<sup>(8)</sup> in 1990 have defined generalized semi-open sets with the help of semi-openness and use them to obtain some characterizations of  $s$ -normal spaces. Devi, Balachandran and Maki<sup>(11)</sup> in 1995 defined two new classes of maps called semi-generalized homeomorphisms and generalized semi-homeomorphisms and also defined two new classes of maps called  $sgc$ -homeomorphisms and  $gsc$ -homeomorphisms. Ahmed and Narli<sup>(15)</sup> in 2007 defined two new classes of maps called  $gsg$ -homeomorphisms and  $sgs$ -homeomorphisms. Garg, Chauhan and Agarwal<sup>(17)</sup> in 2007 introduced two new classes of maps namely  $gs\psi$ -homeomorphisms and  $\psi gs$ -homeomorphisms. Garg et al.<sup>(18)</sup> again in 2007 introduced two new classes of maps called  $sg\psi$ -homeomorphisms and  $\psi sg$ -homeomorphisms. Manoj and Shikha<sup>(19)</sup> in 2007 introduced  $\hat{G}sg$ -and  $sg\hat{G}$ -homeomorphisms. In this paper we introduce two new classes of maps called  $sg\hat{G}$ -homeomorphisms and  $\hat{G}sg$ -homeomorphisms and then study some of their properties.

Throughout the present paper,  $(X, \tau)$  and  $(Y, \sigma)$  denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset  $A$  of a topological space  $(X, \tau)$  the  $cl(A)$ ,  $int(A)$  and  $A^c$  denote the closure of  $A$ , the interior of  $A$  and the complement of  $A$  in  $X$  respectively.

#### Preliminaries

In this section we recall the following definitions.

**Definition 2.01:** A subset  $A$  of a topological space  $(X, \tau)$  is called semi-open<sup>(6)</sup> (resp. semi-closed) if  $A \subseteq cl(int(A))$  (resp.  $int(cl(A)) \subseteq A$ ). Every closed (resp. open) set is semi-closed (resp. semi-open).

**Definition 2.02:** A subset  $A$  of a topological space  $(X, \tau)$  is called semi-generalized closed<sup>(7)</sup> (briefly  $sg$ -closed) if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open. The complement of  $sg$ -closed set is called  $sg$ -open set. Every semi-closed set is  $sg$ -closed set. The family of all  $sg$ -closed sets of any topological space  $(X, \tau)$  is denoted by  $sgc(X, \tau)$ .

**Definition 2.03:** A subset  $A$  of a topological space  $(X, \tau)$  is called generalized semi closed<sup>(8)</sup> (briefly  $gs$ -closed) if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open. The complement of  $gs$ -closed set is called  $gs$ -open set. Every closed (semi-closed,  $g$ -closed and  $sg$ -closed) set is  $gs$ -closed set. The family of all  $gs$ -closed sets of any topological space  $(X, \tau)$  is denoted by  $gsc(X, \tau)$ .

**Definition 2.04:** A subset  $A$  of a topological space  $(X, \tau)$  is called  $\psi$ -closed<sup>(12)</sup> if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $sg$ -open. The complement of  $\psi$ -closed set is called  $\psi$ -open set.

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Every closed (semi-closed) set is  $\psi$ -closed set and every  $\psi$ -closed set is sg-closed (gs-closed) set. The family of all  $\psi$ -closed sets of any topological space  $(X, \tau)$  is denoted by  $\psi c(X, \tau)$ .

**Definition 2.05:** A subset  $A$  of a topological space  $(X, \tau)$  is called  $\hat{g}$ -closed<sup>(13)</sup> if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open. The complement of  $\hat{g}$ -closed set is called  $\hat{g}$ -open set. Every closed set is  $\hat{g}$ -closed set and every  $\hat{g}$ -closed set is  $\psi$ -closed (sg-closed, gs-closed, g-closed) set. The family of all  $\hat{g}$ -closed sets of any topological space  $(X, \tau)$  is denoted by  $\hat{g}c(X, \tau)$ .

**Definition 2.06:** A map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called semi-closed map<sup>(6)</sup> (resp. sg-closed map<sup>(10)</sup>, gs-closed map<sup>(10)</sup>,  $\psi$ -closed map<sup>(16)</sup>,  $\hat{g}$ -closed map<sup>(14)</sup>) if the image of each closed set in  $(X, \tau)$  is semi-closed set (resp. sg-closed set, gs-closed set,  $\psi$ -closed set,  $\hat{g}$ -closed set) in  $(Y, \sigma)$ . Every closed map is semi-closed map. Every semi-closed map is  $\psi$ -closed map. Every  $\psi$ -closed map is sg-closed map, every sg-closed map is gs-closed map and every  $\hat{g}$ -closed map is  $\psi$ -closed map (sg-closed map, gs-closed map, g-closed map).

**Definition 2.07 :** A map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called  $\hat{g}$ -continuous<sup>(14)</sup> (resp.  $\psi$ -continuous<sup>(12)</sup>, sg-continuous<sup>(4)</sup>, gs-continuous<sup>(11)</sup>,  $\psi$ -irresolute<sup>(12)</sup>, sg-irresolute<sup>(9)</sup>, gs-irresolute<sup>(11)</sup>, gsg-irresolute<sup>(15)</sup>, sgs-irresolute<sup>(15)</sup>, gsg $\psi$ -irresolute<sup>(17)</sup>,  $\psi$ gs-irresolute<sup>(17)</sup>, sg $\psi$ -irresolute<sup>(18)</sup>,  $\psi$ sg-irresolute<sup>(18)</sup>,  $\hat{g}$ -irresolute<sup>(13)</sup>,  $\hat{g}g$ s-irresolute<sup>(19)</sup>,  $g$ s $\hat{g}$ -irresolute<sup>(19)</sup>) if the inverse image of every closed (resp. closed, closed, closed,  $\psi$ -closed, sg-closed, gs-closed, gs-closed, sg-closed, gs-closed,  $\psi$ -closed, sg-closed,  $\psi$ -closed,  $\hat{g}$ -closed,  $\hat{g}$ -closed, gs-closed) set in  $(Y, \sigma)$  is  $\hat{g}$ -closed (resp.  $\psi$ -closed, sg-closed, gs-closed,  $\psi$ -closed, sg-closed, gs-closed, sg-closed,  $\psi$ -closed,  $\psi$ -closed, sg-closed,  $\hat{g}$ -closed, gs-closed,  $\hat{g}$ -closed) set in  $(X, \tau)$ .

**Definition 2.08:** A bijective map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called:

- (i) semi-homeomorphism (B)<sup>(15)</sup> (briefly s.h. (B)) if  $f$  is continuous and semi-open map.
- (ii)  $\psi$ -homeomorphism<sup>(16)</sup> if  $f$  is both  $\psi$ -continuous and  $\psi$ -open map.
- (iii)  $\hat{g}$ -homeomorphism<sup>(14)</sup> if  $f$  is both  $\hat{g}$ -continuous and  $\hat{g}$ -open map.
- (iv) semi-generalized homeomorphism<sup>(11)</sup> (briefly sg-homeomorphism) if  $f$  is both sg-continuous and sg-open.
- (v) generalized semi-homeomorphism<sup>(11)</sup> (briefly gs-homeomorphism) if  $f$  is both gs-continuous and gs-open.

- (vi) sgc-homeomorphism<sup>(11)</sup> (resp. gsc-homeomorphism<sup>(11)</sup>,  $\psi^*$ -homeomorphism<sup>(16)</sup>,  $\hat{g}$ c-homeomorphism<sup>(14)</sup>, gsg-homeomorphism<sup>(15)</sup>, sgs-homeomorphism<sup>(15)</sup>, gs $\psi$ -homeomorphism<sup>(17)</sup>,  $\psi$ gs-homeomorphism<sup>(17)</sup>, sg $\psi$ -homeomorphism<sup>(18)</sup>,  $\psi$ sg-homeomorphism<sup>(18)</sup>,  $\hat{g}g$ s-homeomorphism<sup>(19)</sup>,  $g$ s $\hat{g}$ -homeomorphism<sup>(19)</sup>) if  $f$  and  $f^{-1}$  are sg-irresolute (resp. gs-irresolute,  $\psi$ -irresolute,  $\hat{g}$ -irresolute, gsg-irresolute, sgs-irresolute, gs $\psi$ -irresolute,  $\psi$ gs-irresolute, sg $\psi$ -irresolute,  $\psi$ sg-irresolute,  $\hat{g}g$ s-irresolute,  $g$ s $\hat{g}$ -irresolute).

**Definition 2.09:** A space  $(X, \tau)$  is called  $T_{1/2}$ -space<sup>(3)</sup> (resp.  $T_b$ -space<sup>(10)</sup>,  $\hat{T}_b$ -space<sup>(14)</sup>) if every g-closed set (resp. gs-closed set, gs-closed set) is closed set (resp. closed set,  $\hat{g}$ -closed set).

**Proposition 2.10:** In a  $T_{1/2}$ -space every gs-closed set is semi-closed set [10].

### SG $\hat{G}$ -Homeomorphisms

In this section we introduce  $sg\hat{g}$ -homeomorphisms and then investigate the group structure of the set of all  $sg\hat{g}$ -homeomorphisms.

**Definition 3.01:** A map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called a  $sg\hat{g}$ -irresolute map if the set  $f^{-1}(A)$  is  $\hat{g}$ -closed in  $(X, \tau)$  for every sg-closed set  $A$  of  $(Y, \sigma)$ .

**Definition 3.02:** A bijection  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called a  $sg\hat{g}$ -homeomorphism if the function  $f$  and the inverse function  $f^{-1}$  are both  $sg\hat{g}$ -irresolute maps. If there exists a  $sg\hat{g}$ -homeomorphism from  $X$  to  $Y$ , then the spaces  $(X, \tau)$  and  $(Y, \sigma)$  are called  $sg\hat{g}$ -homeomorphic. The family of all  $sg\hat{g}$ -homeomorphism of any topological space  $(X, \tau)$  is denoted by  $sg\hat{g}h(X, \tau)$ .

**Remark 3.03:** The following examples show that the concepts of homeomorphism and  $sg\hat{g}$ -homeomorphism are independent of each other.

**Example 3.04:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ . Define  $f : (X, \tau) \rightarrow (X, \tau)$  by identity mapping then  $f$  is a homeomorphism but not a  $sg\hat{g}$ -homeomorphism for  $f$  and  $f^{-1}$  are not  $sg\hat{g}$ -irresolute maps.

**Example 3.05:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$  and  $\sigma = \{\emptyset, Y\}$ . Define  $f : (X, \tau) \rightarrow (Y, \sigma)$  by identity mapping then  $f$  is  $sg\hat{g}$ -homeomorphism but not homeomorphism.

**Proposition 3.06:** Every  $sg\hat{G}$ -homeomorphism is (i)  $sgc$ -homeomorphism (ii)  $\hat{G}c$ -homeomorphism (iii)  $\psi^*$ -homeomorphism (iv)  $sg\psi$ -homeomorphism (v)  $\psi sg$ -homeomorphism (vi)  $\hat{G}gs$ -homeomorphism (vii)  $sgs$ -homeomorphism (viii)  $\psi gs$ -homeomorphism.

The converse of the above proposition is not true as it can be seen from the following examples.

**Example 3.07:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{\phi, \{a\}, \{a, b\}, X\}$  and  $\sigma = \{\phi, \{a\}, \{a, b\}, \{a, c\}, Y\}$ . Define  $f : (X, \tau) \rightarrow (Y, \sigma)$  by identity mapping then  $f$  is  $sgc$ -homeomorphism but not  $sg\hat{G}$ -homeomorphism for  $f$  is not  $sg\hat{G}$ -irresolute map.

**Example 3.08:** Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\phi, \{a\}, \{a, b, c\}, X\}$  and  $\sigma = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}, Y\}$ . Define  $f : (X, \tau) \rightarrow (Y, \sigma)$  by identity mapping then  $f$  is  $\hat{G}c$ -homeomorphism but not  $sg\hat{G}$ -homeomorphism for  $f$  and  $f^{-1}$  both are not  $sg\hat{G}$ -irresolute maps.

**Example 3.09:** In example (3.07),  $f$  is  $\psi^*$ -homeomorphism but not  $sg\hat{G}$ -homeomorphism.

**Example 3.10:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$  and  $\sigma = \{\phi, \{a\}, Y\}$ . Define  $f : (X, \tau) \rightarrow (Y, \sigma)$  by identity mapping then  $f$  is  $sg\psi$ -homeomorphism but not  $sg\hat{G}$ -homeomorphism for  $f^{-1}$  is not  $sg\hat{G}$ -irresolute map.

**Example 3.11:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{\phi, X\}$  and  $\sigma = \{\phi, \{a, b\}, Y\}$ . Define  $f : (X, \tau) \rightarrow (Y, \sigma)$  by identity mapping then  $f$  is  $\psi sg$ -homeomorphism but not  $sg\hat{G}$ -homeomorphism for  $f^{-1}$  is not  $sg\hat{G}$ -irresolute map. Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{\phi, X\}$  and  $\sigma = \{\phi, \{a, b\}, Y\}$ . Define  $f : (X, \tau) \rightarrow (Y, \sigma)$  by identity mapping then  $f$  is  $\square sg$ -homeomorphism but not  $sg\hat{G}$ -homeomorphism for  $f^{-1}$  is not  $sg\hat{G}$ -irresolute map.

**Example 3.12:** Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\phi, \{a\}, \{a, b, c\}, X\}$  and  $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, Y\}$ . Define  $f : (X, \tau) \rightarrow (Y, \sigma)$  by identity mapping then  $f$  is  $\hat{G}gs$ -homeomorphism but not  $sg\hat{G}$ -homeomorphism for  $f$  is not  $sg\hat{G}$ -irresolute map.

**Example 3.13:** In example (3.10), map  $f$  is  $sgs$ -homeomorphism but not  $sg\hat{G}$ -homeomorphism for  $f^{-1}$  is not  $sg\hat{G}$ -irresolute map.

**Example 3.14:** Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, X\}$  and  $\sigma = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}, Y\}$ . Define  $f : (X, \tau) \rightarrow (Y, \sigma)$  by identity mapping then  $f$  is  $\psi gs$ -homeomorphism but not  $sg\hat{G}$ -homeomorphism for  $f$  and  $f^{-1}$  are not  $sg\hat{G}$ -irresolute map.

**Proposition 3.15:** Every  $gsg$ -homeomorphism is  $sg\hat{G}$ -homeomorphism.

The converse of the above proposition is not true as it can be seen from the following example.

**Example 3.16:** Let  $X = \{a, b, c, d\}$  and  $\tau = \{\phi, \{a\}, \{a, b, c\}, X\}$ . Define  $f : (X, \tau) \rightarrow (X, \tau)$  by identity mapping then  $f$  is  $sg\hat{G}$ -homeomorphism but not  $gsg$ -homeomorphism for  $f^{-1}$  are not  $sg\hat{G}$ -irresolute maps.

**Remark 3.17:**  $sg\hat{G}$ -homeomorphism is independent form  $sg\psi$ -homeomorphism as it can be seen from the following example.

**Example 3.18:** In example (3.05), map  $f$  is  $sg\hat{G}$ -homeomorphism but not  $gsg$ -homeomorphism.

**Example 3.19:** Let  $X = \{a, b, c\}$  and  $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ . Define  $f : (X, \tau) \rightarrow (Y, \sigma)$  by identity mapping then  $f$  is  $sg\psi$ -homeomorphism but not  $sg\hat{G}$ -homeomorphism.

**Remark 3.20:**  $gsc$ -homeomorphism,  $gsc$ -homeomorphism and  $gsg$ -homeomorphism are not necessarily  $sg\hat{G}$ -homeomorphism as it can be seen from the following examples.

**Example 3.21:** Let  $X = \{a, b, c\}$  and  $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ . Define  $f : (X, \tau) \rightarrow (Y, \sigma)$  by identity mapping then  $f$  is  $gsc$ -homeomorphism but not  $sg\hat{G}$ -homeomorphism.

**Example 3.22:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{\phi, \{a\}, \{a, b\}, X\}$  and  $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, Y\}$ . Define  $f : (X, \tau) \rightarrow (Y, \sigma)$  by identity mapping then  $f$  is  $gsc$ -homeomorphism but not  $sg\hat{G}$ -homeomorphism.

**Example 3.23:** Let  $X = \{a, b, c, d\}$  and  $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, X\}$ . Define  $f : (X, \tau) \rightarrow (X, \tau)$  by identity mapping then  $f$  is  $gsg$ -homeomorphism but not  $sg\hat{G}$ -homeomorphism.

**Proposition 3.24:** Every  $gsg$  ( $sgs$ )-homeomorphism from  $\hat{T}_b$ -space onto itself is  $sg\hat{G}$ -homeomorphism.

**Proposition 3.25:** If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is  $sg\hat{G}$ -homeomorphism then every  $sgc(\hat{G}c)$ -homeomorphism from  $X$  to  $Y$  is  $\hat{G}c$  ( $sgc$ )-homeomorphism.

**Proposition 3.26:** If  $f : (X, \tau) \rightarrow (Y, \sigma)$  and  $g : (Y, \sigma) \rightarrow (Z, \eta)$  are  $sg\hat{G}$ -homeomorphism then their composition  $g \circ f : (X, \tau) \rightarrow (Z, \eta)$  is also  $sg\hat{G}$ -homeomorphism.

**Theorem 3.27:** If  $sg\hat{G}h(X, \tau)$  is non-empty then the set  $sg\hat{G}h(X, \tau)$  is a group under the composition of maps.

**Theorem 3.28:** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $sg\hat{g}$ -homeomorphism then  $f$  induces an isomorphism from the group  $sg\hat{g}_h(X, \tau)$  onto the group  $sg\hat{g}_h(Y, \sigma)$ .

**Theorem 3.29:**  $sg\hat{g}$ -homeomorphism is an equivalence relation in the collection of all topological spaces.  
 Proof: Reflexivity and symmetry are immediate and transitivity followed from proposition (3.26).

### $\hat{GSG}$ -Homeomorphisms

In this section we introduce  $\hat{gsg}$ -homeomorphism and investigate its properties.

**Definition 4.01:** A map  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called  $\hat{gsg}$ -irresolute map if the set  $f^{-1}(A)$  is  $sg$ -closed in  $(X, \tau)$  for every  $\hat{g}$ -closed set  $A$  of  $(Y, \sigma)$ .

**Definition 4.02:** A bijection  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called a  $\hat{gsg}$ -homeomorphism if the function  $f$  and the inverse function  $f^{-1}$  are both  $\hat{gsg}$ -irresolute maps. If there exists a  $\hat{gsg}$ -homeomorphism from  $X$  to  $Y$ , then the spaces  $(X, \tau)$  and  $(Y, \sigma)$  are called  $\hat{gsg}$ -homeomorphic.

The family of all  $\hat{gsg}$ -homeomorphism of any topological space is denoted by  $\hat{gsg}_h(X, \tau)$ .

**Proposition 4.03:** Every  $sgc$ -homeomorphism is  $\hat{gsg}$ -homeomorphism.

**Proposition 4.04:** Every (i)  $\hat{g}c$ -homeomorphism (ii)  $gsg$ -homeomorphism (iii)  $\psi^*$ -homeomorphism (iv)  $\psi sg$ -homeomorphism (v)  $sg\psi$ -homeomorphism (vi)  $gs\psi$ -homeomorphism (vii)  $gsg\hat{g}$ -homeomorphism (viii)  $sg\hat{g}$ -homeomorphism is  $\hat{gsg}$ -homeomorphism.

The following examples show that the converse of the above proposition is not true.

**Example 4.05:** Let  $X = Y = \{a, b, c, \}, \tau = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$  and  $\sigma = \{\phi, \{a\}, Y\}$ . Define  $f: (X, \tau) \rightarrow (Y, \sigma)$  by identity mapping then  $f$  is  $\hat{gsg}$ -homeomorphism but not  $\hat{g}c$ -homeomorphism.

**Example 4.06:** In example (3.07), map  $f$  is  $\hat{gsg}$ -homeomorphism but not  $gsg$ -homeomorphism.

**Example 4.07:** In example (3.05), map  $f$  is  $\hat{gsg}$ -homeomorphism but not  $\psi^*$ -homeomorphism for  $f^{-1}$  is not a  $\hat{g}$ -irresolute map.

**Example 4.08:** Let  $X = \{a, b, c, \}, \tau = \{\phi, \{a\}, \{a, b\}, X\}$ . Define  $f: (X, \tau) \rightarrow (X, \tau)$  by identity mapping then  $f$  is  $\hat{gsg}$ -homeomorphism but not  $\hat{g}$ -homeomorphism for  $f$  and  $f^{-1}$  are not  $\hat{g}$ -irresolute maps.

**Example 4.09:** Let  $X = \{a, b, c, \}$  and  $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ . Define  $f: (X, \tau) \rightarrow (X, \tau)$  by identity mapping then  $f$  is  $\hat{gsg}$ -homeomorphism but not  $sg\psi$ -homeomorphism for  $f$  and  $f^{-1}$  are not  $sg\hat{g}$ -irresolute maps.

**Example 4.10:** In example (4.07), map  $f$  is  $\hat{gsg}$ -homeomorphism but not  $gs\psi$ -homeomorphism for  $f$  is not  $gs\hat{g}$ -irresolute map.

**Example 4.11:** Let  $X = Y = \{a, b, c, \}, \tau = \{\phi, \{a\}, X\}$  and  $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$ . Define  $f: (X, \tau) \rightarrow (Y, \sigma)$  by identity mapping then  $f$  is  $\hat{gsg}$ -homeomorphism but not  $gsg\hat{g}$ -homeomorphism.

**Example 4.12:** In example (3.11), map  $f$  is  $\hat{gsg}$ -homeomorphism but not  $sg\hat{g}$ -homeomorphism.

**Proposition 4.13:** Every  $\hat{gsg}$ -homeomorphism is  $\hat{ggs}$ -homeomorphism.

The converse of the above proposition is not true as it can be seen from the following example.

**Example 4.14:** Let  $X = Y = \{a, b, c, \}, \tau = \{\phi, \{a\}, \{a, b\}, X\}$  and  $\sigma = \{\phi, \{a, b\}, Y\}$ . Define  $f: (X, \tau) \rightarrow (Y, \sigma)$  by identity mapping then  $f$  is  $\hat{ggs}$ -homeomorphism but not  $\hat{gsg}$ -homeomorphism for  $f$  is not  $gsg\hat{g}$ -irresolute map.

**Remark 4.15:**  $\hat{gsg}$ -homeomorphism is independent from  $gc$ -homeomorphism  $gsc$ -homeomorphism as it can be seen from the following examples.

**Example 4.16:** In example (3.07), map  $f$  is  $\hat{gsg}$ -homeomorphism but not  $gc$ -homeomorphism.

**Example 4.17:** Let  $X = Y = \{a, b, c, \}, \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$  and  $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$ . Define  $f: (X, \tau) \rightarrow (Y, \sigma)$  by identity mapping then  $f$  is  $gc$ -homeomorphism but not  $\hat{gsg}$ -homeomorphism for  $f^{-1}$  is not  $\hat{gsg}$ -irresolute map.

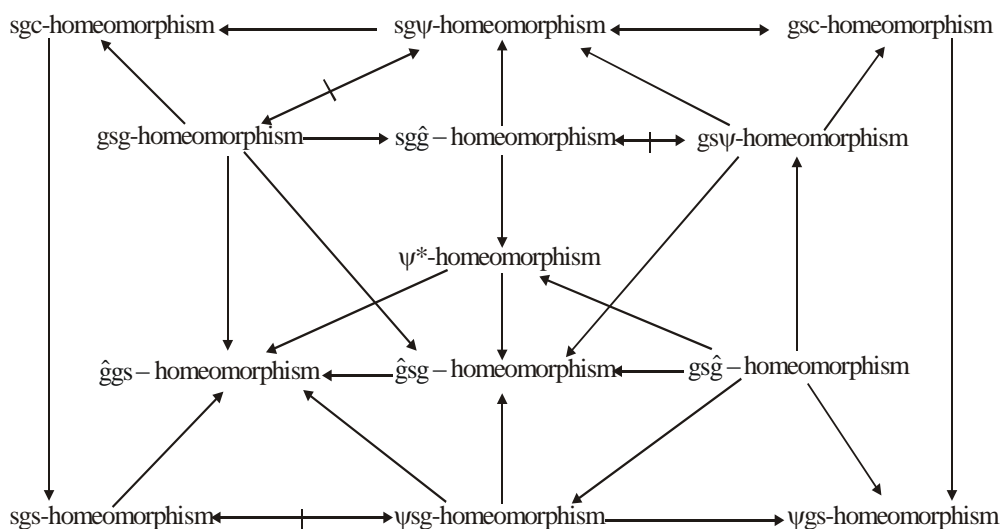
**Example 4.18:** In example (4.11), map  $f$  is  $\hat{gsg}$ -homeomorphism but not  $gsc$ -homeomorphism.

**Example 4.19:** In example (4.22), map  $f$  is  $gsc$ -homeomorphism but not  $\hat{gsg}$ -homeomorphism.

**Remark 4.20:** sgs-homeomorphism and  $\psi$ gs-homeomorphism are not necessarily  $\hat{g}sg$ -homeomorphism as it can be seen from the following example.

**Example 4.21:** In example (3.22), map  $f$  is sgs-homeomorphism and  $\psi$ gs-homeomorphism but not  $\hat{g}sg$ -homeomorphism.

All the above discussion of section (3) and (4) can be summarized by the following diagram.



**Diagram (4.24)**

Where  $A \rightarrow B$  (resp.  $A \leftrightarrow B$ ) represents  $A$  implies  $B$  but not conversely (resp.  $A$  and  $B$  are independent).

**Theorem 4.22:** Every  $\hat{g}sg$ -homeomorphism from a  $T_b$ -space onto itself is a homeomorphism..

**Theorem 4.23 :** Every  $\hat{g}sg$ -homeomorphism from a  $\hat{T}_b$ -space onto itself is a  $sg\hat{g}$ -homeomorphism.

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**How to cite this article:**

Manoj Garg (2017) '  $\hat{SGG}$  -Homeomorphisms And -  $\hat{GSG}$  Homeomorphisms In Topological Spaces', *International Journal of Current Advanced Research*, 06(03), pp. 2543-2548.  
DOI: <http://dx.doi.org/10.24327/ijcar.2017.2548.0046>

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