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Research Article

ON NON- HOMOGENEOUS QUINTIC DIOPHANTINE EQUATION WITH FIVE UNKNOWNS $2(x-y)(x^3+y^3)=4^n\left(z^2-w^2\right)T^3$

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ABSTRACT

The non-homogeneous quintic diophantine equation with five unknowns given by $2(x-y)(x^3+y^3)=4^n\left(z^2-w^2\right)T^3$ is analyzed for its non-zero distinct integer solutions .

INTRODUCTION

The Diophantine equations are rich in variety and offer an unlimited field for research [1-3]. In particular refer [4-14] for a few problems on Biquadratic equation with 2, 3,4 and 5 unknowns. In [15-19], problems on quintic equations with three and five unknowns are analysed for their corresponding integer solutions. This paper concerns with yet another interesting non-homogeneous quintic diophantine equation with five variables given by $2(x-y)(x^3+y^3)=4^n\left(z^2-w^2\right)T^3$ for determining its infinitely many non-zero distinct integral solutions.

Method of Analysis

The non-homogeneous quintic diophantine equation with five variables under consideration is

$$2(x-y)(x^3+y^3)=4^n\left(z^2-w^2\right)T^3 \tag{1}$$

The process of obtaining non-zero distinct integer solutions to (1) are illustrated below:

Illustration 1:
Introducing the linear transformations

$$x=4^n(u+v),\ y=4^n(u-v),\ T=4^n v,\ z=2u+v,\ w=2u-v,\ u\neq v \tag{2}$$

in (1), it reduces to the equation

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$$u^2+3v^2=v^3 \tag{3}$$

whose solutions may be taken as

$$v=Q^2+3,\ u=Q(Q^2+3) \tag{4}$$

In view of (2), the corresponding integer solutions to (1) are given by

$$x=4^n(Q^3+Q^2+3Q+3)$$
$$y=4^n(Q^3-Q^2+3Q-3)$$
$$T=4^n(Q^2+3)$$
$$z=(2Q^3+Q^2+6Q+3)$$
$$w=(2Q^3-Q^2+6Q-3)$$

Note:1

Apart from (2),one may consider the following transformations

$$x=4^n(u+v),\ y=4^n(u-v),\ T=4^n v,\ z=u+2v,\ w=u-2v,\ u\neq v$$
$$x=4^n(u+v),\ y=4^n(u-v),\ T=4^n v,\ z=2uv+1,\ w=2uv-1,\ u\neq v$$

leading to two different solutions to (1).

Illustration 2:
Introducing the linear transformations

$$x = 4^n(u+v), y = 4^n(u-v), T = 4^n u, z = 2u+v, w = 2u-v, u \neq v \quad (5)$$

in (1), it reduces to the equation

$$u^2 + 3v^2 = u^3 \quad (6)$$

whose solutions may be taken as

$$v = Q(3Q^2+1), u = 3Q^2+1 \quad (7)$$

In view of (5), the corresponding integer solutions to (1) are given by

$$x = 4^n(3Q^3+3Q^2+Q+1)$$
$$y = 4^n(-3Q^3+3Q^2-Q+1)$$
$$T = 4^n(3Q^2+1)$$
$$z = (3Q^3+6Q^2+Q+2)$$
$$w = (-3Q^3+6Q^2-Q+2)$$

Note:2

Apart from (5),one may consider the following transformations

$$x = 4^n(u+v), y = 4^n(u-v), T = 4^n u, z = u+2v, w = u-2v, u \neq v$$
$$x = 4^n(u+v), y = 4^n(u-v), T = 4^n u, z = 2uv+1, w = 2uv-1, u \neq v$$

leading to two different solutions to (1).

Illustration 3:

Introducing the linear transformations

$$x = 4^n(u+v), y = 4^n(u-v), T = 4^n P, z = 2u+v, w = 2u-v, u \neq v \quad (8)$$

in (1), it is written as

$$u^2 + 3v^2 = P^3 = P^3 * 1 \quad (9)$$

Assume

$$P = a^2 + 3b^2 \quad (10)$$

Write 1 on the R.H.S. of (9) as

$$1 = \frac{(1+i\sqrt{3})(1-i\sqrt{3})}{4} \quad (11)$$

Substituting (10) & (11) in (9) and employing the method of factorization, consider

$$u + i\sqrt{3}v = \frac{(a+i\sqrt{3}b)^3(1+i\sqrt{3})}{2} \quad (12)$$

Equating the real &imaginary parts in (12), the values of u and v are obtained.

Since our interest is on finding integer solutions, replace a by 2A ,b by 2B in the above resulting values of u, v and (10).In view of (8), the corresponding integer solutions to (1) are as follows:

$$x = 4^{n+1} * (2A^3 - 18AB^2 - 6A^2B + 6B^3),$$
$$y = 4^{n+1} * (-12A^2B + 12B^3),$$
$$T = 4^{n+1} * (A^2 + 3B^2),$$
$$z = 4(3A^3 - 27AB^2 - 15A^2B + 15B^3),$$
$$w = 4(A^3 - 9AB^2 - 21A^2B + 21B^3),$$

Note :3

In addition to (11),1 on the R.H.S. of (9) may expressed as below:

$$1 = \frac{(1+i4\sqrt{3})(1-i4\sqrt{3})}{49},$$
$$1 = \frac{(11+i4\sqrt{3})(11-i4\sqrt{3})}{169},$$
$$1 = \frac{(11+i5\sqrt{3})(11-i5\sqrt{3})}{196},$$

Following the above analysis,one has three more sets of integer solutions to (1).

Illustration 4:

Introducing the linear transformations

$$x = 4^n(u+v), y = 4^n(u-v), T = 4^n P, z = 8u+v, w = 8u-v, u \neq v \quad (13)$$

in (1), it is written as

$$u^2 + 3v^2 = 4P^3 \quad (14)$$

Write 4 on the R.H.S. of (14) as

$$4 = (1+i\sqrt{3})(1-i\sqrt{3}) \quad (15)$$

Substituting (10) & (15) in (14) and employing the method of factorization, consider

$$u + i\sqrt{3}v = (a+i\sqrt{3}b)^3(1+i\sqrt{3}) \quad (16)$$

Equating the real &imaginary parts in (16), the values of u and v are obtained.

In view of (13),the corresponding integer solutions to (1) are as follows:

$$x = 4^n * (2a^3 - 18ab^2 - 6a^2b + 6b^3),$$
$$y = 4^n * (-12a^2b + 12b^3),$$
$$T = 4^n * (a^2 + 3b^2),$$
$$z = (9a^3 - 81ab^2 - 69a^2b + 69b^3),$$
$$w = (7a^3 - 63ab^2 - 75a^2b + 75b^3)$$

Illustration 5:

Introducing the linear transformations

$$x=4^n(u+v), y=4^n(u-v), T=4^n v, z=8u+v, w=8u-v, u\neq v \quad (17)$$

in (1), it reduces to the equation

$$u^2+3v^2=4v^3 \quad (18)$$

whose solutions may be taken as

$$v=k^2\pm k+1, u=(k^2\pm k+1)(2k+1) \quad (19)$$

In view of (17), the corresponding integer solutions to (1) are given by

$$x=4^n(k^2\pm k+1)(2k+2)$$

$$y=4^n(k^2\pm k+1)(2k)$$

$$T=4^n(k^2\pm k+1)$$

$$z=(k^2\pm k+1)(16k+9)$$

$$w=(k^2\pm k+1)(16k+7)$$

Illustration 6:
Introducing the linear transformations

$$x=4^n(u+v), y=4^n(u-v), T=4^n u, z=8u+v, w=8u-v, u\neq v \quad (20)$$

in (1), it reduces to the equation

$$u^2+3v^2=4u^3 \quad (21)$$

whose solutions may be taken as

$$u=3k^2\pm 3k+1, v=(3k^2\pm 3k+1)(2k-1) \quad (22)$$

In view of (20), the corresponding integer solutions to (1) are given by

$$x=4^n(3k^2\pm 3k+1)(2k)$$

$$y=4^n(3k^2\pm 3k+1)(2-2k)$$

$$T=4^n(3k^2\pm 3k+1)$$

$$z=(3k^2\pm 3k+1)(2k+7)$$

$$w=(3k^2\pm 3k+1)(-2k+9)$$

CONCLUSION

An attempt has been made to obtain non-zero distinct integer solutions to the non-homogeneous quintic diophantine equation with five unknowns given by $2(x-y)(x^3+y^3)=4^n(z^2-w^2)T^3$.One may search for other sets of integer solutions to the considered equation as well as other choices of the fourth degree diophantine equations with multivariables.

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