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Research Article

A COMMON FIXED POINT THEOREM IN FUZZY METRIC SPACE

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A R T I C L E I N F O

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In the present paper we prove common fixed point theorem for occasionally weakly compatible mapping in fuzzy metric space. The theorem is a new version of literature many fixed point theorem in fuzzy metric spaces.

INTRODUCTION

Zadeh in 1965 defined the fuzzy set. The concept of fuzzy metric space was introduced by Kramosil and Michalek in 1975. Grorge and Veermani in 1994 modified the notion of fuzzy metric spaces with the help of continuous t-norms. Pant in 1998 introduced the concept of reciprocally continuous mappings and proved some common fixed point theorem. Jungck generalized the idea of commuting mapping to compatible mappings and then to weakly compatible mapping. Malhotra et al in 2011proved some fixed point theorem on occasionally weakly compatible mappings.

The aim of this paper is to obtain a common fixed point theorem for occasionally weakly compatible mappings in fuzzy metric space.

Preliminaries

Definition: A binary operation * : [0, 1] → [0, 1] is a continuous t-norms if it satisfies the following conditions:

(i) * is associative and commutative;

(ii) * is continuous;

(iii) a*1 = a for all a ϵ [0, 1];

(iv) a*b ≤ c*d whenever a ≤ c and b ≤ d where a, b, c, d ϵ [0, 1]

Definition: A fuzzy set A in X is a function with domain X and values in [0, 1].

Definition: A triples (X, M, *) is said to be a fuzzy metric space (FM space) if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set of X^2 x (0, ∞) satisfying the following conditions, for all x, y, z ϵ X and s, t > 0;

(i) M(x, y, t) > 0;
(ii) M(x, y, t) = 1 for all t > 0 if and only if x = y.
(iii) M(x, y, t) = M(y, x, t);
(iv) M(x, y, t) * M(y, z, s) ≤ M(x, z, t + s);
(v) M(x, y,.) : (0, ∞) → (0, 1) is left continuous.

(X, M, *) denotes a fuzzy metric space, M (x, y, t) can be thought of as degree of nearness between x and y with respect to t. We identify x = y with M(x, y, t) = 1 for all t > 0.

Definition: Two self mappings f and g of a fuzzy metric space (X, M, *) are called compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$ wherever, $\{x_n\}$ is sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some x in X.

Definition: Two self mappings f and g of a fuzzy metric space (X, M, *) are called reciprocally continuous on X if $\lim_{n\to\infty} fgx_n = f_x$ and $\lim_{n\to\infty} gfx_n = x$ wherever, $\{x_n\}$ is sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some x in X.

Definition: Let (X, M, *) be a fuzzy metric space. Then

(i) A sequence $\{x_n\}$ in X is said to converge to x in X if for each ϵ > 0 and each t > 0, there exist n_0 ϵ N such that $M(x_n, x, t) > 1 - ϵ$ for all $n \geq n_0$.
(ii) A sequence $\{x_n\}$ in X is said to be cauchy in X if for each ϵ > 0 and each t > 0, there exist n_0 ϵ N such that $M(x_n, x_m, t) > 1 - ϵ$ for all $n, m \geq n_0$.
(iii) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition: Two self maps f and g of a set X are occasionally weakly compatible (owe) iff there is a point in X which is a coincidence point of f and g at which f and g commute.

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Lemma: Let X be a set and f, g owe self maps of X. If f and g have a unique point of coincidence, w = f_x = g_x, then w is the unique common fixed point of f and g.

Lemma: Let (X, M, *) be a fuzzy metric space. If there exist q ϵ (0, 1) such that M(x, y, qt) ≥ M(x, y, t) for all x, y ϵ X and t > 0 then x = y.

Main Theorem

Theorem: Let (X, M, *) be a complete fuzzy metric space and let P, Q, S and T are self-mappings of X. Let the pairs {P, S} and {Q, T} be owe. If there exist q ϵ (0, 1) such that $M(Px, Qy, qt) \geq \min\{M(Sx, Ty, t), \{\frac{M(Sx, Px, t) + M(Qy, Ty, t)}{2}\}, M(Px, Ty, t), M(Qy, Sx, t)$ (1)

For all x, y ϵ X and for all t > 0, then there exists a unique point w ϵ X such that Pw = Sw = w and a unique point z ϵ X such that Qz = Tz = z. Moreover z = w, then there exists a unique common fixed points of P, Q, S and T.

Proof: Let the pair {P, S} and {Q, T} be owc so there are points x, y ϵ X such that Px = Sx and Qy = Ty. We claim that Px = Qy. By inequality (1)

$$M(Px, Qy, qt) \geq \min\{M(Sx, Ty, t), \{\frac{M(Sx, Px, t) + M(Qy, Ty, t)}{2}\}, M(Px, Ty, t), M(Qy, Sx, t)\}$$

$$\geq \min\{M(Px, Qy, t), \{\frac{M(Px, Px, t) + M(Qy, Qy, t)}{2}\}, M(Px, Qy, t), M(Qy, Px, t)$$

$$\geq M(Px, Qy, t)$$

So Px = Qy i.e. Px = Sx = Qy = Ty.

Suppose that there is another point z such that Pz = Sz then by (1) we have Pz = Sz = Qy = Ty so Px = Pz and w = Px = Sx is unique point of coincidence of P and S.

Similarly there is a unique point z ϵ X such that z = Qz = Tz. Assume that w ≠ z. We have

$$M(w, z, qt) = M(Pw, Qz, qt)$$

$$\geq \min\{M(Sw, Tz, t), \{\frac{M(Sw, Pz, t) + M(Qz, Tz, t)}{2}\}, M(Pw, Tz, t), M(Qz, Sw, t)\}$$

$$\geq \min\{M(w, z, t), \{\frac{M(w, z, t) + M(z, z, t)}{2}\}, M(w, z, t), M(z, w, t)\} \geq M(w, z, t)$$

Therefore by Lemma 2.9 we get z = w and z is a common fixed point of P, Q, S and T. The uniqueness of fixed point can be easily proved by equation (1).

Theorem: Let (X, M, *) be a complete fuzzy metric space and let P, Q, S and T are self-mappings of X. Let the pairs {P, S} and {Q, T} be owe. If there exist q ϵ (0, 1) such that $M(Px, Qy, qt) \geq \Phi\ [\min\{M(Sx, Ty, t), \{\frac{M(Sx, Px, t) + M(Qy, Ty, t)}{2}\}, M(Px, Ty, t), M(Qy, Sx, t)\}]$ (2)

for all x, y ϵ X and Φ ϵ [0, 1] → [0, 1] such that Φ(t) > t for all 0 < t < 1, then there exists a unique common fixed point of P, Q, S and T.

Proof: Let the pair {P, S} and {Q, T} be owc so there are points x, y ϵ X such that Px = Sx and Qy = Ty. We claim that Px = Qy. By inequality (2)

$$M(Px, Qy, qt) \geq \Phi\ [\min\{M(Sx, Ty, t), \{\frac{M(Sx, Px, t) + M(Qy, Ty, t)}{2}\}, M(Px, Ty, t), M(Qy, Sx, t)\}]$$

$$\geq \Phi[M(Px, Qy, t)]$$

Now proof follows from theorem (3.1).

References

1. C.T. Aage, J.N. Salunke : Common fixed point theorems in fuzzy metric spaces, Int. Jour. Pure Appl. Math., 56(2), 2009, 155-164.
2. C.T. Aage, J.N. Salunke : Some fixed point theorems in fuzzy metric spaces, Int. Jour. Pure Appl. Math., 56(3), 2009, 311-320.
3. A. Al-Thagafi and Naseer Shahzad : Generalized I-Nonexpansive selfmaps and invariant approximations, Acta Mathematica Sinica, English series May, 24(5), 2008, 867-876.
4. P. Balasubrmaniam, S. Murlisankar, R. P. Pant : Common fixed point of four mappings in fuzzy metric spaces, J. Fuzzy Math., 10(2), 2002.
5. Y. J. Cho, H. K. Pathak, S. M. Kang, J. S. Jung : Common fixed points of compatible maps of type (A) on fuzzy metric spaces, Fuzzy sets and systems, 93, 1998, 99-111.
6. A George, P. Veeramani : On some results in fuzzy metric spaces, Fuzzy sets and systems, 1988, 64, 395-399.
7. M. Grabiec : Fixed points in fuzzy metric spaces, Fuzzy sets and systems, 1988, 385-389.
8. O. Hadzic : Common fixed point theorems for families of mapping in complete metric space, Math. Japan, 1984, 127-134.
9. G. Jungck : Compatible mappings and common fixed points (2), Int. J. Math. Sci., 1988, 285-288.
10. G. Jungck and B. E. Rhoades : Fixed point for set valued functions without continuity, Indian jour. Pure appl. Math., 29(3), 1998, 771-779.
11. S. K. Malhotra, N. Verma and R. Sen : Some common fixed point theorems for occasionally weakly compatible mappings in fuzzy metric spaces, Int. Jour. Appl. Eng. Res. Dind., 1(4), 2011, 933-941.
12. G. Jungck and B. E. Rhoades: Fixed point theorems for occasionally weakly compatible mappings, Fixed point theory, 7(2), 2006, 287-296.
13. G. Jungck and B. E. Rhoades: Fixed point theorems for occasionally weakly compatible mappings, Erratum, Fixed point theory, 9(1), 2008, 283-284.
14. O. Kramosiland, J. Michalek : Fuzzy metric and statistical metric spaces, Kybernetka, 11, 1975, 326-334.
15. S. Kutukcu : A fixed point theorem for contraction type mappings in Manger spaces, Am. J. Appl. Sci., 4(6), 2007, 371-373.

16. S. K utukcu, S. Sharma and H. Tokogoz, A fixed point theorem in fuzzy metric spaces, Int. Jour. Math. Anal., 1(18), 2007, 861-872.
17. R. P. Pant: Common fixed points of four mappings, Bull. Math. Soc., 90, 1998, 281-286.
18. R. P. Pant: Common fixed points of four contractive mappings, J. Math. Anal. Appl., 226, 1998, 251-258.
19. R. P. Pant and K. Jha: Aremark on common fixed points of four mappings in a fuzzy metric space, J. Fuzzy Math., 12(2), 2004, 433-437.
20. H. K. Pathak and P. Singh: Common fixed point theorem for weakly compatible mappings, Int. Math. Forum, 2(57) 2007, 2831-2839.
21. B. Schweizer and A. Sklar : Statistical metric spaces, Pacific J. Math., Pecific J. Math., 1960, 1960.
22. Seong Hoon Cho: On common fixed point in fuzzy metric space, Int. Math. Forum, 1(10), 2006, 471-479.

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