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Research Article

ENTROPY APPROACH FOR UTILITY ASSIGNMENT IN DECISION PROCESS

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ABSTRACT

The object of the present paper is to show the equivalence between the maximum utility and maximum entropy principle based on Bose-Einstein and Fermi-Dirac Entropies.

INTRODUCTION

The transportation problem is one of the subclasses of linear programming problem(LPP). A traditional transportation planning process consists of four major components: trip generation, trip distribution, modal split and trip assignment. The problem is called transportation problem because it involves the transportation or physical distribution of goods from several supply points to a number of demand points. The wide class of transportation demand models has been derived from the laws of physics. The best known of these models is the gravity model (generated from an analogy with Newton's gravitational law) and the entropy model (derived from the second thermodynamic principle. The solution of the transportation problem requires the determination of how many units of physical goods should be shipped from each origin to various destinations, satisfying source availability and destination demand, while minimizing the total cost of transportation.

This paper is designed with the following plans and programmes: In section (2) we have considered trip distribution based on maximum entropy (Shannon) model and derived same distributions from maximum-utility model with different cost functions. In section (3) we have the similar task with Bose-Einstein and Fermi-Dirac form of entropy.

During the present decade a special focus of attention and query has perpetually been made on transportation problem in urban and environmental modelling. This area has been drawing the interest and attention of many researchers who firmly consider this area to be an active field of studies. Therefore, rigorous investigation and research are constantly being accomplished to by a good number of scientists of different background in order to improve and enrich this field. With this end in view they employ different approaches. A recent and more general approach is based on the concept of entropy [7, 12] and the principle of maximum entropy estimation [4]. This approach was developed profoundly by Wilson [9, 10, 11] and others [12].

The concept of the economic model of utility paves another way to meet the problem. The interrelation between the maximum entropy and maximum utility was pointed out by Wilson [10]. The Chief aim and goal of this present paper is to explore the equivalence or interrelation between these two principles in the context of trip distribution. The approach is, however, different from that of Wilson [10] and is better in line with that of Beckmann [1].

Shannon's Entropy Approach

Let us consider a model city with a Central Business District (CBD), and a set of location or site $\{i, i = 1, 2, \ldots, n\}$ and a set of working places or sectors $\{j, j = 1, 2, \ldots, m\}$ of the C.B.D.
Let T_{ij} be the number of trips originating from the ith location place to the jth working place (destination). Then the entropy

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of the trip distribution is given by (analogous to Shannon entropy).

$$S = -\sum_{i=1}^{n}\sum_{j=1}^{m} T_{ij} ln T_{ij} \quad (1)$$

Let us assume that the total trips originating from the site i and the total number of persons employed in the jth working place be prescribed.

$$\sum_{j=1}^{m} T_{ij} = O_i \quad (i = 1, 2, \ldots, n)$$

$$\sum_{i=1}^{n} T_{ij} = D_j \quad (j = 1, 2, \ldots, m) \quad (2)$$

and further that the total cost of transportation be fixed.

$$\sum_{i=1}^{n}\sum_{j=1}^{m} T_{ij} C(r_{ij}) = \hat{C} \quad (3)$$

Where r_{ij} is the distance of the jth sector of the C.B.D. from the ith site of location. $C(r_{ij})$ is the cost function. The constraints (2) and (3) are insufficient to establish the trip distribution T_{ij}. This can, however, be estimated by Jaynes maximum-entropy principle [4].

According to this principle the least biased distribution will be that which will maximize the entropy S given by (1) subject to the constraints (2) and (3). The maximization yield [9].

$$T_{ij} = o_i d_j e^{-\mu c(r_{ij})} \quad (4)$$

Where the parameters o_i, d_j and μ are to be determined by the equation :

$$\sum_{j=1}^{m} o_i d_j e^{-\mu c(r_{ij})} = O_i \ (i = 1, 2, \ldots, n)$$

$$\sum_{i=1}^{n} o_i d_j e^{-\mu c(r_{ij})} = D_j \ (j = 1, 2, \ldots, m) \quad (5)$$

and $$\sum_{i=1}^{n}\sum_{j=1}^{m} o_i d_j C(r_{ij}) e^{-\mu c(r_{ij})} = \hat{C} \quad (6)$$

Utility Approach

According to the considerations discussed so far the distribution has been determined as a problem of the theory of information. Since the information (or constraints) available to us are insufficient to uniquely determine the exact distribution, we have used the maximum entropy principle of statistical inference. Let us now try to set up an economic model of the choice behaviour that underlines trip making. Here we are to distinguish the trips by purposes. In any city work trips are really the result of some basic choices. Generally, the first and prime importance to a person is either to accept and acknowledge a job or to decide to form a business center and afterwards he must seek for an adequate habitable place to live in. He always tries to maximize his utility as far as possible with limited available resources.

A person looking at various potential residences associates a rating or utility index with each residence. Let the probability density of utility x for a particular location i be $f_i(x)$, $x \geq a > 0$. Then the probability of something in such a location having utility equal to z or greater (better) than $z$is given by

$$P(z) = \int_{z}^{\infty} f_i(x)\, dx \quad (7)$$

The utility z includes rent, access to school, hospitals, gas connection, local shopping facilities, Electricity board etc. except the distance from C. B. D. So, the net utility at a place is the utility minus the transportation cost (z-βr_{ij}) where r_{ij} is the distance of the ith sector to the jth working place at the C. B. D. and β is the conversion factor of distance to utility.

A person will now try to maximize the net utility. Since the economic power of different persons are different, so he will accept a place of living with a satisfying level of utility t (say) so that,

$$z - \beta r_{ij} \geq t \text{ or } z \geq t + \beta r_{ij} \quad (8)$$

Now, if O_i be the total number of housing supply in the ith sector at a distance $r_{ij}$from jth working place, then probability of person living in this level is

$$O_i P(t + \beta r_{ij}) \quad (9)$$

If the total number of persons employed in the jth working place in C. B. D. be $D_j$then [1]

$$T_{ij} \approx O_i D_j P(t + \beta r_{ij}) \quad (10)$$

Comparing (4) and (10) we observe that

$$o_i d_j e^{-\mu c(r_{ij})} \approx O_i D_j P(t + \beta r_{ij})$$

or $$C(r_{ij}) \approx lnP(t + \beta r_{ij}) + ln(O_i D_j) - ln(o_i d_j) \quad (11)$$

So, it shows that cost can always be found so that entropy maximization and utility maximization became equivalent. Let us now illustrate the equivalence with some specific example by finding the appropriate cost functions.

Illustrative Examples

First, suppose that the distribution of utility to be a negative exponential

$$f_i(x) = e^{-\alpha x}; x \geq 0 \quad (12)$$

then from (10)

$$T_{ij} \approx O_i D_j \int_{t+\beta r_{ij}}^{\infty} e^{-\alpha x} dx \approx \frac{O_i D_j e^{-\alpha(t+\beta r_{ij})}}{\alpha}$$

Now from (4) we see that if the cost of travel $C(r_{ij})$ be of the form :

$$C(r_{ij}) \approx t + \beta r_{ij} \quad (13)$$

Then two method lead to the same type of distribution.

$$f_i(x) = \frac{e^{-x} x^{m-1}}{\Gamma(m)}, x \geq 0 \quad (14)$$

Then

$$T_{ij} \approx O_i D_j \int_{t+\beta r_{ij}}^{\infty} \frac{e^{-x} x^{m-1}}{\Gamma(m)} dx$$

$$= O_i D_j e^{-(t+\beta r_{ij})}(O_1 + O_2 r_{ij} + \ldots\ldots\ldots . O_m r_{ij}^{m-1})$$

This leads to entropy maximization distribution if we take

$$C(r_{ij}) \approx \left[(t + \beta r_{ij}) - \ln \sum_{\mu=1}^{m-1} O_\mu r^{\mu-1} \right] \quad (15)$$

If the distribution of utility to be a Pareto

$$f_i(x) = \frac{ak^a}{x^{a+1}} \; ; x \geq k > 0, \; a > 0 \quad (16)$$

Then from (10)

$$T_{ij} \approx O_i D_j \int_{t+\beta r_{ij}}^{\infty} \left(\frac{ak^a}{x^{a+1}} \right) dx \approx O_i D_j \frac{k^a}{(t+\beta r_{ij})^a}$$

We observe that if we take the cost function

$$C(r_{ij}) \approx ln(t + \beta r_{ij})^a \quad (17)$$

The maximum-entropy distribution leads to the utility distribution.

Bose-Einstein and Fermi-Dirac Entropy Approach

In this section we shall try to obtain some trip distribution based on entropies other than that of Shannon and examine the role of utility functional in generating these types of distributions.

As before let T_{ij} be the number of trips from the ith sector (origin) to the jth working place and the constraints or information available are the same as those of (2) and (3).

Now, the problem is to estimate T_{ij} on the basis of information (constraints (2) and (3)). We apply Jaynes maximum entropy principle with quantum measure of entropy.

$$\hat{S} = -\sum_{i=1}^{n} \sum_{j=1}^{m} T_{ij} ln T_{ij} + a \sum_{i=1}^{n} \sum_{j=1}^{m} (1+aT_{ij}) \, ln(1+aT_{ij}) \quad (18)$$

where $a = +1$ for Bose-Einstein entropy and $a = -1$ for Fermi-Dirac entropy. The maximization of the entropy $\hat{S}$ subject to the constraints (2) and (3) leads to the distribution [6]

$$T_{ij} = \frac{1}{o_i d_j e^{\mu c(r_{ij})} - a} \quad (19)$$

The value $a = 1$ in (19) corresponds to Fermi-Dirac distribution of trips. In this case, at most one trip end is permitted per destination (job). The value $a = -1$ in (19) corresponds to Bose-Einstein distribution of trips. This corresponds to unlimited and per destination [3]. We shall discuss the feasibility of the distribution (19) in reality later on.

Utility Approach

We have seen in section (2) that

$$T_{ij} \approx O_i D_j P(t + \beta r_i) \quad (20)$$

$$P(z) = \int_z^{\infty} f_i(x) \, dx$$

where $f_i(x)$ is some kind of utility functional for the ith location. This may be the potential functional of the jth work place or may be some functional which depends on the utility of the ith origin (sector-living place) and attraction of jth work place. O_i and D_j are same prescribed values related to the ith origin (living place) and jth destination (work place). Again we observe from (10) and (19)

$$O_i D_j P(t + \beta r_{ij}) \approx \frac{1}{o_i d_j e^{\mu c(r_{ij})} - a}$$

or, $$o_i d_j e^{\mu c(r_{ij})} \approx \frac{1}{O_i D_i P(t + \beta r_{ij})} + a$$

or, $$C(r_{ij}) \approx ln \left[\frac{a}{o_i d_j} + \frac{1}{O_i o_i D_j d_j P(t + \beta r_{ij})} \right] \quad (21)$$

So, costs can also be found which can make entropy maximization and utility concept equivalent. The equivalence has also been illustrated with some specific examples.

Illustrative Examples

Let us consider an utility function as $f_i(x) = \frac{ae^x}{(e^x + a)^2}$; $x > a$ (22)

Then $$T_{ij} \approx O_i D_j \int_{t+\beta r_{ij}}^{\infty} \frac{ae^x}{(e^x + a)^2} dx \approx O_i D_j \frac{a}{e^{t+\beta r_{ij}} + a} = \frac{a \; O_i D_j}{e^{s+kr_{ij}} + a} \quad (23)$$

We observe that, for a = $\pm$ 1, the distribution (23) so obtained resembles to quantum distribution of trips with $C(r_{ij}) \approx (t + \beta r_{ij})$.

We have already seen that the utility distribution follows a negative exponential as $f_i(x) = e^{-\alpha x}; x \geq 0$. T_{ij} comes out to be of the form

$$T_{ij} \approx \frac{O_i D_j e^{-\alpha(t+\beta r_{ij})}}{\alpha} \quad (24)$$

which leads to the quantum distribution of the trip if

$$C(r_{ij}) \approx ln \left(e^{\alpha(t+\beta r_{ij})} \pm 1 \right) \quad (25)$$

CONCLUSIONS

In fine, this paper, as already mentioned earlier, deals with two approaches to the problem of decision making in the transportation system. The two approaches are based on the concepts of entropy and utility. In the present paper we have tried to show the interrelation between the above two approaches mathematically in the case of trip distribution. From this point of view this paper appears to be of hypothetical and theoretical type. But, to be honest, it is not out and out hypothetical, because some of the functions exploited in the paper have been successfully applied in some other models. For example, the exponential utility function has been used in risk-sharing problem [6]. The choice of the utility functions $f(x)$ is made for a particular purpose only. So it is somewhat of ad hoc nature. It will depend on different economic problems or situations and its success will also be based on the proper choice of the utility function [8]. In this paper we have selected a number of utility function $f(x)$ and have shown how the maximum-utility method which is of great commercial importance can be converted into a well-established statistical decision theory based on maximum-entropy principle [10, 5]. Regarding the applicability of Bose-Einstein and Fermi-Dirac entropy we state that though Shannon entropy is of wide range of applicability, Bose-Einstein and Fermi-Dirac entropy has been applied successfully in case of work-trip distribution and commodity distribution respectively [3]. Ours is the first approach to show the equivalence between the maximum utility and maximum-entropy principle based on Bose-Einstein and Fermi-Dirac entropies.

References

Beckmann, M. J. (1974) : “Entropy Gravity and Utility in Transportation Modelling”. “Information, Inference and Decision” Edited by Gunter Menges, D. Reidel transl. To Dordrecht, Hollands.

Abbas, Ali E, Maximum entropy utility, *Operation Research*, 54 (2006), 277-290.

Evans, A. W.(1970) : “Some properties of trip distribution models.” Transportation Research 4, p. 19-36.

Fisk, C.(1985) : “Entropy and information Theory : Are we missing something?” Env. And planning, 14A, p.679-710.

Jaynes, E. T.(1957) : “Information theory and statistical Mechanics”, Physical Reviews 106, p.620-630.

Jumarie, G(1990) : “Relative Information and Application” Springer-Verlag, Berlin.

Kapur, J.N.(1990) : “Maximum Entropy Models in Science and Engineering”, Wiley Eastern, New Delhi, India.

Shannon, C. E., and Weaver, W. (1949): “Mathematical Theory of Communication”, University of Illioois Press, Urbana, Chicago, U.S.A.

Tribus. M. (1969): Rational Description, Decision and Designs”, Pergamon Press, New York, U. S. A.

Wilson, A. G. (1967): “A Statistical Theory of Spatial Distribution Models”. Transportation Research 1, p.253-269.

Wilson, A. G. (1970): “Entropy in Urban and Regional Modelling”. Pion, London, U. K.

Wilson, A. G. (1979): “Optimization in Locational and Transport Analysis. John Wiley

Chichester U.K. Webber, M.J.(1979): “Information Theory an Urban spatial structures”. CroomHelm, London., U.K.

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