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Research Article

EVALUATION AND FORMULATION OF RALEIGH- RITZ BASED PECULIAR TOTAL POTENTIAL ENERGY FUNCTIONAL FOR DOUBLY SYMMETRIC MULTI (DSM) CELL AND MONO SYMMETRIC MULTI (MSM) CELL THIN- WALLED BOX COLUMN (TWBC) CROSS- SECTIONS

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A B S T R A C T

Knowing the critical buckling load would put the designer in better stead to carry out efficient design. This work is concerned with the evaluation and formulation of peculiar Total Potential Energy Functional (TPEF)for Doubly Symmetric Multi (DSM) cell and Mono Symmetric Multi (MSM) cell Thin -walled Box Column (TWBC) cross-sections in preparation for the eventual stability analysis in subsequent papers. This present study is a follow up of the works by Nwachukwu and others (2017) and Nwachukwu and others (2021). For each of the cross – section under consideration, the cross –sectional properties are evaluated first to obtain the cross-sectional areas and moment of inertia. This is followed by the formulation of the TPEF for the cross- sections for different boundary conditions..The Raleigh- Ritz based formulated TPEF equations are found suitable, handy and simple to be used in the proper stability/buckling analysis (in future publications) where critical bucking loads will be obtained. The audience is therefore advised to look forward towards future publications, where these equations will be transformed in the stability analysisof DSM and MSM TWC.

INTRODUCTION

DSM and MSM are common examples of TWC cross –sections .According to Simao and Simoes da silva (2004), the use of very slender thin-walled cross-sections members have become increasingly common due to their high stiffness/weight ratio, in recent years. In general, according to Ezeh and Osadebe (2010), thin-walled structures consist of a wide and growing field of engineering application which seek efficiency in strength and cost by minimizing material. For about a century many branches of the industry have sought stronger and at the same time lighter structural solutions which optimize the effectiveness and the cost of the structures (Andreassen, 2012). Among these are the civil, mechanical, naval, and aerospace industries. This has led to an increasing use of thin-walled structures such as cold-formed steel beams and columns, steel and concrete box girders, ship hulls, trapezoidal steel sheeting and other structures in which one dimension is small compared to the other dimensions. Thin-walled structures such as beams, columns, plates, shells, sheeting, among others are frequently used in civil, naval, space, offshore and aerospace constructions.

The present study is an attempt to evaluate and formulate the TPEF for DSM and MSM cross-sections. It is the follow up of the works by Nwachukwu and others (2017) and Nwachukwu and others (2021) where the governing equation for the TPEF for a TWBC applicable to RRM and peculiar TPEF for DSS cross – section were derived respectively . Before now, many authors have carried out one form of analysis or the other on thin- walled box columns and related topics, but none has used RRM approach with polynomial shape function. For example, Krolak and others (2009) presented a theoretical, numerical and

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experimental analysis of the stability and ultimate load of multi-cell thin-walled columns of rectangular and square cross-sections subjected to axial compression. Shanmugam and others (1989) presented a numerical method to investigate the ultimate strength behavior of thin-walled steel box columns subjected to axial loads and biaxial end moments.

The work of Ezeh (2009) involved a theoretical formulation based on Vlasov's theory as modified by Varbanov, in analyzing flexural, flexural-torsional, and flexural-torsional-distortional buckling modes of thin-walled closed columns. Chidolue and Osadebe (2012), also used Vlasov's theory to carryout Torsional- Distortional analysis of thin- walled box girder bridges. Chidolue and Aginam (2012) investigated the effects of shape factor on the Flexural- Torsional-Distortional behavior of thin-walled box girder structures using Vlasov's Theory. Ezeh (2010) also investigated the buckling behavior of axially compressed multi- cell doubly symmetric thin- walled column using Vlasov's theory. The works of Osadebe and Chidolue (2012a), Osadebe and Chidolue (2012b), Osadebe and Ezeh (2009a), Osadebe and Ezeh (2009b) were also based on Vlasov's method.

Thus in the area of stability analysis of thin-walled box (closed) columns, little or no effort has been done to use the method of Raleigh- Ritz for analysis. Henceforth, it has become important to further the frontier of knowledge by developing the individual TPEF for different DSM and MSM boundary conditions in line with RRM..This presentwork is therefore an attempt to formulate a working TPEF for DSM and MSM cross- sections based on the developed governing TPEF Equations. The formulated energy functional will now be used toanalyze a DSM and MSM thin- walled box (closed) columns of different boundary conditions in subsequent papers .

Evaluation and Formulation of TPEF For DSM TWBC Cross-Section

Evaluation of DSM Cross- Sectional Properties

Let us recall the general(governing) Total Potential Energy Functional derived by Nwachukwu and others (2017) as stated in Eqn.(1).

$$\pi = k_1 \int_L v^2 x^2 (2L-x)^2 dx + k_2 \int_L (v^I)^2 dx + k_3 \int_L (v^{II})^2 dx - k_4 \int_L (v^I)^2 dx. \quad (1)$$

$$k_1 = \frac{Ap^2}{8EI^2}; \; k_2 = \frac{AG}{2}; k_3 = \frac{EI}{2}; \; and \;\; k_4 = \frac{P}{2} \quad \text{2(a-d)}$$

P is critical buckling load, A is Cross sectional area, E is young modulus of elasticity, G is shear modulus, I is moment of inertia, and L is length of the column.

Here,v = the displacement function, which is a function of polynomial shape function, ϕ

$$v = \sum_i^n c_i \phi_i = c_1\phi_1 + c_2\phi_2 + c_3\phi_3 + \ldots\ldots + c_n\phi_n \quad (3)$$

according to Raleigh- Ritz Theory
ϕ = Polynomial shape function
c = undetermined coefficient / unknown constant.

Note that Nwachukwu and others (2021) have generated the Polynomial shape function used in the formulation. Again, Nwachukwu and others (2021) have demonstrated the efficacy of Eqn. (1)by using it to formulate the TPEF for Doubly Symmetric Single (DSS) cell TWBC for different boundary conditions. For instance, for S-S boundary condition, the peculiarTPEF is given as

$$\pi_{DSS}^{S-S} = k_1{}^{DSS}\varphi_1{}^{S-S} + k_2{}^{DSS}\varphi_2{}^{S-S} + k_3{}^{DSS}\varphi_3{}^{S-S} - k_4{}^{DSS}\varphi_4{}^{S-S}. \quad (4)$$

$$= k_1{}^{DSS}[360c_1{}^2 L^{12} - 1575c_1{}^2L^{13} + 2870c_1{}^2L^{14} - 2772c_1{}^2L^{15} + \frac{16380c_1{}^2L^{16}}{11} - 420c_1{}^2L^{17} + \frac{630c_1{}^2L^{18}}{13} - \frac{72c_1c_2\sqrt{53900}\,L^{12}}{7} + 63c_1c_2\sqrt{53900}\,L^{13} - 162c_1c_2\sqrt{53900}\,L^{14} + \frac{2028c_1c_2\sqrt{53900}\,L^{15}}{10} - \frac{1836c_1c_2\sqrt{53900}\,L^{16}}{11} + 87c_1c_2\sqrt{53900}\,L^{17} - 24c_1c_2\sqrt{53900}\,L^{18} + \frac{36c_1c_2\sqrt{53900}\,L^{19}}{14} + 3960c_2{}^2 L^{12} - 31185c_2{}^2L^{13} + 105490c_2{}^2 L^{14} - 1995840c_2{}^2L^{15} + 230580c_2{}^2 L^{16} - 166320c_2{}^2L^{17} + \frac{949410c_1{}^2L^{18}}{13} - 17820c_2{}^2L^{19} + 1848c_2{}^2L^{20}]$$

$$+k_2{}^{DSS}[840c_1{}^2L^2 - 3780c_1{}^2L^3 + 6552c_1{}^2L^4 - 5040c_1{}^2L^5 + 1440c_1{}^2L^6 - 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 + 9240c_2{}^2L^2 - 83160c_2{}^2L^3 + 310464c_2{}^2L^4 - 600600c_2{}^2L^5 + 633600c_2{}^2L^6 - 346500c_2{}^2L^7 + 77000c_2{}^2L^8]$$

$$+ k_3{}^{DSS}[\frac{2520c_1{}^2}{L^2} - \frac{8820c_1{}^2}{L} + 40320c_1{}^2 - 45360c_1{}^2L + 18144c_1{}^2L^2 - \frac{72c_1c_2\sqrt{53900}}{L^2} + \frac{648c_1c_2\sqrt{53900}}{L} - 2592c_1c_2\sqrt{53900} + 4896c_1c_2\sqrt{53900}\,L - 4320c_1c_2\sqrt{53900}L^2 +$$

$1440c_1c_2 \quad \sqrt{53900}L^3 + \frac{27720{c_2}^2}{L^2} - \frac{332640{c_2}^2}{L} + 1884960{c_2}^2 - 5266800{c_2}^2L +$
$7650720{c_2}^2L^2 - 5544000{c_2}^2L^3 + 1584000{c_2}^2L^4$]
$-{k_4}^{DSS}$ $[840{c_1}^2L^2 - 3780{c_1}^2L^3 + 6552{c_1}^2L^4 - 5040{c_1}^2L^5 + 1440{c_1}^2L^6$
$- 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 +$
$90c_1c_2\sqrt{53900}L^7 + 9240{c_2}^2L^2 - 83160{c_2}^2L^3 + 310464{c_2}^2L^4 -$
$600600{c_2}^2L^5 + 633600{c_2}^2L^6 - 346500{c_2}^2L^7 + 77000{c_2}^2L^8$] (5)

Where k_1, k_2, k_3 and k_4 are all defined in Eqns.2 (a-d) respectively. The difference between DSS and DSM TPEF formulation is in the computation of their cross section properties. In order to compute the cross-section properties in k_1, k_2, k_3 and k_4, for DSM , we consider the cross- section shown in Fig.1.This is the same cross-section used by Ezeh (2009) . The aim is to allow for future comparison when the critical buckling loads will be determined using the present study formulations.

Thus, our interest in evaluating the cross sectional properties are to determine the Cross- Sectional Area for DSM, A^{DSM} and its Moment of Inertia, I^{DSM}

Cross- Sectional Area, A^{DSM} And Centroid

Applying the thin- walled assumptions, we have in Fig.2 as follows:

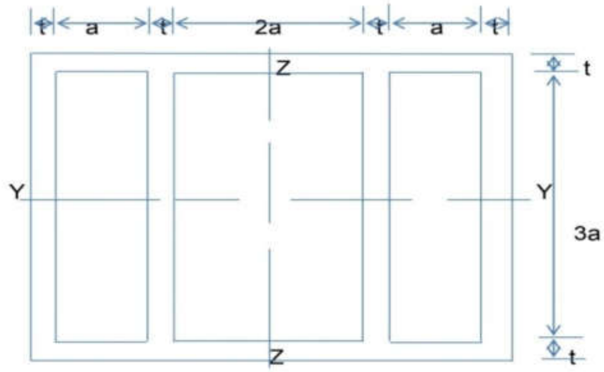

Fig.1: A DSM(Three cell) thin- walled column cross section

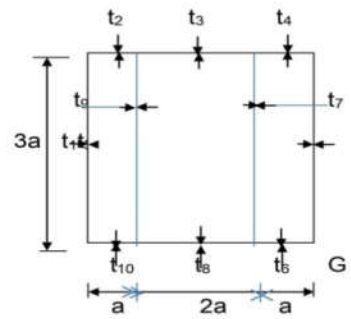

Fig.2: Thin – Walled Assumption for DSM

The evaluation of the centroid and the cross- sectional area, are validated using Table 1.

Table 1 Centroid and Cross- Sectional Area For DSM

Section	$\bar{y}_i$	$\bar{z}_i$	A_i
1	0	$\frac{-3a}{2}$	$3at_1$
2	$\frac{a}{2}$	0	at_2
3	$\frac{2a}{2}$	0	$2at_3$
4	$\frac{a}{2}$	0	at_4
5	0	$\frac{-3a}{2}$	$3at_5$
6	$\frac{a}{2}$	0	at_6
7	0	$\frac{-3a}{2}$	$3at_7$
8	$\frac{2a}{2}$	0	$2at_8$
9	0	$\frac{-3a}{2}$	$3at_9$
10	$\frac{a}{2}$	0	at_{10}
SUM	$\sum \bar{y}_i = 4a$	$\sum \bar{z}_i = -6a$	$\sum A_i = 20at$

Where $t_1 = t_2 = t_3 = t_4 = t_5 = t_6 = t_7 = t_8 = t_9 = t_{10} = t$ (6)

Thus $\sum A_i = \mathbf{A}^{DSM} = 20at$(7)

$Similarly, \sum \bar{y}_i A_i = 4a \times 20at = 80a^2t$ (8)

$\sum \bar{z}_i A_i = -6a \times 20at = -120a^2t$ (9)

Thus ,

$$\bar{y}_G = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{80a^2t}{20at} = 4a \quad (10)$$

$$\bar{z}_G = \frac{\sum \bar{z}_i A_i}{\sum A_i} = \frac{-120a^2t}{20at} = -6a \quad (11)$$

Moment of Inertia, I^{DSM}

Evaluation process of moment of inertiaI^{DSM} is shown in Table 2.

Table 2 Moment of Inertia for DSM

Section	I_{yci}	I_{Zci}
1	$\frac{t(3a)^3}{12}$	$\frac{(t)^3 3a}{12}$
2	$\frac{t(a)^3}{12}$	$\frac{(t)^3 a}{12}$
3	$\frac{t(2a)^3}{12}$	$\frac{(t)^3 2a}{12}$
4	$\frac{t(a)^3}{12}$	$\frac{(t)^3 a}{12}$
5	$\frac{t(3a)^3}{12}$	$\frac{(t)^3 3a}{12}$
6	$\frac{t(a)^3}{12}$	$\frac{(t)^3 a}{12}$
7	$\frac{t(3a)^3}{12}$	$\frac{(t)^3 3a}{12}$
8	$\frac{t(2a)^3}{12}$	$\frac{(t)^3 2a}{12}$

9	$\frac{t(3a)^3}{12}$	$\frac{(t)^3 3a}{12}$
10	$\frac{t(a)^3}{12}$	$\frac{(t)^3 a}{12}$

* Moment of Inertia, $\boldsymbol{I_{YY}}$

$I_{YY} = 4I_{YY1} + 4I_{YY2} + 2\, I_{YY3}$ (12)

Using thin- walled assumptions

$I_{YY1} = I_{Yc1} + A_1 Y_1 = I_{Yc1} + A_1[-(\frac{3a}{2} - \bar{z}_G)]^2$

$= \frac{27a^3\ t}{12} + 3\text{at}\ [-(\frac{3a}{2} - 6\text{a})]^2 = \frac{27a^3\ t}{12} + \frac{243a^3\ t}{3} = 63\ a^3\ t$

∴ $4\, I_{YY1} = 324a^3\ t$(13)

Similarly, $I_{YY2} = I_{Yc2} + A_2[0 - \bar{z}_G)]^2 = \frac{a^3\ t}{12} + \text{at}\ [6\text{a}]^2 = \frac{433a^3\ t}{12}$

∴ $4\, I_{YY2} = \frac{433a^3\ t}{3}$ (14)

Again, $I_{YY3} = \frac{8a^3\ t}{12} + 2\text{at}\ [6\text{a}]^2 = \frac{218a^3\ t}{3}$

∴ $2\, I_{YY3} = \frac{436a^3\ t}{3}$ (15)

Thus, $I_{YY} = 324a^3\ t + \frac{433a^3\ t}{3} + \frac{436a^3\ t}{3} = \frac{1841a^3\ t}{3}$ (16)

*** Moment of Inertia, $\boldsymbol{I_{ZZ}}$**

$I_{ZZ} = 4\, I_{zz1} + 4\, I_{zz2} + 2\, I_{zz3}$ (17)

Where ,

$I_{ZZ1} = I_{Zc1} + A_1[-(\bar{y}_G)]^2$

$= \frac{3at^3}{12} + 3\text{at}\ [4\text{a}]^2 = \frac{3at^3}{12} + 48\text{a}^3\ t$

∴ $4\, I_{ZZ1} = at^3 + 192a^3\ t$ (18)

$I_{ZZ2} = I_{Zc2} + A_2[\frac{a}{2} - \bar{y}_G)]^2$

$= \frac{at^3}{12} + \text{at}\ [\frac{a}{2} - 4\text{a}]^2 = \frac{at^3}{12} + \frac{49a^3\ t}{4}$

∴ $4\, I_{ZZ2} = \frac{at^3}{3} + 49a^3\ t$ (19)

Again, $I_{ZZ3} = I_{Zc3} + A_3[\frac{2a}{2} - 4\text{a})]^2$

$= \frac{2at^3}{12} + 2\text{at}\ [9\, a^2\ ] = \frac{at^3}{6} + 18\text{a}^3\ t$

∴ $2\, I_{ZZ3} = \frac{at^3}{3} + 36a^3\ t$

(20)

Thus, $I_{ZZ} = \frac{5at^3}{3} + 277a^3\ t$ (21)

We select I_{ZZ} as our working moment of inertia since it is less than I_{YY}.

Thus, $\mathbf{I}^{DSM} = I_{ZZ} = \frac{5at^3}{3} + 277a^3\ t$ (22)

Formulation of TPEF for DSM Different Boundary Condition Cases
(a).case 1: Pinned-Pinned(s-s)- DSM Thin Walled Column

From Eqns.(4) and (5), the total potential energy functional for DSM-[S-S] thin-walled box column can be evaluated. That is:

$\pi_{DSM}^{S-S} = k_1{}^{DSM}\varphi_1{}^{S-S} + k_2{}^{DSM}\varphi_2{}^{S-S} + k_3{}^{DSM}\varphi_3{}^{S-S} - k_4{}^{DSM}\varphi_4{}^{S-S}$ (23)

$= k_1{}^{DSM}\ [\ 24c_1{}^2\ L^{10} - 60c_1{}^2L^{11} + \frac{390c_1{}^2L^{12}}{7} - 10c_1{}^2L^{13} + \frac{10c_1{}^2L^{14}}{3} - \frac{8c_1c_2\sqrt{6300}\ L^{10}}{5}$

$+ \frac{20c_1c_2\sqrt{6300}\ L^{11}}{3} - \frac{74c_1c_2\sqrt{6300}\ L^{12}}{7} + 8\, c_1c_2\sqrt{6300}\ L^{13} - \frac{26c_1c_2\sqrt{6300}\ L^{14}}{9} +$

$\frac{2c_1c_2\sqrt{6300}\ L^{15}}{5} + 168c_2{}^2\ L^{10} - 980c_2{}^2L^{11} + 2310c_2{}^2\ L^{12} - \frac{5565c_2{}^2L^{13}}{2} + \frac{5390c_2{}^2L^{14}}{3} - 588c_2{}^2\ L^{15} + \frac{840c_2{}^2L^{16}}{11}\ ]$

$+ k_2{}^{DSM}[30c_1{}^2 \quad - 60c_1{}^2L \quad + 40c_1{}^2L^2 \quad - 2\, c_1c_2\sqrt{6300}$

$+8\,c_1c_2\sqrt{6300}\,L - 12c_1c_2\sqrt{6300}L^2 +$
$6\,c_1c_2\sqrt{6300}L^3 + 210{c_2}^2 - 1260{c_2}^2L + 3360{c_2}^2L^2 - 3780{c_2}^2L^3 + 1512{c_2}^2L^4\,]$

$+ {k_3}^{DSM}[\frac{120{c_1}^2}{L^2} - \frac{24\,c_1c_2\sqrt{6300}}{L^2} + \frac{24\,c_1c_2\sqrt{6300}}{L}$
$+\frac{7560{c_2}^2}{L^2} - \frac{15120{c_2}^2}{L} + 10080{c_2}^2]$

$-{k_4}^{DSM}[30{c_1}^2 - 60{c_1}^2L + 40{c_1}^2L^2 - 2\,c_1c_2\sqrt{6300}$
$+8\,c_1c_2\sqrt{6300}\,L - 12c_1c_2\sqrt{6300}L^2 + 6\,c_1c_2\sqrt{6300}L^3 + 210{c_2}^2 - 1260{c_2}^2L$
$+3360{c_2}^2L^2 - 3780{c_2}^2L^3 + 1512{c_2}^2L^4\,]$ (24)

Where

$${k_1}^{DSM} = \frac{A^{DSM}p^2}{8EI^{2(DSM)}},\ {k_2}^{DSM} = \frac{A^{DSM}G}{2},\ {k_3}^{DSM} = \frac{E\,I^{DSM}}{2} \& {k_4}^{DSM} = \frac{P}{2} \qquad 25(a\text{-}d)$$

A^{DSM} and I^{DSM} are defined in Eqns.(7) and (22) respectively.

Case 2: Fixed-Fixed[c-c]- DSM Thin Walled Column

From what Nwachukwu and others (2021) formulated for Fixed-Fixed[C-C]- DSS Thin Walled Column, the total potential energy functional(TPEF) for DSM-[C-C] Thin-Walled Box Column can be obtained as follows. That is:

$$\pi_{DSM}^{C-C} = {k_1}^{DSM}{\varphi_1}^{C-C} + {k_2}^{DSM}{\varphi_2}^{C-C} + {k_3}^{DSM}{\varphi_3}^{C-C} - {k_4}^{DSM}{\varphi_4}^{C-C} \qquad (26)$$

$= {k_1}^{DSM}[360{c_1}^2L^{12} - 1575{c_1}^2L^{13} + 2870{c_1}^2L^{14} - 2772{c_1}^2L^{15} + \frac{16380{c_1}^2L^{16}}{11}$
$- 420{c_1}^2L^{17} + \frac{630{c_1}^2L^{18}}{13} - \frac{72c_1c_2\sqrt{53900}\,L^{12}}{7} + 63c_1c_2\sqrt{53900}\,L^{13} - 162c_1c_2\sqrt{53900}\,L^{14} + \frac{2028c_1c_2\sqrt{53900}\,L^{15}}{10} -$
$\frac{1836c_1c_2\sqrt{53900}\,L^{16}}{11} + 87c_1c_2\sqrt{53900}\,L^{17} - 24c_1c_2\sqrt{53900}\,L^{18} + \frac{36c_1c_2\sqrt{53900}\,L^{19}}{14} + 3960{c_2}^2L^{12} -$
31185c22L13 + 105490c22L14−1995840c22L15 + 230580c22L16− 166320c22L17+ 949410c12L1813–17820c22L19+ 1848c22L20]

$+{k_2}^{DSM}[840{c_1}^2L^2 - 3780{c_1}^2L^3 + 6552{c_1}^2L^4 - 5040{c_1}^2L^5 + 1440{c_1}^2L^6$
$- 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 +$
$90c_1c_2\sqrt{53900}L^7 + 9240{c_2}^2L^2 - 83160{c_2}^2L^3 + 310464{c_2}^2L^4 -$
$600600{c_2}^2L^5 + 633600{c_2}^2L^6 - 346500{c_2}^2L^7 + 77000{c_2}^2L^8]$
$+ {k_3}^{DSM}[\frac{2520{c_1}^2}{L^2} - \frac{8820{c_1}^2}{L} + 40320{c_1}^2 - 45360{c_1}^2L + 18144{c_1}^2L^2 - \frac{72c_1c_2\sqrt{53900}}{L^2} + \frac{648c_1c_2\sqrt{53900}}{L} -$
$2592c_1c_2\sqrt{53900} + 4896c_1c_2\sqrt{53900}\,L - 4320c_1c_2\sqrt{53900}L^2 + 1440c_1c_2\sqrt{53900}L^3 +$
$\frac{27720{c_2}^2}{L^2} - \frac{332640{c_2}^2}{L} + 1884960{c_2}^2 - 5266800{c_2}^2L +$
$7650720{c_2}^2L^2 - 5544000{c_2}^2L^3 + 1584000{c_2}^2L^4\,]$
$-{k_4}^{DSM}[840{c_1}^2L^2 - 3780{c_1}^2L^3 + 6552{c_1}^2L^4 - 5040{c_1}^2L^5 + 1440{c_1}^2L^6$
$- 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 +$
$9240{c_2}^2L^2 - 83160{c_2}^2L^3 + 310464{c_2}^2L^4 -$
$600600{c_2}^2L^5 + 633600{c_2}^2L^6 - 346500{c_2}^2L^7 + 77000{c_2}^2L^8\,]$(27)

Where ${k_1}^{DSM}, {k_2}^{DSM}, {k_3}^{DSM}$ and ${k_4}^{DSM}$ are defined in Eqns.25(a-d) respectively.

Case 3: Fixed-Pinned[C-S]- DSM Thin- Walled Column

Again, from the work of Nwachukwu and others (2021) for Fixed-Pinned[C-S]- DSS Thin Walled Column, the total potential energy functional(TPEF) for DSM-[C-S] Thin-Walled Box Column can be obtained as follows:

$$\pi_{DSM}^{C-S} = {k_1}^{DSM}{\varphi_1}^{C-S} + {k_2}^{DSM}{\varphi_2}^{C-S} + {k_3}^{DSM}{\varphi_3}^{C-S} - {k_4}^{DSM}{\varphi_4}^{C-S} \qquad (28)$$

$= {k_1}^{DSM}[\frac{22680{c_1}^2L^{12}}{133} - \frac{98280{c_1}^2L^{13}}{152} + \frac{174510{c_1}^2L^{14}}{171} - \frac{162540{c_1}^2L^{15}}{190} + \frac{83790{c_1}^2L^{16}}{209} - \frac{22680{c_1}^2L^{17}}{228} + \frac{2520{c_1}^2L^{18}}{247} -$
$\frac{216}{7}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{12} + \frac{10728}{8}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{13} - \frac{39078}{9}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{14} + \frac{58500}{10}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{15} - \frac{44640}{11}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{16} +$
$\frac{18000}{12}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{17} - \frac{3546}{13}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{18} + \frac{252}{14}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{19} + \frac{27720{c_2}^2L^{12}}{1729}$
$- \frac{2633400{c_2}^2L^{13}}{1976} + \frac{66784410{c_2}^2L^{14}}{2223} - \frac{203312340{c_2}^2L^{15}}{2470} + \frac{250637310{c_2}^2L^{16}}{2717} - \frac{151295760{c_2}^2L^{17}}{2964} + \frac{45952830{c_2}^2L^{18}}{3211} - \frac{6500340{c_2}^2L^{19}}{3458} + \frac{339570{c_2}^2L^{20}}{3705}$
]

$$+k_2^{DSM}[\frac{22680c_1^2L^2}{57}-\frac{113400c_1^2L^3}{76)}+\frac{202230c_1^2L^4}{95}-\frac{151200c_1^2L^5}{114}+\frac{40320c_1^2L^6}{133}-$$

$$\frac{216}{3}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^2+\frac{15768}{4}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^3-\frac{61254}{5}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^4+\frac{81324}{6}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^5-\frac{39978}{7}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^6+$$

$$\frac{5040}{8}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^7$$

$$+\frac{27720c_2^2L^2}{741}-\frac{3908520c_2^2L^3}{988}+\frac{143497970c_2^2L^4}{1235}-\frac{415273320c_2^2L^5}{1482}$$

$$+\frac{379861020c_2^2L^6}{1729}-\frac{102841200c_2^2L^7}{1976}+\frac{8489250c_2^2L^8}{2223}]$$

$$+k_3^{DSM}[\frac{22680c_1^2}{19L^2}-\frac{226800c_1^2}{38L}+\frac{748440c_1^2}{57}-\frac{907200c_1^2L}{76}+\frac{362880c_1^2L^2}{95}-\frac{216}{L^2}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}+\frac{31536}{2L}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}-$$

$$\frac{221832}{3}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}+\frac{480384}{4}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L-\frac{350352}{5}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^2$$

$$+\frac{60480}{6}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^3+\frac{24640c_2^2}{247L^2}-\frac{7817040c_2^2}{494L}+\frac{568731240c_2^2}{741}-\frac{2489699520c_2^2L}{988}$$

$$+\frac{3350350080c_2^2L^2}{1235}-\frac{123094400c_2^2L^3}{1482}+\frac{135828000c_2^2L^4}{1729}]$$

$$-k_4^{DSM}[\frac{22680c_1^2L^2}{57}-\frac{113400c_1^2L^3}{76)}+\frac{202230c_1^2L^4}{95}-\frac{151200c_1^2L^5}{114}+\frac{40320c_1^2L^6}{133}-\frac{216}{3}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^2+\frac{15768}{4}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^3-$$

$$\frac{61254}{5}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^4+\frac{81324}{6}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^5-\frac{39978}{7}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^6+\frac{5040}{8}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^7$$

$$+\frac{27720c_2^2L^2}{741}-\frac{3908520c_2^2L^3}{988}+\frac{143497970c_2^2L^4}{1235}-\frac{415273320c_2^2L^5}{1482}$$

$$+\frac{379861020c_2^2L^6}{1729}-\frac{102841200c_2^2L^7}{1976}+\frac{8489250c_2^2L^8}{2223}] \quad (29)$$

Where $k_1^{DSM}, k_2^{DSM}, k_3^{DSM}$ and k_4^{DSM} are defined in Eqns.25 (a-d) respectively.

Evaluation and formulation of TPEF for MSM TWBC cross-section

Evaluation of MSM Cross- sectional properties

An MSM thin- walled box column cross section is shown in Fig.3.

Cross- Sectional Area,A^{MSM} and Centroid.

Applying the same thin- walled assumptions used in the previous section, we have in Fig.4 as follows:

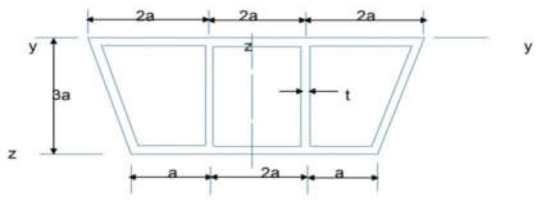

Fig.3 : An MSM thin-walled box column cross section

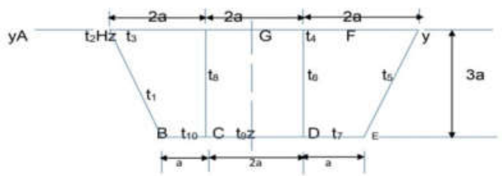

Fig.4: Thin – Walled Assumption for MSM

From Fig.4,

AB = EF = $\sqrt{a^2 + (3a)^2}$ = $\sqrt{10a^2}$ = $a\sqrt{10}$(30)

The evaluation of the centroid and the cross- sectional area, are validated using Table 3.

Table 3 Centroid and Cross- Sectional Area For MSM

Section	$\bar{y}_i$	$\bar{z}_i$	A_i
1	0	$-\frac{a\sqrt{10}}{2}$	$at\sqrt{5}$
2	a	0	2at
3	a	0	2at
4	a	0	2at
5	0	$-\frac{a\sqrt{10}}{2}$	$at\sqrt{5}$
6	0	$-\frac{3a}{2}$	3at
7	$\frac{a}{2}$	0	at
8	0	$-\frac{3a}{2}$	3at
9	a	0	2at
10	$\frac{a}{2}$	0	at
SUM	$\sum \bar{y}_i = 5a$	$\sum \bar{z}_i = -3a - a\sqrt{10}$	$\sum A_i = 15at + 2at\sqrt{10}$

Note t1 = t2 = t3 = t4 = t5 = t6 = t7 = t8 =t9 = t10 = t (31)

Thus $\boldsymbol{A}^{MSM} = \sum A_i = 15at + 2at\sqrt{10}$= 21.325at (32)

$\bar{y}_G$= 5a (33)

And $\bar{z}_G = -3a - a\sqrt{10}$ (34)

Moment of Inertia, I^{MSM}

The moment of inertia will be about Z-Z, and the following table, Table 4 is set up as follows:

Table 4 Moment of Inertia for MSM

Section	I_{Zci}
1	$\frac{(t)^3 a\sqrt{10}}{12}$
2	$\frac{(t)^3 2a}{12}$
3	$\frac{(t)^3 2a}{12}$
4	$\frac{(t)^3 2a}{12}$
5	$\frac{(t)^3 a\sqrt{10}}{12}$
6	$\frac{(t)^3 3a}{12}$
7	$\frac{(t)^3 a}{12}$
8	$\frac{(t)^3 3a}{12}$
9	$\frac{(t)^3 2a}{12}$
10	$\frac{(t)^3 a}{12}$

.

Thus, $I_{ZZ} = 2\, I_{zz1} + 4I_{zz2} + 2I_{zz6} + 2I_{zz7}$

Where

$$I_{ZZ1} = I_{Zc1} + A_1[-(\bar{y}_G)]^2$$

$= \frac{at^3\sqrt{10}}{12} + at\sqrt{10}[5a]^2$

$\therefore \quad 2I_{ZZ1} = 0.527at^3 + 158.12a^3 t$ (35)

Similarly,

$I_{ZZ2} = I_{Zc2} + A_2[\bar{y}_2 - \bar{y}_G]^2 = \frac{2at^3}{12} + 2at\ [4a]^2$

$\therefore\ 4I_{ZZ2} = 0.67at^3 + 128a^3 t$ (36)

Again

$I_{ZZ6} = \frac{3at^3}{12} + 3at\ [5a]^2$

$\therefore\ 2I_{ZZ6} = 0.5at^3 + 150a^3 t$ (37)

And

$I_{ZZ7} = I_{Zc7} + A_7[\bar{y}_7 - \bar{y}_G]^2 = \frac{at^3}{12} + at\ [a - 5a]^2 = \frac{at^3}{12} + 16a^3 t$

$\therefore\ 2I_{ZZ7} = 0.167at^3 + 32a^3 t$ (38)

Thus, $\mathbf{I}^{MSM} = I_{ZZ} = 1.864at^3 + 468.12a^3 t$ (39)

Formulation of TPEF for MSM different boundary condition cases

Case 1: pinned-pinned(s-s)- MSM thin walled column

The total potential energy functional (TPEF) for MSM-[S-S] thin-walled box column can be evaluated as follow:

$\pi_{MSM}^{S-S} = k_1{}^{MSM}\varphi_1{}^{S-S} + k_2{}^{MSM}\varphi_2{}^{S-S} + k_3{}^{MSM}\varphi_3{}^{S-S} - k_4{}^{MSM}\varphi_4{}^{S-S}$ (40)

$= k_1{}^{MSM}\ [24c_1{}^2\ L^{10} - 60c_1{}^2L^{11} + \frac{390c_1{}^2L^{12}}{7} - 10c_1{}^2L^{13} + \frac{10c_1{}^2L^{14}}{3} - \frac{8c_1c_2\sqrt{6300}\ L^{10}}{5} + \frac{20c_1c_2\sqrt{6300}\ L^{11}}{3} - \frac{74c_1c_2\sqrt{6300}\ L^{12}}{7} + 8c_1c_2\sqrt{6300}\ L^{13} - \frac{26c_1c_2\sqrt{6300}\ L^{14}}{9} + \frac{2c_1c_2\sqrt{6300}\ L^{15}}{5} + 168c_2{}^2\ L^{10} - 980c_2{}^2L^{11} + 2310c_2{}^2\ L^{12} - \frac{5565c_2{}^2L^{13}}{2} + \frac{5390c_2{}^2L^{14}}{3} - 588c_2{}^2\ L^{15} + \frac{840c_2{}^2L^{16}}{11}\ ]$

$+k_2{}^{MSM}[30c_1{}^2 - 60c_1{}^2L + 40c_1{}^2L^2 - 2c_1c_2\sqrt{6300} + 8c_1c_2\sqrt{6300}\ L - 12c_1c_2\sqrt{6300}L^2 + 6c_1c_2\sqrt{6300}L^3 + 210c_2{}^2 - 1260c_2{}^2L + 3360c_2{}^2L^2 - 3780c_2{}^2L^3 + 1512c_2{}^2L^4\ ]$

$+ k_3{}^{MSM}\ [\frac{120c_1{}^2}{L^2} - \frac{24c_1c_2\sqrt{6300}}{L^2} + \frac{24c_1c_2\sqrt{6300}}{L} + \frac{7560c_2{}^2}{L^2} - \frac{15120c_2{}^2}{L} + 10080c_2{}^2]$

$-k_4{}^{MSM}[30c_1{}^2 - 60c_1{}^2L + 40c_1{}^2L^2 - 2c_1c_2\sqrt{6300} + 8c_1c_2\sqrt{6300}\ L - 12c_1c_2\sqrt{6300}L^2 + 6c_1c_2\sqrt{6300}L^3 + 210c_2{}^2 - 1260c_2{}^2L + 3360c_2{}^2L^2 - 3780c_2{}^2L^3 + 1512c_2{}^2L^4\ ]$ (41)

Where

$k_1{}^{MSM} = \frac{A^{MSM}p^2}{8EI^{2(MSM)}}, k_2{}^{MSM} = \frac{A^{MSM}G}{2}, k_3{}^{MSM} = \frac{E\,I^{MSM}}{2} \& k_4{}^{MSM} = \frac{P}{2}$ 42(a-d)

A^{MSM} and I^{MSM} are defined in Eqns.(32) and (39) respectively.

Case 2: Fixed-Fixed[C-C]- MSM Thin Walled Column

The total potential energy functional for MSM-[C-C] thin-walled box column can be obtained as follows:

$\pi_{MSM}^{C-C} = k_1{}^{MSM}\varphi_1{}^{C-C} + k_2{}^{MSM}\varphi_2{}^{C-C} + k_3{}^{MSM}\varphi_3{}^{C-C} - k_4{}^{MSM}\varphi_4{}^{C-C}$ (43)

$= k_1{}^{MSM}[360c_1{}^2\ L^{12} - 1575c_1{}^2L^{13} + 2870c_1{}^2L^{14} - 2772c_1{}^2L^{15} + \frac{16380c_1{}^2L^{16}}{11} - 420c_1{}^2L^{17} + \frac{630c_1{}^2L^{18}}{13} - \frac{72c_1c_2\sqrt{53900}\ L^{12}}{7} + 63c_1c_2\sqrt{53900}\ L^{13} - 162c_1c_2\sqrt{53900}\ L^{14} + \frac{2028c_1c_2\sqrt{53900}\ L^{15}}{10} - \frac{1836c_1c_2\sqrt{53900}\ L^{16}}{11} + 87c_1c_2\sqrt{53900}\ L^{17} - 24c_1c_2\sqrt{53900}\ L^{18} + \frac{36c_1c_2\sqrt{53900}\ L^{19}}{14} + 3960c_2{}^2\ L^{12} - 31185c_2{}^2L^{13} + 105490c_2{}^2\ L^{14} - 1995840c_2{}^2L^{15} + 230580c_2{}^2\ L^{16} - 166320c_2{}^2L^{17} + \frac{949410c_1{}^2L^{18}}{13} - 17820c_2{}^2L^{19} + 1848c_2{}^2L^{20}$

$+k_2{}^{MSM}[840c_1{}^2L^2 - 3780c_1{}^2L^3 + 6552c_1{}^2L^4 - 5040c_1{}^2L^5 + 1440c_1{}^2L^6 - 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 + 9240c_2{}^2L^2 - 83160c_2{}^2L^3 + 310464c_2{}^2L^4 -$

$600600{c_2}^2L^5 + 633600{c_2}^2L^6$- $346500{c_2}^2L^7 + 77000{c_2}^2L^8$]

$+ {k_3}^{MSM}$ [$\frac{2520{c_1}^2}{L^2}$-$\frac{8820{c_1}^2}{L}$ $+ 40320{c_1}^2$- $45360{c_1}^2L +\ 18144{c_1}^2L^2$- $\frac{72c_1c_2\ \ \sqrt{53900}}{L^2}$ $+ \frac{648c_1c_2\ \ \sqrt{53900}}{L}$ -

$2592c_1c_2\ \ \sqrt{53900} + 4896c_1c_2\ \ \sqrt{53900}$ L -$4320c_1c_2\ \ \sqrt{53900}L^2 +\ \ 1440c_1c_2\ \ \sqrt{53900}L^3$ +

$\frac{27720{c_2}^2}{L^2}$-$\frac{332640{c_2}^2}{L}$ $+ 1884960{c_2}^2$–$5266800{c_2}^2L + 7650720{c_2}^2L^2$ - $5544000{c_2}^2L^3 + 1584000{c_2}^2L^4$]

$-{k_4}^{MSM}$ [$840{c_1}^2L^2$– $3780{c_1}^2L^3 + 6552{c_1}^2L^4$– $5040{c_1}^2L^5 + 1440{c_1}^2L^6$

- $24c_1c_2\sqrt{53900}L^2$ $+ 171c_1c_2\sqrt{53900}L^3$ - $432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5$ -$360c_1c_2\sqrt{53900}L^6$ +

$90c_1c_2\sqrt{53900}L^7 + 9240{c_2}^2L^2$– $83160{c_2}^2L^3 + 310464{c_2}^2L^4$ –

$600600{c_2}^2L^5 + 633600{c_2}^2L^6$- $346500{c_2}^2L^7 + 77000{c_2}^2L^8$] (44)

Where ${k_1}^{MSM}, {k_2}^{MSM}, {k_3}^{MSM}$ and ${k_4}^{MSM}$ are defined in Eqns. 42(a-d) respectively.

Case 3: Fixed-Pinned[C-S]- MSM Thin- Walled Column

The total potential energy functional for MSM-[C-S] thin-walled box column can be obtained as follows:

$\pi_{MSM}^{C-S} = {k_1}^{MSM}{\varphi_1}^{C-S} + {k_2}^{MSM}{\varphi_2}^{C-S} + {k_3}^{MSM}{\varphi_3}^{C-S} - {k_4}^{MSM}{\varphi_4}^{C-S}$ (45)

$= {k_1}^{MSM}$[$\frac{22680{c_1}^2L^{12}}{133}$-$\frac{98280{c_1}^2L^{13}}{152}$ $+ \frac{174510{c_1}^2L^{14}}{171}$ - $\frac{162540{c_1}^2L^{15}}{190}$ $+ \frac{83790{c_1}^2L^{16}}{209}$ $-$ $\frac{22680{c_1}^2L^{17}}{228}$ $+ \frac{2520{c_1}^2L^{18}}{247}$ -

$\frac{216}{7}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^{12} $+ \frac{10728}{8}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^{13}- $\frac{39078}{9}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^{14} $+ \frac{58500}{10}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^{15}- $\frac{44640}{11}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^{16} +

$\frac{18000}{12}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^{17}–$\frac{3546}{13}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^{18} $+ \frac{252}{14}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^{19}+$\frac{27720{c_2}^2L^{12}}{1729}$

- $\frac{2633400{c_2}^2L^{13}}{1976}$ $+ \frac{66784410{c_2}^2L^{14}}{2223}$-$\frac{203312340{c_2}^2L^{15}}{2470}$ $+ \frac{250637310{c_2}^2L^{16}}{2717}$ $-$ $\frac{151295760{c_2}^2L^{17}}{2964}$ $+ \frac{45952830{c_2}^2L^{18}}{3211}$-$\frac{6500340{c_2}^2L^{19}}{3458}$ $+ \frac{339570{c_2}^2L^{20}}{3705}$

]

$+{k_2}^{MSM}$[$\frac{22680{c_1}^2L^2}{57}$-$\frac{113400{c_1}^2L^3}{76)}$ $+ \frac{202230{c_1}^2L^4}{95}$-$\frac{151200{c_1}^2L^5}{114}$ $+ \frac{40320{c_1}^2L^6}{133}$ - $\frac{216}{3}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^2 $+ \frac{15768}{4}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^3-

$\frac{61254}{5}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ $L^4 + \frac{81324}{6}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^5- $\frac{39978}{7}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ $L^6 + \frac{5040}{8}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^7

$+\frac{27720{c_2}^2L^2}{741}$-$\frac{3908520{c_2}^2L^3}{988}$ $+ \frac{143497970{c_2}^2L^4}{1235}$ $-$ $\frac{415273320{c_2}^2L^5}{1482}$

$+ \frac{379861020{c_2}^2L^6}{1729}$-$\frac{102841200{c_2}^2L^7}{1976}$ $+ \frac{8489250{c_2}^2L^8}{2223}$]

$+ {k_3}^{MSM}$ [$\frac{22680{c_1}^2}{19L^2}$-$\frac{226800{c_1}^2}{38L}$ $+ \frac{748440{c_1}^2}{57}$-$\frac{907200{c_1}^2L}{76}$ $+ \frac{362880{c_1}^2L^2}{95}$ $-$ $\frac{216}{L^2}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ $+ \frac{31536}{2L}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ -

$\frac{221832}{3}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ $+ \frac{480384}{4}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L- $\frac{350352}{5}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ $L^2 + \frac{60480}{6}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^3

$+\frac{24640{c_2}^2}{247L^2}$-$\frac{7817040{c_2}^2}{494L}$ $+ \frac{568731240{c_2}^2}{741}$-$\frac{2489699520{c_2}^2L}{988}$

$+ \frac{3350350080{c_2}^2L^2}{1235}$ - $\frac{123094400{c_2}^2L^3}{1482}$ $+ \frac{135828000{c_2}^2L^4}{1729}$]

$-{k_4}^{MSM}$ [$\frac{22680{c_1}^2L^2}{57}$-$\frac{113400{c_1}^2L^3}{76)}$ $+ \frac{202230{c_1}^2L^4}{95}$-$\frac{151200{c_1}^2L^5}{114}$ $+ \frac{40320{c_1}^2L^6}{133}$ - $\frac{216}{3}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^2 $+ \frac{15768}{4}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^3-

$\frac{61254}{5}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ $L^4 + \frac{81324}{6}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^5- $\frac{39978}{7}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ $L^6 + \frac{5040}{8}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}$ L^7

$+\frac{27720{c_2}^2L^2}{741}$-$\frac{3908520{c_2}^2L^3}{988}$ $+ \frac{143497970{c_2}^2L^4}{1235}$ $-$ $\frac{415273320{c_2}^2L^5}{1482}$

$+ \frac{379861020{c_2}^2L^6}{1729}$-$\frac{102841200{c_2}^2L^7}{1976}$ $+ \frac{8489250{c_2}^2L^8}{2223}$] (46)

Where ${k_1}^{MSM}, {k_2}^{MSM}, {k_3}^{MSM}$ and ${k_4}^{MSM}$ are defined in Eqns.42 (a-d) respectively.

CONCLUSIONS

The study was able to evaluate cross sectional properties of DSM and MSM cross sections, which are moment of Inertia and cross sectional area, given in Eqns.(7), (22), (32) &(39). The study also formulated peculiar TPEF for DSM and MSM cross-sections. The formulated Raleigh- Ritz based DSM and MSM TPEF given in Eqns.(24), (27), (29), (41),(44), & (46) are found handy and convenient to be used in the bulking/stability analysis of DSM and MSM cross- sections. The developed expressions will now be used to formulate series of stability matrices in subsequent publications where the critical bulk ling load will be evaluated.

Reference

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